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Cambridge Lower Secondary Mathematics

LEARNER'S BOOK 7

Lynn Byrd, Greg Byrd & Chris Pearce

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Cambridge Lower Secondary

Mathematics

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Introduction

Welcome to Cambridge Lower Secondary Mathematics Stage 7

The *Cambridge Lower Secondary Mathematics* course covers the Cambridge Lower Secondary Mathematics curriculum framework and is divided into three stages: 7, 8 and 9.

During your course, you will learn a lot of facts, information and techniques. You will start to think like a mathematician.

This book covers all you need to know for Stage 7.

The curriculum is presented in four content areas:

- Number
- Algebra
- Geometry and measures
- Statistics and probability.

This book has 16 units, each related to one of the four content areas. However, there are no clear dividing lines between these areas of mathematics; skills learned in one unit are often used in other units. The book encourages you to understand the concepts that you need to learn, and gives opportunity for you to practise the necessary skills.

Many of the questions and activities are marked with an icon that indicates that they are designed to develop certain *thinking and working mathematically* skills.

There are eight characteristics that you will develop and apply throughout the course:

- Specialising – testing ideas against specific criteria;
- Generalising – recognising wider patterns;
- Conjecturing – forming questions or ideas about mathematics;
- Convincing – presenting evidence to justify or challenge a mathematical idea;
- Characterising – identifying and describing properties of mathematical objects;
- Classifying – organising mathematical objects into groups;
- Critiquing – comparing and evaluating ideas for solutions;
- Improving – Refining your mathematical ideas to reach more effective approaches or solutions.

Your teacher can help you develop these skills, and you will also develop your ability to apply these different strategies.

We hope you will find your learning interesting and enjoyable.

Greg Byrd, Lynn Byrd and Chris Pearce

Contents

How to use this book

In this book you will find lots of different features to help your learning.

Questions to find out what you know already.

Getting started

1 Put these numbers in order, from smallest to largest: 9, −7, 6, −5, 3, 0.
2 Find the multiples of 9 that are less than 50.
3 Find the factors of 15.
4 Work out $13^2 - 12^2$. Write your answer as a square number.

What you will learn in the unit.

In this section you will …

- use letters to represent numbers
- use the correct order of operations in algebraic expressions
- write and use expressions.

Important words to learn.

Key words

integers
inverse
inverse operation
number line
negative integers
positive integers

Step-by-step examples showing how to solve a problem.

Worked example 2.3

Simplify each expression.

a $2x + 3x$ b $7y - 2y$ c $4p + 3q + 2p - q$ d $5t + 7 - 3t + 3$

Answer

a $2x + 3x = 5x$	$2x$ and $3x$ are like terms, so add them to get $5x$.
b $7y - 2y = 5y$	$7y$ and $2y$ are like terms, so subtract to get $5y$.
c $4p + 3q + 2p - q = 6p + 2q$	$4p + 2p = 6p$ and $3q - q = 2q$, but $6p$ and $2q$ are not like terms so you cannot simplify any further.
d $5t + 7 - 3t + 3 = 2t + 10$	$5t - 3t = 2t$ and $7 + 3 = 10$, but $2t$ and 10 are not like terms so you cannot simplify any further.

These questions will help you develop your skills of thinking and working mathematically.

6 This is part of Bethan's homework. Bethan has made a mistake in every answer. Explain what Bethan has done wrong. Work out the correct answers.

Question

Multiply out the brackets.

a $4(x + 4)$ *b* $2(6x - 3)$

These investigations, to be carried out with a partner or in a group, will help develop skills of thinking and working mathematically.

Think like a mathematician

11 **a** Copy and complete this addition table.
b Add the four answers inside the addition table.
c Add the four integers on the side and the top of the addition table.
d What do you notice about the answers to parts **b** and **c**? Is this true for any addition table? Give evidence to justify your answer.

+	−5	7
4		
−3		

Questions to help you think about how you learn.

Look again at Question **6**. What method did you use to answer this question? Do you think this was the best method or would you use a different method if you had to answer the question again?

This is what you have learned in the unit.

Summary checklist

- I can add positive and negative integers.
- I can subtract positive and negative integers.

Questions that cover what you have learned in the unit. If you can answer these, you are ready to move on to the next unit.

Check your progress

1 Work out:
a $3 - 7$ **b** $-3 + -7$
c $-2 \times (2 - -4)$ **d** $(-9 + -6) \div 3$

2 **a** Find two integers that add up to 2 and multiply to make −15.
b Find two integers that add up to 3 and multiply to make −70.

3 Find the missing numbers.
a $5 \times \square = -8 - 7$ **b** $-12 \div \square = 4 + -6$

4 Find all the common factors of 16 and 24.

5 **a** Find all the multiples of 6 between 50 and 70.
b Find the lowest common multiple of 6 and 15.

At the end of several units, there is a project for you to carry out, using what you have learned. You might make something or solve a problem.

Mixed-up properties

Here are nine property cards:

Their difference is a factor of their sum	Their highest common factor (HCF) is 1	Their product has exactly 4 factors
Their difference is prime	Their sum is a square number	Their lowest common multiple (LCM) is 12
They are both factors of 30	They are both prime	Their product is a cube number

Here are six number cards:

> Acknowledgements

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1 Integers

Getting started

1 Put these numbers in **order**, from smallest to largest: 9, −7, 6, −5, 3, 0.
2 Find the multiples of 9 that are less than 50.
3 Find the **factors** of 15.
4 Work out $13^2 - 12^2$. Write your answer as a **square number**.

When you count objects, you use the **positive whole numbers** 1, 2, 3, 4, …

Whole numbers are the first numbers that humans invented.

You can use these numbers for more than counting.

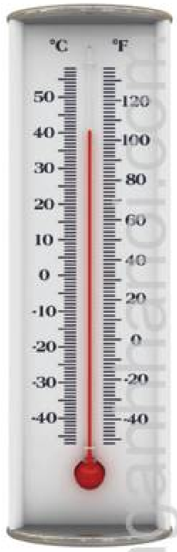
For example, to measure temperature it is useful to have the number 0 (zero) and **negative whole numbers** −1, −2, −3, …

You can put these numbers on a **number line**.

1, 2, 3, 4, … are sometimes called positive numbers to distinguish them from the negative numbers −1, −2, −3, −4, …

Positive and negative whole numbers together with zero are called **integers**.

In this unit you will learn about integers and their properties.



1.1 Adding and subtracting integers

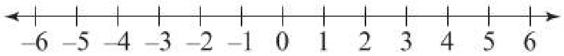
In this section you will …

- add and subtract with positive and negative integers.

Key words

integers
inverse
inverse operation
number line
negative integers
positive integers

Integers are positive and negative whole numbers, together with zero.
You can show integers on a number line.



Integers greater than zero are **positive integers**: 1, 2, 3, 4, …
Integers less than zero are **negative integers**: −1, −2, −3, −4, …
You can use a number line to help you to add integers.

Tip

The '…' (called an ellipsis) shows that the lists continue forever.

Worked example 1.1

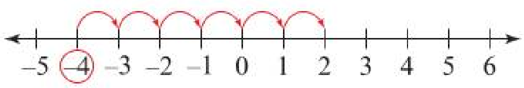
Work out:

a $-4 + 6$ **b** $8 + -3$ **c** $-3 + -5$

Answer

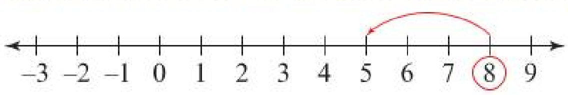
a You can use a number line to help you.

Start at −4. Move 6 to the right.



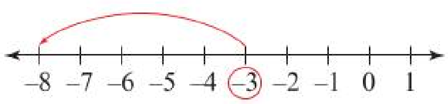
$-4 + 6 = 2$

b Start at 8. Move 3 to the left. You move to the left because it is −3.



$8 + -3 = 5$

c Start at −3. Move 5 to the left.



$-3 + -5 = -8$

Subtraction is the **inverse operation** of addition.

The **inverse** of 3 is -3. The inverse of -5 is 5.

To subtract an integer, you add the inverse.

You can draw a number line to help you.

Worked example 1.2

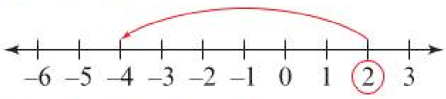
Work out:

a $2 - 6$ **b** $-4 - -3$ **c** $2 - -4$

Answer

a $2 - 6 = 2 + -6$

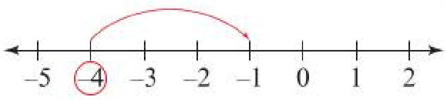
Add the inverse of 6. The inverse of 6 is -6.



$2 - 6 = 2 + -6 = -4$

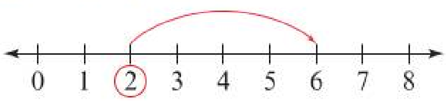
b $-4 - -3 = -4 + 3$

Add the inverse of -3. The inverse of -3 is 3.



$-4 - -3 = -4 + 3 = -1$

c $2 - -4 = 2 + 4 = 6$



Worked example 1.3

Estimate the answers to calculations by **rounding** the numbers.

a $-48 + -73$ **b** $123 - 393$ **c** $6.15 - -4.87$

Answer

a $-48 + -73$ is approximately $-50 + -70 = -120$

This is rounding to the nearest 10.

b $123 - 393$ is approximately $100 - 400 = -300$

This is rounding to the nearest hundred.

c $6.15 - -4.87$ is approximately $6 - -5 = 6 + 5 = 11$

This is rounding to the nearest whole number.

Exercise 1.1

1 Do these additions.

a $-3+4$ **b** $3+-7$ **c** $-4+-4$ **d** $9+-5$

2 Do these subtractions.

a $-1-5$ **b** $3--5$ **c** $-3-7$ **d** $-4--6$

3 Work out:

a $4+-6$ **b** $4--6$ **c** $-4+6$ **d** $-4-6$

4 Work out the missing integers.

a $6+\square=10$ **b** $6+\square=4$ **c** $6+\square=-4$ **d** $6+\square=0$

5 Two integers add up to −4. One of the integers is 5. Work out the other integer.

Tip

A 'pair of integers' means 'two integers'.

6 −1 and 7 is a pair of integers that add up to 6.

a Find four pairs of integers that add up to 1.

b How can you see immediately that two integers add up to 1?

7 ● and ▲ are two integers.

a Show that ● + ▲ and ▲ + ● have the same value.

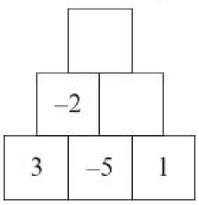
b Do ● − ▲ and ▲ − ● have the same value? Give evidence to **justify** your answer.

8 Copy and complete this addition table.

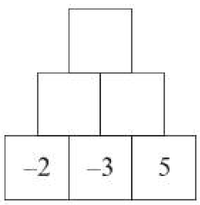
+	−4	6	−2
3		9	
−5			

9 Copy and complete these addition pyramids. The first one has been started for you.

a



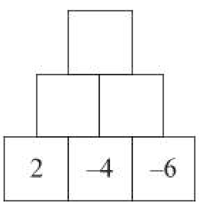
b



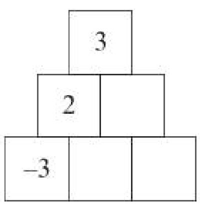
Tip

In part **a**, $3+-5=-2$.

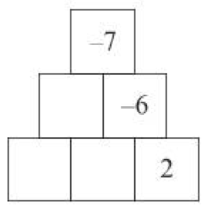
c



d



e



How are parts **d** and **e** different from parts **a**, **b** and **c**?

10 Estimate the answers to these questions. Round the numbers to the nearest whole number.

a $-3.14 + 8.26$ b $-5.93 - 6.37$

c $3.2 - -6.73$ d $-13.29 + -5.6$

11 Estimate the answers to these questions.

a $-67 + 29$ b $-82 - 47$

c $688 - -512$ d $-243 + -514$

12 a Work out:

i $-3 + 4 + -5$ ii $-5 + 4 + -3$

iii $-3 + -5 + 4$ iv $-3 + 4 + -5$

b What do the answers to part **a** show? Is this true for any three integers?

Tip

For part **i**, first add −3 and 4. Then add −5 to the answer.

Think like a mathematician

13 a Copy and complete this addition table.

b Add the four answers inside the addition table.

c Add the four integers on the side and the top of the addition table.

d What do you notice about the answers to parts **b** and **c**? Is this true for any addition table? Give evidence to justify your answer.

+	−5	7
4		
−3		

How did you do the investigation in part **d**? Could you improve your method?

14 Three integers are equally spaced on a number line. Two of the integers are −3 and 7.

a What is the other integer? Is there more than one possible answer?

b **Compare** your answer with a partner's. Critique each other's method.

Summary checklist

- [ ] I can add positive and negative integers.
- [ ] I can subtract positive and negative integers.

1.2 Multiplying and dividing integers

In this section you will …

- multiply and divide with positive and negative integers.

Key word

product

$3 \times 4 = 3 + 3 + 3 + 3 = 12$

In a similar way, $-3 \times 4 = -3 + -3 + -3 + -3 = -12$.

$5 \times 2 = 2 + 2 + 2 + 2 + 2 = 10$

In a similar way, $5 \times -2 = -2 + -2 + -2 + -2 + -2 = -10$.

Tip

You say that '12 is the product of 3 and 4' and that '−12 is the product of −3 and 4'.

Worked example 1.4

Work out:

a 6×-4 **b** -9×3

Answer

a $6 \times 4 = 24$

So $6 \times -4 = -24$.

b $9 \times 3 = 27$

So $-9 \times 3 = -27$.

Division is the inverse operation of multiplication.

$3 \times 4 = 12$ So $12 \div 4 = 3$.

This is also true when you **divide** a negative integer by a positive integer.

$-3 \times 4 = -12$ So $-12 \div 4 = -3$.

Worked example 1.5

Work out:

a $-20 \div 5$ **b** $-20 \div 10$ **c** $5 \times (1 + -4)$

Answer

a $20 \div 5 = 4$, so $-20 \div 5 = -4$.

b $20 \div 10 = 2$, so $-20 \div 10 = -2$.

c $1 + -4 = -3$

$5 \times (1 + -4) = 5 \times -3 = -15$.

First, do the addition in the **brackets**.
Then multiply the answer by 5.

Exercise 1.2

1 Work out:

a 3×-2 **b** 5×-7 **c** 10×-4 **d** 6×-6

2 Work out:

a $-15 \div 3$ **b** $-30 \div 6$ **c** $-24 \div 4$ **d** $-27 \div 9$

3 Work out the missing numbers.

a $9 \times \square = -18$ **b** $5 \times \square = -30$

c $-2 \times \square = -14$ **d** $-8 \times \square = -40$

4 Work out the missing numbers.

a $-12 \div \square = -3$ **b** $-18 \div \square = -9$

c $\square \div 4 = -4$ **d** $\square \div 10 = -2$

5 The product of two integers is -10.
Find the possible values of the two integers.

How can you be sure you have found all the possible answers?

6 Copy and complete this multiplication table.

×	−3	−5
5		
7		

7 Estimate the answers to these calculations by rounding to the nearest whole number.

a -3.2×-6.8 **b** 9.8×-5.35

c $-16.1 \div 1.93$ **d** $-7.38 \div -1.86$

8 Estimate the answers to these calculations by rounding the numbers.

a -53×-39 **b** 32×-61 **c** -38×9.3 **d** $-493 \div -5.1$

9 Work out these calculations. Do the calculation in the brackets first.

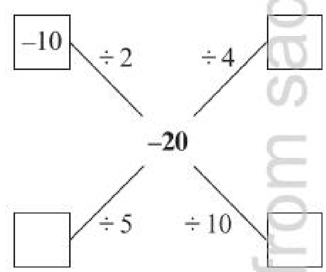
a $3 \times (-6 + 2)$ **b** $-4 \times (-1 + 7)$ **c** $5 \times (-2 - 4)$ **d** $-2 \times (3 - -7)$

10 **a** Copy and complete these divisions.
For example, $-20 \div 2 = -10$.

b Can you add any more lines to the diagram? You must divide by a positive integer. The answer must be an integer.

c Draw a similar diagram with -28 in the centre.

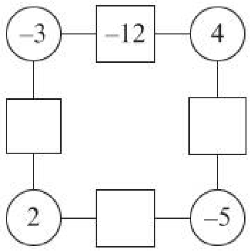
d Compare your answer to part **c** with a partner's. Do you agree?



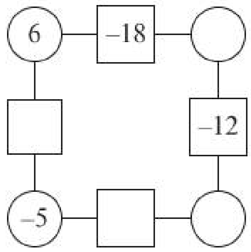
11 In these diagrams, the integer in a square is the product of the integers in the circles next to it. For example, $-3 \times 4 = -12$.

Copy and complete the diagrams.

a



b

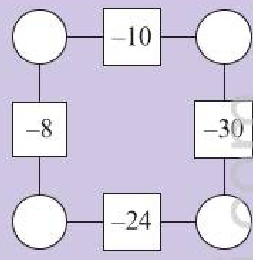


Think like a mathematician

12 This diagram is similar to the diagrams in Question **11**.

The numbers in the circles must be integers.

Copy and complete the diagram. Are there different ways to do this?



Summary checklist

- ☐ I can multiply a negative integer by a positive integer.
- ☐ I can divide a negative integer by a positive integer.

1.3 Lowest common multiples

In this section you will …

- find out about lowest common multiples.

Key words

common multiple
digit
lowest common multiple
multiple

The **multiples** of 4 are 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, …
The multiples of 6 are 6, 12, 18, 24, 30, 36, 42, …
The **common multiples** of 4 and 6 are 12, 24, 36, …
The **lowest common multiple** (LCM) of 4 and 6 is 12.

Tip

4 × 1, 4 × 2, 4 × 3, and so on.

12 is the smallest number that is a multiple of both 4 and 6.

Worked example 1.6

Find the lowest common multiple of 6 and 10.

Answer

The multiples of 6 are 6, 12, 18, 24, 30, 36, …

The last **digit** of a multiple of 10 is 0, so 30 is a multiple of 10 and it must be the LCM of 6 and 10.

Exercise 1.3

1 Write the first five multiples of:
   a 5 b 10 c 7 d 12

2 a Write the multiples of 3 that are less than 40.
   b Write the multiples of 5 that are less than 40.
   c Find the common multiples of 3 and 5 that are less than 40.

3 a Find the common multiples of 4 and 3 that are less than 50.
   b Complete this sentence: The common multiples of 4 and 3 are multiples of …

4 Find the LCM of 8 and 12.

5 Find the LCM of 10 and 15.

6 Find the LCM of 7 and 8.

Think like a mathematician

7 Look at this statement: If A and B are two whole numbers, then $A \times B$ is a common multiple of A and B.

a Show that the statement is true when $A = 4$ and $B = 7$.

b Show that the statement is true when $A = 6$ and $B = 5$.

c Is the statement always true? Give evidence to justify your answer.

d Look at this statement: If A and B are two whole numbers, then $A \times B$ is the lowest common multiple of A and B.

Is this statement true? Give evidence to justify your answer.

8 Find the LCM of 3, 4 and 6.

9 Find the LCM of 18, 9 and 4.

10 21 is the LCM of two numbers. What are the numbers?

11 30 is the LCM of two numbers. What are the numbers?

How did you answer questions **10** and **11**? If you were asked another question similar to questions **10** and **11**, would you do it the same way?

Summary checklist

☐ I can find the lowest common multiple of two numbers by listing the multiples of each number.

1.4 Highest common factors

In this section you will …

- find out about highest common factors.

Key words

common factor
factor
highest common factor

The factors of 18 are 1, 2, 3, 6, 9 and 18.
The factors of 27 are 1, 3, 9 and 27.
The **common factors** of 18 and 27 are 1, 3 and 9.
The **highest common factor** (HCF) of 18 and 27 is 9.

Tip

$18 = 1 \times 18$ or 2×9 or 3×6.

9 is the largest factor of both 18 and 27.

Worked example 1.7

Find the highest common factor of 28 and 42.

Answer

Find pairs of whole numbers that have a product of 28.

$28 = 1 \times 28$ $28 = 2 \times 14$ $28 = 4 \times 7$

The factors of 28 are 1, 2, 4, 7, 14 and 28.

Find pairs of whole numbers that have a product of 42.

$42 = 1 \times 42 = 2 \times 21 = 3 \times 14 = 6 \times 7$

The factors of 42 are 1, 2, 3, 6, 7, 14, 21 and 42.

The common factors are 1, 2, 7 and 14.

The highest common factor of 28 and 42 is 14.

The common factors are in both of the lists of factors.

You can use a highest common factor to simplify a **fraction** as much as possible.

Worked example 1.8

a Find the HCF of 16 and 40.

b Use your answer to part **a** to write the fraction $\frac{16}{40}$ as simply as possible.

Answer

a The factors of 16 are 1, 2, 4, 8 and 16.

The largest number in this list that is a factor of 40 is 8 (because $8 \times 5 = 40$).

So, the HCF of 16 and 40 is 8.

b Simplify the fraction by dividing 16 and 40 by the HCF of 16 and 40.

From part **a**, the HCF of 16 and 40 is 8.

So, divide both 16 and 40 by 8.

$$\frac{16}{40} = \frac{2}{5}$$

Exercise 1.4

1 Find the factors of:

a 24 **b** 50 **c** 45 **d** 19

2 Find the factors of:

a 33 **b** 34 **c** 35 **d** 36 **e** 37

3 **a** Find the common factors of 18 and 48.

b Find the highest common factor of 18 and 48.

4 Find the highest common factor of:

a 12 and 28 **b** 12 and 30 **c** 12 and 36

5 Find the highest common factor of:

a 18 and 24 **b** 19 and 25 **c** 20 and 26 **d** 21 and 28

6 Find the highest common factor of:

a 60 and 70 **b** 60 and 80 **c** 60 and 90

7 **a** Find the highest common factor of 35 and 56.

b Use your answer to part **a** to simplify the fraction $\frac{35}{56}$ as much as possible.

How did knowing the highest common factor help you to answer part **b**?

8 a Find the highest common factor of 25 and 36.

b Explain why the fraction $\frac{25}{36}$ cannot be simplified.

9 Find the highest common factor of 54, 72 and 90.

10 Two numbers have a highest common factor of 4. One of the numbers is between 10 and 20. The other number is between 20 and 40.

a What are the two numbers? Find all the possible answers.

b How can you be sure you have all the possible answers?

Think like a mathematician

11 a Find the HCF of 8 and 12.

b Find the LCM of 8 and 12.

c Find the product of 8 and 12.

d Find the product of the HCF and the LCM of 8 and 12.

e What do you notice about the answers to parts **c** and **d**?

f Can you generalise the result in part **e** for different pairs of numbers? Investigate.

12 The HCF of two numbers is 3. The LCM of the two numbers is 45.

a Explain why each number is a multiple of 3.

b Explain why each number is a factor of 45.

c Find the two numbers.

d Check with a partner to see if you have the same answers. Did you both answer the question in the same way?

Summary checklist

- [ ] I can find the highest common factor of two numbers by listing the factors of each number.

1.5 Tests for divisibility

In this section you will …

- learn tests of divisibility to find factors of large numbers.

Key words

divisible

tests for divisibility

2, 3 and 5 are all factors of 30.

You say that '30 is **divisible** by 2' because $30 \div 2$ does not have a remainder.

30 is divisible by 3 and 30 is divisible by 5.

30 is not divisible by 4 because $30 \div 4 = 7$ with remainder 2 (which can be written as 7 r 2).

87 654 is a large number.

Is 87 654 divisible by 2? By 3 By 4? By 5?

Here are some **rules for divisibility**:

- A number is divisible by 2 when the last digit is 0, 2, 4, 6 or 8.

  87 654 is divisible by 2 because the last digit is 4.
- A number is divisible by 3 when the **sum** of the digits is a multiple of 3.

  $8+7+6+5+4=30$ and $30=10\times3$, so 87 654 is divisible by 3.
- A number is divisible by 4 when the number formed by the last two digits is divisible by 4.

  The last two digits of 87 654 are 54 and $54 \div 4 = 13$ r 2. So 87 654 is not divisible by 4.
- A number is divisible by 5 when the last digit is 0 or 5.

  The last digit of 87 654 is 4, so it is not divisible by 5.
- A number is divisible by 6 when it is divisible by 2 and 3.

  87 654 is divisible by 6.
- To **test for divisibility** by 7, remove the last digit, 4, to leave 8765
  - Subtract twice the last digit from 8765, that is: $8765 - 2 \times 4 = 8765 - 8 = 8757$
  - If this number is divisible by 7, so is the original number.
  - $8757 \div 7 = 1252$ with no remainder and so 87 654 is divisible by 7.
- A number is divisible by 8 when the number formed by the last three digits is divisible by 8.

  $654 \div 8 = 81$ r 6, so 87 654 is not divisible by 8.

Tip

A whole number is divisible by 2 when 2 is a factor of the number.

- A number is divisible by 9 when the sum of the digits is divisible by 9.

  $8+7+6+5+4=30$ and 30 is not divisible by 9. So 87 654 is not divisible by 9.
- A number is divisible by 10 when the last digit is 0.

  The last digit of 87 654 is 4, so 87 654 is not divisible by 10.
- A number is divisible by 11 when the difference between the sum of the odd digits and the sum of the even digits is 0 or a multiple of 11.

  The sum of the odd digits of 87 654 is $4+6+8=18$.

  The sum of the even digits of 87 654 is $5+7=12$.

  $18-12=6$, so 87 654 is not a multiple of 11.

Worked example 1.9

The number *7 258 has one digit missing.

a Find the missing digit when:

  i the number is divisible by 6 **ii** the number is divisible by 11

b A number is divisible by 66 when it is divisible by 6 and 11. Could *7 258 be divisible by 66? Give a reason for your answer.

Answer

a **i** The number must be a multiple of 2 and 3.

The last digit is 8, so the number is divisible by 2.

The sum of the digits is $*+7+2+5+8=*+22$.

If this is a multiple of 3, then * is 2 or 5 or 8.

There are three possible values for *.

ii The sum of the odd digits is $8+2+*=10+*$.

The sum of the even digits is $5+7=12$.

When $*=2$ the difference between these will be zero, so 27 258 is divisible by 11.

b The answer to part **a** shows that the number is divisible by 66 when $*=2$. This is the only possibility.

Exercise 1.5

1 **a** Show that the number 28 572 is divisible by 3 but not by 9.

b Change the final digit of 28 572 to make a number that is divisible by 9.

2 **a** Show that 57 423 is divisible by 3 but not by 6.

b The number 57 42* is divisible by 6. Find the possible values of the digit *.

3 **a** Show that 25 764 is divisible by 2 and by 4.

b Is 25 764 divisible by 8? Give a reason for your answer.

4 **a** Show that 3 and 4 are factors of 25 320.

b Find two more factors of 25 320 that are between 1 and 12.

5 **a** Choose any four digits.

b If it is possible, arrange your digits to make a number that is divisible by

i 2 **ii** 3 **iii** 4 **iv** 5 **v** 6

c Can you arrange your digits to make a number that is divisible by all five numbers in part **a**? If not, can you make a number that is divisible by four of the numbers?

d Give your answers to a partner to check.

6 **a** Show that 924 is divisible by 11.

b Is 161 084 divisible by 11? Give a reason for your answer.

7 Use a test for divisibility test to show that:

a 2583 is divisible by 7. **b** 3852 is not divisible by 7.

8 **a** Show that only two numbers between 1 and 10 are factors of 22 599.

b What numbers between 1 and 10 are factors of 99 522?

9 Copy and complete this table. The first line has been done for you.

Number	Factors between 1 and 10
12	2, 3, 4, 6
123	
1234	
12 345	
123 456	

10 Use the digits 4, 5, 6 and 7 to make a number that is a multiple of 11.
How many different ways can you find to do this?

11 a Show that 2521 is not divisible by any integer between 1 and 12.
b Rearrange the digits of 2251 to make a number divisible by 5.
c Rearrange the digits of 2251 to make a number divisible by 4.
d Rearrange the digits of 2251 to make a number divisible by 8.
e Find the smallest integer larger than 2521 that is divisible by 6.
f Find the smallest integer larger than 2521 that is divisible by 11.

12 44 and 44 444 are numbers where every digit is 4.
a Explain why any positive integer where every digit is 4 must be divisible by 2 and by 4.
b Here are two facts about a number:
Every digit is 4.
It is divisible by 5.
Explain why this is impossible.
c Here are two facts about a number:
Every digit is 4.
It is divisible by 3.
i Find a number with both these properties.
ii Is there more than one possible number? Give a reason for your answer.
d Here are two facts about a number:
Every digit is 4.
It is divisible by 11.
i Find a number with both these properties.
ii Is there more than one possible number? Give a reason for your answer.

Think like a mathematician

13 a $2 \times 4 = 8$
Look at this statement: A number is divisible by 8 when it is divisible by 2 and by 4.
Do you think the statement is correct? Give evidence to justify your answer.
b $2 \times 5 = 10$
Look at this statement: A number is divisible by 10 when it is divisible by 2 and by 5.
Do you think the statement is correct? Give evidence to justify your answer.

Continued

c $3 \times 5 = 15$

Look at this statement: A number is divisible by 15 when it is divisible by 3 and by 5.

Do you think the statement is correct? Give evidence to justify your answer.

Summary checklist

- [ ] I can use a test to see if a number is divisible by 2, 3, 4, 5, 6, 8, 9, 10 or 11.

1.6 Square roots and cube roots

In this section you will …

- find out how square numbers and cube numbers are related to square roots and cube roots.

Key words

cube number
cube root
consecutive
equivalent
index
square number
square root

$1 \times 1 = 1$ $2 \times 2 = 4$ $3 \times 3 = 9$ $4 \times 4 = 16$ $5 \times 5 = 25$

The square numbers are 1, 4, 9, 16, 25, …

You use an **index** of 2 to show square numbers.

$1^2 = 1$ $2^2 = 4$ $3^2 = 9$ $4^2 = 16$ $5^2 = 25$

You read 1^2 as '1 squared' and you read 2^2 as '2 squared'.

$4^2 = 16$ is **equivalent** to $4 = \sqrt{16}$, which is read as '4 is the **square root** of 16'.

The symbol for square root is $\sqrt{\ }$.

Worked example 1.10

Work out $\sqrt{100} - \sqrt{81}$.

Answer

$10^2 = 10 \times 10 = 100$ and $9^2 = 81$.

So $\sqrt{100} = 10$ and $\sqrt{81} = 9$.

$\sqrt{100} - \sqrt{81} = 10 - 9 = 1$

$1 \times 1 \times 1 = 1$ $2 \times 2 \times 2 = 8$ $3 \times 3 \times 3 = 27$

The **cube numbers** are 1, 8, 27, …

You use an index of 3 and write $1^3 = 1$, $2^3 = 8$, $3^3 = 27$, …

You read 1^3 as '1 cubed' and you read 2^3 as '2 cubed'.

You say '2 is the **cube root** of 8', which is written as $2 = \sqrt[3]{8}$.

Tip

The symbol for cube root is $\sqrt[3]{\ }$.

Worked example 1.11

Work out $\sqrt{64} \div \sqrt[3]{64}$.

Answer

$8^2 = 64$ and so $\sqrt{64} = 8$.

$4^3 = 64$ and so $\sqrt[3]{64} = 4$.

Hence, $\sqrt{64} \div \sqrt[3]{64} = 8 \div 4 = 2$.

You can estimate the square roots of integers that are not square numbers.

Worked example 1.12

a Show that 9 is the closest integer to $\sqrt{79}$.

b Show that $\sqrt{215}$ is between 14 and 15.

Answer

a $9^2 = 91$ and $8^2 = 64$.

79 is between 64 and 81 so $\sqrt{79}$ is between 8 and 9.

79 is much closer to 81 than to 64 so 9 is the closest integer to $\sqrt{79}$

b $14^2 = 196$ and $15^2 = 225$.

210 is between 196 and 225 and so $\sqrt{215}$ is between 14 and 15.

Exercise 1.6

1 Copy and complete the following.

a $3^2 = \square$ **b** $5^2 = \square$ **c** $8^2 = \square$

d $10^2 = \square$ **e** $15^2 = \square$

2 An equivalent statement to $7^2 = 49$ is $\sqrt{49} = 7$. Write equivalent statements to your answers to Question **1**.

3 Find:

a $\sqrt{36}$ **b** $\sqrt{81}$ **c** $\sqrt{121}$ **d** $\sqrt{144}$

4 Copy and complete the following.

a $1^3 = \square$ **b** $2^3 = \square$ **c** $3^3 = \square$

d $4^3 = \square$ **e** $5^3 = \square$

5 An equivalent statement to $6^3 = 6 \times 6 \times 6 = 216$ is $\sqrt[3]{216} = 6$.
Write equivalent statements to your answers to Question **4**.

6 Work out the integer that is closest to

a $\sqrt{15}$ **b** $\sqrt{66}$ **c** $\sqrt{150}$

7 **a** Show that $\sqrt{90}$ is between 9 and 10.

b Find two **consecutive** integers to complete this sentence: $\sqrt{180}$ is between … and …

c Find two consecutive integers to complete this sentence: $\sqrt[3]{90}$ is between … and …

8 **a** Use a calculator to find 17^2.

b Complete this statement: $\sqrt{\square} = 17$

9 Complete the following statements.

a $\sqrt{\square} = 18$ **b** $\sqrt{\square} = 20$ **c** $\sqrt{\square} = 23$ **d** $\sqrt{\square} = 26$

10 Complete the following statements.

a $\sqrt[3]{\square} = 7$ **b** $\sqrt[3]{\square} = 9$ **c** $\sqrt[3]{\square} = 10$ **d** $\sqrt[3]{\square} = 12$

11 **a** Show that 36 has nine factors.

b Find the factors of these square numbers.

i 9 **ii** 16 **iii** 25

c Explain why every square number has an odd number of factors.

d Find a number that is <u>not</u> square that has an odd number of factors.

e Does every cube number have an odd number of factors? Give a reason for your answer.

f Investigate how many factors different square numbers have.

How did you do part **f**? Would it be helpful to work with a partner?

Think like a mathematician

12 $1^2 = 1$ and $2^2 = 4$ and so $2^2 - 1^2 = 3$.

1^2 and 2^2 are consecutive square numbers. The difference between 1^2 and 2^2 is 3.

a Copy and complete this diagram, showing the differences between consecutive square numbers.

Square numbers: 1^2 2^2 3^2 4^2 5^2 6^2

Difference: 3 ☐ ☐ ☐ ☐

b Describe any pattern in your answers.

c Investigate the differences between consecutive cube numbers.

Cube numbers: 1^3 2^3 3^3 4^3 5^3 6^3

Difference: 7 ☐ ☐ ☐ ☐

13 **a** Work out:

i $\sqrt{1^3}$ ii $\sqrt{1^3 + 2^3}$ iii $\sqrt{1^3 + 2^3 + 3^3}$

b What do you notice about your answers to part **a**?

c Does the pattern continue when you add more cube numbers? Give a reason for your answer.

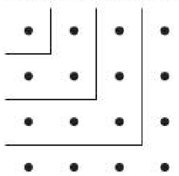
d Compare your answer to part **c** with a partner's. Can you improve your answer?

14 **a** Add up the first three odd numbers and find the square root of the answer.

b Add up the first four odd numbers and find the square root of the answer.

c Can you generalise the results of parts **a** and **b**?

d Look at this diagram.



How is this diagram connected with the earlier parts of this question?

Summary checklist

☐ I can find square numbers and their corresponding square roots.

☐ I can find cube numbers and their corresponding cube roots.

Check your progress

1 Work out:

a $3-7$ **b** $-3+-7$

c $-2\times(2--4)$ **d** $(-9+-6)\div 3$

2 **a** Find two integers that add up to 2 and multiply to make −15.

b Find two integers that add up to 3 and multiply to make −70.

3 Find the missing numbers.

a $5\times\square=-8-7$ **b** $-12\div\square=4+-6$

4 Find all the common factors of 16 and 24.

5 **a** Find all the multiples of 6 between 50 and 70.

b Find the lowest common multiple of 6 and 15.

6 **a** Find the highest common factor of 26 and 65.

b Simplify the fraction $\frac{26}{65}$.

7 The integer N is less than 100. $\sqrt{N}$ and $\sqrt[3]{N}$ are both integers.

a Explain why N must be a square number.

b Find the value of N.

8 The number 96*32 has a digit missing.

a Explain why the number is divisible by 4.

b Find the missing digit if the number is divisible by 3.

c Find the missing digit if the number is divisible by 11.

9 Copy and complete the following.

$1^3=1^2$

$4^3=8^2$

$\square^3=\square^2$

$16^3=\square^2$

› Project 1

Mixed-up properties

Here are nine property cards:

Their difference is a factor of their sum	Their highest common factor (HCF) is 1	Their product has exactly 4 factors
Their difference is prime	Their sum is a square number	Their lowest common multiple (LCM) is 12
They are both factors of 30	They are both prime	Their product is a cube number

Here are six number cards:

2	3	4	5	6	7

Can you find a way to arrange the property cards and the number cards in a grid, so that each property card describes the pair of numbers at the top of the column and on the left of the row?

For example, the cell marked * could contain the card 'They are both prime' because 2 and 5 are both prime.

	4	5	7
2		*	
3			
6			

Can you find more than one way to arrange the cards?

Which cards could go in lots of different places?

Which cards can only go in a few places?

Could you replace the six numbers with other numbers and still complete the grid?

2 Expressions, formulae and equations

Getting started

1 Work out:

a $14 + 5$ **b** $21 - 7$ **c** $-6 + 4$ **d** $12 - 15$

e 4×8 **f** $45 \div 9$ **g** -8×2 **h** $36 \div -3$

2 Work out the following. Remember to use the correct order of operations.

a $7 + 2 \times 3$ **b** $(8 + 4) \times 7$ **c** $9 + 12 \div 3$ **d** $(8 - 5) \div 3$

e $\frac{15}{5} \times 7$ **f** $\frac{18-6}{3}$ **g** $-6 \times 2 - 8$ **h** $-5 + \frac{10}{2}$

3 **Fill in** the missing numbers in each of these calculations.
Choose from the numbers in the circle on the right.

a $\square + 6 = 8$ **b** $\square \times 4 = 24$ **c** $\square \div 2 = 5$

d $\square - 2 = 9$ **e** $4 \times \square = -20$ **f** $-18 \div \square = -6$

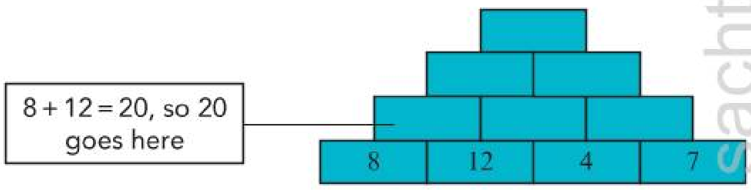
11 2 3 −5 10 6

4 Work out the answers to these problems. Show all your working.

a Karin buys three pizzas that cost \$2.75 each and a drink that costs \$0.89. What is the total cost?

b Izzy and Sam go for a meal in a restaurant. They order food that costs \$8.70, \$3.45 and \$12.25. They **share** the cost of the meal between them. How much do they each pay?

5 In this number pyramid, you find the number in each block by adding the numbers in the two blocks below it. Complete the number pyramid.



Algebra is the part of mathematics in which you use letters and other symbols to represent numbers.

It is the most important part of mathematics because it links together the other strands such as geometry, statistics and number.

Algebra allows you to take a situation and make it more general.

You can use **formulae** (rules) that work in every case. This can help you in your everyday life, such as working out the **area** of your garden, calculating the best deals at a supermarket, applying for a bank loan or hiring a car.

Algebra is used in all modern technology jobs, from creating websites on the internet to making cell phones and smart televisions. As well as technology, algebra is used in many other jobs, such as engineering, medicine, economics, food science and traffic management.

Studying algebra helps you to think logically. It teaches you to solve problems. This will help you in all aspects of life, not just in your algebra lessons.

2.1 Constructing expressions

In this section you will ...

- use letters to represent numbers
- use the correct order of operations in algebraic expressions
- write and use expressions.

Key words

coefficient
constant
expression
equation
equivalent expression
term
unknown
variable

In algebra you can use a letter to represent an **unknown** number.

An **expression** contains numbers and letters, but not an equals sign.

An **equation** contains numbers and letters and an equals sign.

Example: $5n+4$ is an expression.

$5n+4=19$ is an equation.

In the expression $5n+4$, there are two **terms**. $5n$ is one term. The other term is 4.

The letter n is called the **variable** because it can have different values.

The **coefficient** of n is 5 because it is the number that multiplies the variable.

In the equation $5n+4=19$, n is the unknown number, 5 is the coefficient of n, and the numbers 4 and 19 are **constants**. A constant may also be written as a letter, such as π. π is the **ratio** of a circle's circumference to its diameter. It is approximately 3.14.

You can use a letter to represent an unknown number to solve problems.

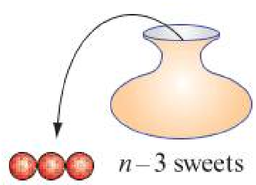
Example: Shown is a bag of sweets. You don't know how many sweets are in the bag.

Tip

$5n+4$ means $5 \times n+4$. Use the correct order of operations. Do the multiplication before the addition.

You will learn more about π later in your studies.

n represents the unknown number of sweets in the bag.



Three sweets are taken out of the bag.

Now there are $n-3$ sweets left in the bag.

Worked example 2.1

Mathew is x years old. David is 4 years older than Mathew. Adam is 2 years younger than Mathew. Kathryn is three times Matthew's age. Ella is half Mathew's age.

Write down an expression for each person's age.

Continued

Answer

Mathew is x years old.	This is the information you have to start with.
David is $x+4$ years old.	David is 4 years older than Mathew, so add 4 to x.
Adam is $x-2$ years old.	Adam is 2 years younger than Mathew, so subtract 2 from x.
Kathryn is $3x$ years old.	Kathryn is 3 times Matthew's age so multiply 3 by x. You write $3 \times x$ as $3x$. Always write the number before the letter.
Ella is $\frac{x}{2}$ years old.	Ella is half Mathew's age. You need to divide x by 2. You write $x \div 2$ as $\frac{x}{2}$.

Exercise 2.1

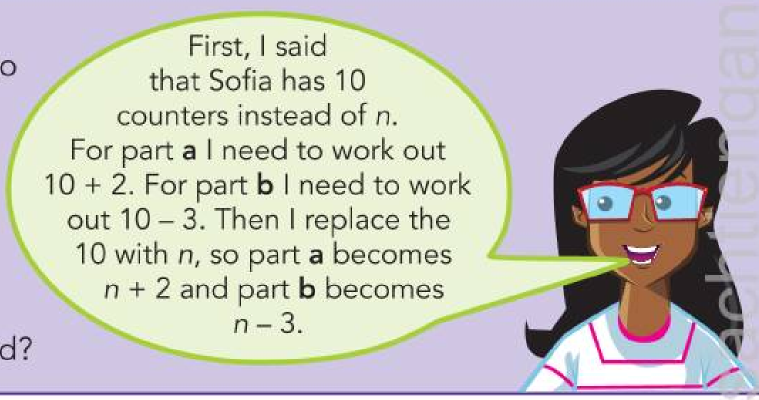
1 Sofia has a bag that contains n counters.
Write an expression for the total number of counters she has in the bag when:

- **a** she puts in two more counters
- **b** she takes out three counters.

Think like a mathematician

2 Discuss in pairs or groups.
Zara uses the following method to answer Question **1**.

- **a** What do you think of Zara's method?
- **b** Do you think that this method will help you write expressions?
- **c** Can you improve her method?



3 The temperature on Tuesday was t °C.
Write an expression for the temperature when it is:

- **a** 2 °C higher than it was on Tuesday
- **b** twice as warm as it was on Tuesday
- **c** half as warm as it was on Tuesday

Tip

Twice means × 2.
Half means ÷ 2.

4 Write an expression for the answer to each of these.

a Dravid has x DVDs. He buys six more.
How many DVDs does Dravid now have?

b Molly is m years old and Barney is b years old.
What is the total of their ages?

c Ted can store g photographs on one memory card.
How many photographs can he store on three memory cards of the same size?

Think like a mathematician

5 Discuss in pairs or groups.
How would you write an expression for each of these?

a I think of a number x. I multiply the number by 6, then add 1.

b I think of a number x. I multiply the number by 4, then subtract 9.

c I think of a number x. I divide the number by 6, then subtract 1.

d I think of a number x. I divide the number by 2, then add 7.

e I think of a number x. I multiply the number by 2, then subtract the result from 25.

6 Maliha thinks of a number, y.

a Write an expression for the number Maliha gets when she:

i multiplies the number by 3

ii divides the number by 2

iii multiplies the number by 4, then adds 1

iv multiplies the number by 2, then subtracts 5

v multiplies the number by 5, then subtracts the result from 52

vi divides the number by 4, then adds 3

b Check that your expressions are correct by replacing the y with a number.

Tip

For part **iii**, if my number is 5, I need to work out $4 \times 5 + 1 = 20 + 1 = 21$.

Four times my number is 20, and 20 add 1 equals 21.

So, my expression is $4y + 1$.

Activity 2.1

Work with a partner to take it in turns to make up an 'I think of a number' question, like those in Question **5**.

For example, 'I think of a number. I multiply by 3, then subtract 2.'

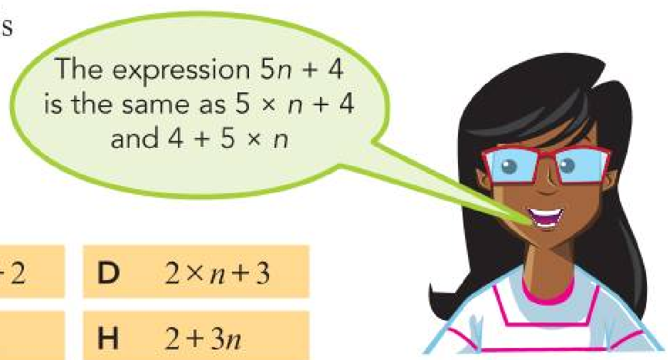
Your partner must write down the expression correctly, using a letter of their choice.

Check that their expression is correct. If it is correct, then your partner scores 1 point.

Do this five times each, then check your scores.

7 **a** Zara is **classifying** some expression cards into groups of **equivalent expressions**.
Explain why Zara is correct.

b Classify these cards into groups of equivalent expressions.



A	$2n+3$	**B**	$2\times n-3$	**C**	$3\times n+2$	**D**	$2\times n+3$
E	$3-2n$	**F**	$3-2\times n$	**G**	$3+2n$	**H**	$2+3n$
I	$2n-3$	**J**	$3n+2$	**K**	$3+2\times n$	**L**	$2+3\times n$

8 This is part of Pedro's classwork.

Question
In a clothes shop, T-shirts cost \$t and shirts cost \$s.
Write an expression for the total cost of:
a one T-shirt and one shirt   b four T-shirts and two shirts
Answers
a $t\times s$ b $2t+4s$

Explain the mistakes that Pedro has made and write down the correct answers.

9 Write an expression for each of these situations. You can choose your own letters, but make sure that you explain what your letters represent.

a the total cost of two tacos and four burritos
b the total cost of eight lemon cakes and five carrot cakes
c the total value of six gold coins is doubled
d the total value of five silver coins is tripled

10 Write an expression for each of these.

a x more than y
b x less than y
c m more than two times n
d a less than three times b
e p multiplied by q
f four times g multiplied by h

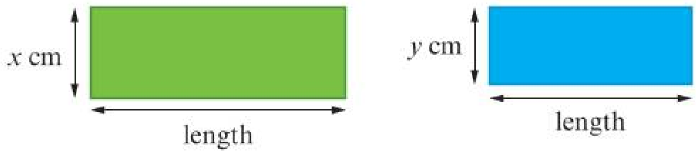
Tips

In question **9** part **a**, you could start by explaining 'Let the cost of one taco be \$$t$ and the cost of one burrito be \$$b$.

'Doubled' means × 2.

'Tripled' means × 3.

11 The diagram shows two rectangles.



The green rectangle on the left has a width of x cm. The length is six times the width.

The blue rectangle on the right has a width of y cm. The length is 3 cm more than two times the width.

Write an expression for the difference in the lengths of the rectangles.

Summary checklist

- ☐ I can use letters to represent unknown numbers.
- ☐ I can use the correct order of operations in algebraic expressions.
- ☐ I can write and understand expressions.

2.2 Using expressions and formulae

In this section you will …

- substitute numbers into expressions
- write and use formulae.

Key words

derive
formula
formulae
substitute

A formula is a mathematical rule that shows the relationship between two quantities (variables).

You can write a formula using words:

Area of rectangle = length × width or using letters: $A = l \times w$

You can **substitute** numbers into expressions and formulae.

When $l = 5$ cm and $w = 4$ cm, $A = 5 \times 4 = 20\text{ cm}^2$.

You can write or **derive** your own formulae, to help you solve problems.

Worked example 2.2

a Work out the value of these expressions.

i $a+b$ when $a=2$ and $b=4$

ii $2w-3v$ when $w=12$ and $v=5$

b Write a formula for the number of days in any number of weeks, in:

i words **ii** letters

c Use the formula in part **b** to work out the number of days in eight weeks.

Answer

a **i** $a+b=2+4$	Substitute 2 for a and 4 for b in the expression.
$=6$	Add 2 and 4 together to give a total of 6.
ii $2w-3v=2\times 12-3\times 5$	Substitute 12 for w and 5 for v in the expression.
$=24-15$	Use the correct order of operations and do the multiplications before the subtraction.
$=9$	
b **i** number of days = $7\times$ number of weeks	There are seven days in a week, so multiply the number of weeks by 7.
ii $d=7w$	Choose d for days and w for weeks and write $7\times w$ as $7w$.
	Always write the number before the letter, so write $7w$ not $w7$.
c $d=7\times 8$	Substitute $w=8$ into the formula.
$=56$	

Exercise 2.2

1 Copy and complete the workings to find the value of each expression.

a $x+9$ when $x=13$

$x+9=13+9$

$=\square$

b $\frac{x}{5}$ when $x=40$

$\frac{x}{5}=\frac{40}{5}$

$=\square$

Tip

$\frac{x}{5}$ means $x\div 5$.

c $10 - x$ when $x = 3$

$10 - x = 10 - 3$

$= \square$

d $2m + n$ when $m = 7$ and $n = 6$

$2m + n = 2 \times 7 + 6$

$= \square + 6$

$= \square$

e $7x$ when $x = 5$

$7x = 7 \times 5$

$= \square$

f $ab - c$ when $a = 9$, $b = 8$ and $c = 32$

$ab - c = 9 \times 8 - 32$

$= \square - 32$

$= \square$

2 Work out the value of each expression.

a $a + 5$ when $a = 3$

b $x - 9$ when $x = 20$

c $f + g$ when $f = 7$ and $g = 4$

d $m - n$ when $m = 100$ and $n = 25$

e $3k$ when $k = 5$

f $p + 2q$ when $p = 5$ and $q = 3$

g $2h + 3t$ when $h = 8$ and $t = 5$

h $\frac{y}{4}$ when $y = 32$

i $\frac{30}{c} - 2$ when $c = 6$

j $\frac{x + y}{2}$ when $x = 19$ and $y = 11$

3 Raul writes a formula for the number of hours in any number of days.
He writes: number of hours = 24 × number of days

a Explain why this formula is correct.

b Write the formula using letters. Use h for hours and d for days.

c Use your formula to work out the number of hours in five days.

4 **a** Write a formula for the number of minutes in any number of hours, in:

i words

ii letters

b Use your formula in part **a ii** to work out the number of minutes in five hours.

5 Kristina goes out for a meal at a restaurant with four friends. They share the total cost of the meal equally between the five of them.

a Write a formula to work out the amount they each pay, in:

i words

ii letters

b Use your formula in part **a ii** to work out the amount they each pay when the total cost of the meal is $85.

Think like a mathematician

Work with a partner, or in a small group, to answer questions **6** and **7**.

When you have answered the questions, discuss your answers with other groups.

6 Sanjay is paid \$9 per hour. He writes this formula $T = 9h$.

a What do you think the letters h and T stand for?

b Write the formula in words.

c Work out the value of T when $h = 30$.

7 Every week, Yim pays for petrol and insurance for her car. She writes the formula $C = p + i$.

a What do you think the letters C, p and i stand for?

b Write the formula in words.

c Work out the value of C when $p = \$25$ and $i = \$7$.

8 Jan calculates how much money he has left each week, using the formula:

$M = P - E$, where:	M is the money he has left over
	P is the money he is paid for working
	E are his expenses

Tip

Jan's expenses are things that he must pay for, such as food, transport, bills, etc.

a Find the value of M when:

i $P = \$225$ and $E = \$72$ ii $P = \$178$ and $E = \$36$

b Work out the value of P when $M = \$160$ and $E = \$45$.

Explain how you worked out your answer.

9 A formula used in Science is:

$V = IR$, where:	V is the voltage
	I is the current
	R is the resistance

Tip

IR means $I \times R$.

Work out the value of V when:

a $I = 3$ and $R = 7$ b $I = 4$ and $R = 9$

You can use inverse operations to work out:

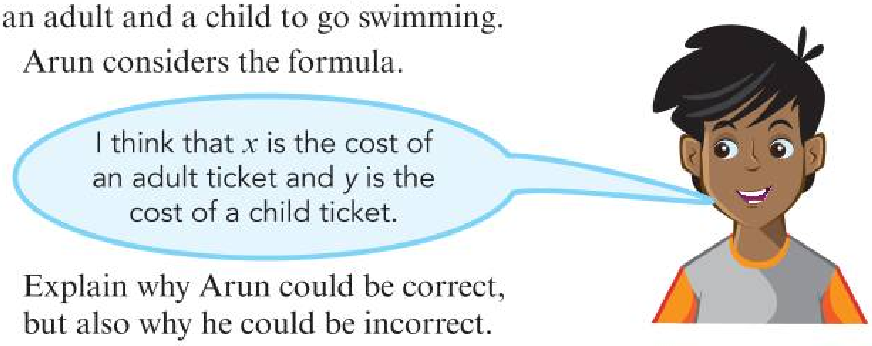
- the value of I if you know V and R
- the value of R if you know V and I.

c Work out the value of I when $V = 30$ and $R = 6$

d Work out the value of R when $V = 40$ and $I = 5$.

10 Yola uses the formula $C = x + y$ to work out the total cost of tickets for an adult and a child to go swimming.

a Arun considers the formula.

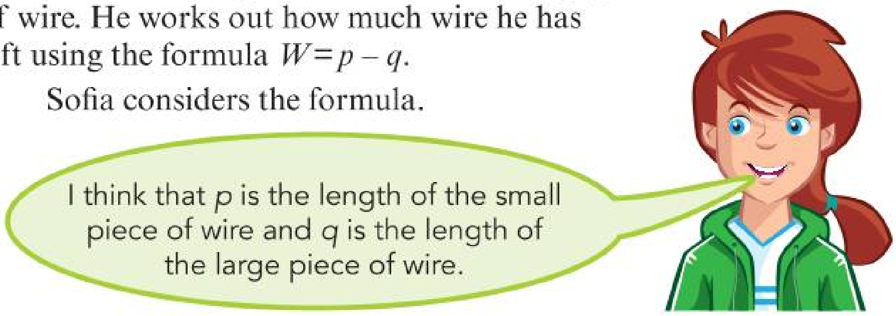


Explain why Arun could be correct, but also why he could be incorrect.

b How could you improve Yola's formula to make it easier to use?

11 Mateo cuts a small piece of wire from a large piece of wire. He works out how much wire he has left using the formula $W = p - q$.

a Sofia considers the formula.



Is Sofia correct? Explain your answer.

b How could you improve Mateo's formula to make it easier to use?

12 What value of k can you substitute into these expressions to give you the same answer?

$k + 10$	$3k$	$20 - k$

Tip

Start by trying different values for k.

How well do you think you understood this section on formulae?

Summary checklist

- ☐ I can substitute numbers into expressions.
- ☐ I can write formulae.
- ☐ I can understand and use formulae.

2.3 Collecting like terms

In this section you will …

- collect like terms.

Key words

collecting like terms

like terms

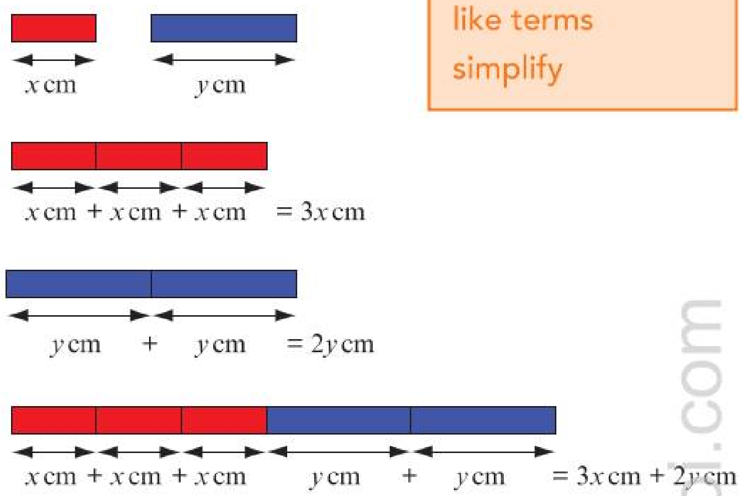
simplify

Here are two different bricks.

The length of the red brick is x cm.

The length of the blue brick is y cm.

When you join together three red bricks, the total length is $3x$ cm. When you join together two blue bricks, the total length is $2y$ cm. When you join together three red bricks and two blue bricks the total length is $3x\,\text{cm} + 2y\,\text{cm}$.



You can add, subtract or combine **like terms**.

You cannot combine terms that contain different letters.

You can **simplify** an expression by **collecting like terms**.
This means that you rewrite the expression in as short a way as possible.

This table shows examples of good practice when you write simplified expressions.

✓	✗
a	$1a$, $1\times a$, a^1
$2a$	$a+a$, $2\times a$, a^2, $a2$
ab	$a\times b$, ba, $b\times a$
$\frac{a}{b}$	$a\div b$
a^2	$a\times a$
a^3	$a\times a\times a$

✓ is the best way to write simplified expressions.
✗ is not the best way to write simplified expressions.

Worked example 2.3

Simplify each expression.

a $2x + 3x$ **b** $7y - 2y$ **c** $4p + 3q + 2p - q$ **d** $5t + 7 - 3t + 3$

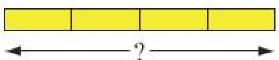
Answer

a $2x + 3x = 5x$	$2x$ and $3x$ are like terms, so add them to get $5x$.
b $7y - 2y = 5y$	$7y$ and $2y$ are like terms, so subtract to get $5y$.
c $4p + 3q + 2p - q = 6p + 2q$	$4p + 2p = 6p$ and $3q - q = 2q$, but $6p$ and $2q$ are not like terms so you cannot simplify any further.
d $5t + 7 - 3t + 3 = 2t + 10$	$5t - 3t = 2t$ and $7 + 3 = 10$, but $2t$ and 10 are not like terms so you cannot simplify any further.

Exercise 2.3

1 Erik has yellow, green and blue bricks.

The length of a yellow brick is a.
The length of a green brick is b.
The length of a blue brick is c.
Work out the total length of these arrangements of bricks.
Give your answer in its **simplest form**.

a


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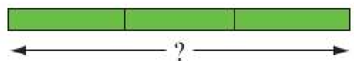
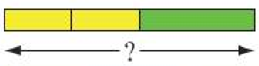
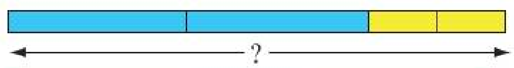
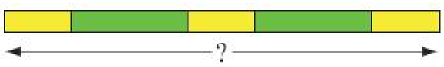
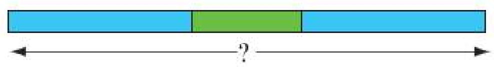


c


d


e


f


2 Match each expression, **a** to **f**, to its correctly simplified expression, **i** to **vi**.

The first one has been done for you: **a** and **v**.

a	$x + x$	**i**	$7x$
b	$5x + 4x$	**ii**	x
c	$x + 6x$	**iii**	$4x$
d	$5x - 2x$	**iv**	$9x$
e	$2x - x$	**v**	$2x$
f	$8x - 4x$	**vi**	$3x$

Tip

Remember that x is the same as $1x$.

3 Simplify each of these expressions.

a $x+x+x+x+x$ **b** $2y+4y$ **c** $5d+3d$

d $6t+3t+4t$ **e** $8g+5g+g$ **f** $9p+p+6p$

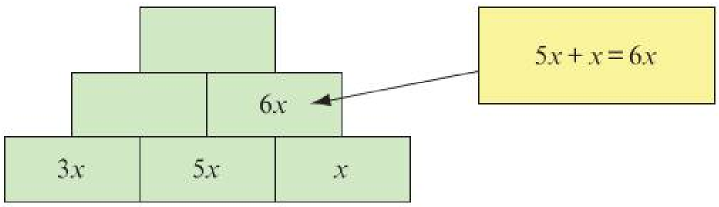
g $7w-4w$ **h** $8n-n$ **i** $9b^2-5b^2$

j $6f+2f-3f$ **k** $9j+j-7j$ **l** $8k^3-5k^3-2k^3$

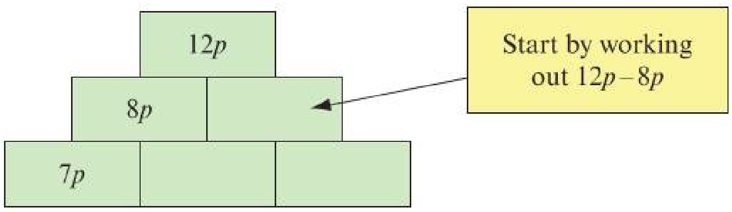
4 In an algebraic pyramid, you find the expression in each block by adding the expressions in the two blocks below it.

Copy and complete these pyramids.

a



b



5 Copy and complete the workings to simplify these expressions.

a $2x+8x+6y+9y=10x+\square y$

b $9d-7d+3h-h=\square d+2h$

c $2g+8+3g-5=2g+3g+8-5=\square g+\square$

d $8p+12t-3p+t=8p-3p+12t+t=\square p+\square t$

e $3a+4b+5c-2b-8c=3a+4b-2b+5c-8c=\square a+\square b-\square c$

6 Simplify these expressions by collecting like terms.

a $2a+3a+5b$ **b** $3c+5c+2d+d$ **c** $4t+1+3t+9$

d $6m-2m+7n-3n$ **e** $9k+5f-3k-2f$ **f** $10q-5q+17-9$

g $7r+2s+3t-2r+s+2t$ **h** $12+6h+8k-6-3h-3k$

Think like a mathematician

7 In pairs or groups, discuss what the terms xy and yx mean.

Also discuss how you can simplify these expressions:

a $4xy+3yx$ **b** $8pq+2ed-3qp+2de$

What can you say, in general, about terms that have the same letters but are in a different order?

8 Write each expression in its simplest form.
The first one has been done for you.

- **a** $2ab + 3ab + 5pq + 7qp = 5ab + 12pq$
- **b** $3st + 5st + 9pu + 7up$
- **c** $4vb + 2bv + 6ad - 4da$
- **d** $11rt + 9gh - 2rt - 7hg$
- **e** $8xy + 12xz + 3yx - 9zx$
- **f** $6a + 7ac - 2a + ca$
- **g** $4mn - 3nm + 7gh - 7hg$

> **Tip**
>
> In part **a**, start by rewriting $7qp$ as $7pq$.
>
> In part **d**, start by rewriting the expression with like terms together and all of the letters in alphabetical order, like this: $11rt - 2rt + 9gh - 7gh$.

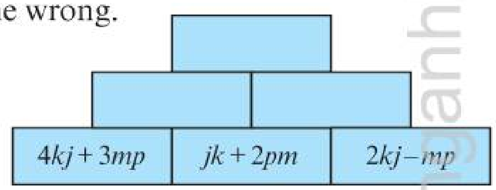
9 This is part of Dai's homework.

Question
Write these expressions in their simplest form.
a $2x + 8 + 6x - 4$ *b* $3bc + 5bd - 2bc + 3db$

Solution
a $2x + 8 + 6x - 4 = 8x + 4 = 12x$
b $3bc + 5bd - 2bc + 3db = 5bc + 5bd + 3db$

Dai has made several mistakes. Explain what Dai has done wrong.
Work out the correct answers.

10 Copy and complete this algebraic pyramid.
Remember that you find the expression in each block by adding the expressions in the two blocks below it.



Activity 2.2

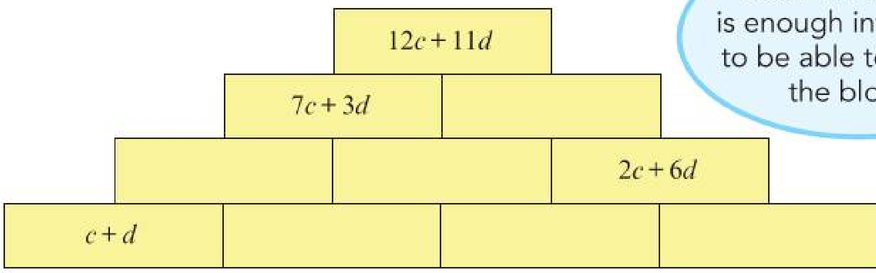
Design an algebraic pyramid like the one in Question **10**, for a partner to complete. On a piece of paper, draw your pyramid and fill in the bottom row of the pyramid.

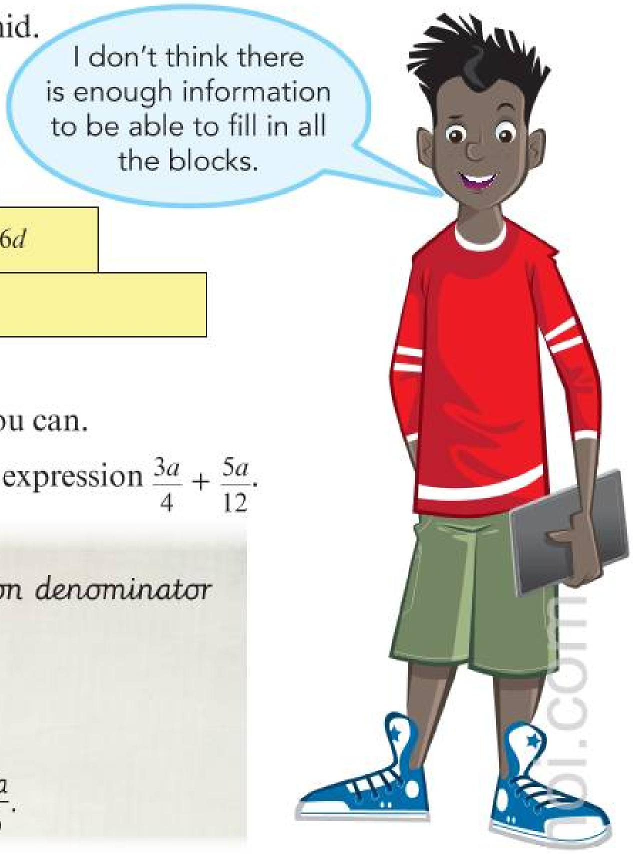
You may use single or double letter terms. You may use addition and subtraction signs.

Before you ask your partner to complete your pyramid, make sure you write down the answers on a different piece of paper.

Swap pyramids with a partner, complete their pyramid, then swap back and mark each other's work. Discuss any mistakes that have been made.

11 Marcus is trying to complete this algebraic pyramid.





Is Marcus correct? Explain your answer.

Copy the pyramid and fill in as many blocks as you can.

12 This is the method Giovanna uses to simplify the expression $\frac{3a}{4} + \frac{5a}{12}$.

> *To work out $\frac{3a}{4} + \frac{5a}{12}$, I need to use a common denominator of 12.*
>
> $\frac{3a}{4} = \frac{3 \times 3a}{3 \times 4} = \frac{9a}{12}$, *so* $\frac{9a}{12} + \frac{5a}{12} = \frac{14a}{12}$.
>
> *I can now write $\frac{14a}{12}$ in its simplest form as $\frac{7a}{6}$.*

a Critique Giovanna's method. Is it easy to follow?

Do you think this is the best method to use to answer this type of question or can you improve the method? Discuss your answers with a partner.

b Simplify these expressions. Write your answers in their simplest form.

i $\frac{a}{2} + \frac{5a}{8}$ **ii** $\frac{y}{2} - \frac{y}{6}$ **iii** $\frac{2x}{3} - \frac{5x}{12}$

13 Simplify these expressions.

Write your answers in their simplest form and as an **improper fraction**.

a $\frac{3a}{4} + \frac{a}{2}$ **b** $\frac{3b}{5} + \frac{7b}{15}$ **c** $2c + \frac{c}{3}$

> **Tip**
>
> An improper fraction is a fraction where the **numerator** is larger than the **denominator**.

Summary checklist

☐ I can collect like terms.

2.4 Expanding brackets

In this section you will …

- expand brackets.

Key words

brackets
expand

Some algebraic expressions include brackets.

To **expand** a term with brackets, you multiply each term inside the brackets by the term outside the brackets. Expanding a term with brackets is sometimes called 'expanding the brackets' or 'multiplying out the brackets'.

Tip

$4(n+3)$ means $4 \times (n+3)$, but you usually write an expression like this without the $\times$.

Worked example 2.4

Expand the brackets.

a $4(n+3)$ **b** $2(x-5)$ **c** $3(2g+h-7)$

Answer

a $4(n+3) = 4 \times n + 4 \times 3$

$= 4n + 12$

Multiply the 4 by the n, then multiply the 4 by the 3.

Simplify the $4 \times n$ to $4n$ and simplify the 4×3 to 12. Add the two terms together.

b $2(x-5) = 2 \times x - 2 \times 5$

$= 2x - 10$

There is a minus sign before the 5, so you need to take away the 10 from the $2x$.

c $3(2g+h-7) = 3 \times 2g + 3 \times h - 3 \times 7$

$= 6g + 3h - 21$

The first term is $3 \times 2g$, which is the same as $3 \times 2 \times g$, which simplifies to $6g$.

There are three terms. You need to add the first two terms and then subtract the third term.

Exercise 2.4

1 Copy and complete the following. Expand the brackets first.

a $2(x+9) = 2 \times x + 2 \times 9$
$= 2x + \square$

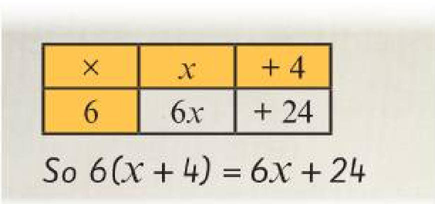
b $3(y-1) = 3 \times y - 3 \times 1$
$= 3y - \square$

c $4(7+p) = 4 \times 7 + 4 \times p$
$= \square + \square$

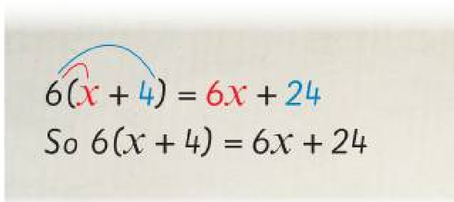
d $5(q-3) = 5 \times q - 5 \times 3$
$= \square - \square$

2 Su and Li compare the methods they use to expand $6(x+4)$.

Su uses a multiplication box like this:



Li uses multiplication arcs like this:



- **a** Write down the **advantages** and **disadvantages** of Su's method.
- **b** Write down the advantages and disadvantages of Li's method.
- **c** Write down the advantages and disadvantages of the method used in Question **1**.
- **d** Which method do you think is best for expanding brackets correctly? Explain why.

3 Expand the brackets.

a	$3(y+6)$	**b**	$4(w+2)$	**c**	$5(z+5)$	**d**	$3(b-1)$
e	$6(d-9)$	**f**	$2(e-8)$	**g**	$6(2+f)$	**h**	$2(1+g)$
i	$9(3+i)$	**j**	$6(2-x)$	**k**	$2(1-y)$	**l**	$5(7-p)$

4 Expand these brackets. Copy and complete the workings.

a $2(2x+1) = 2 \times 2x + 2 \times 1$
$= 4x + \square$

b $5(3y-2) = 5 \times 3y - 5 \times 2$
$= 15y - \square$

c $7(2g+9p) = 7 \times 2g + 7 \times 9p$
$= \square + \square$

d $4(4q-11+r) = 4 \times 4q - 4 \times 11 + 4 \times r$
$= \square - \square + \square$

5 Multiply out the brackets.

a	$3(2x+1)$	**b**	$4(3y+5)$	**c**	$5(2w+3)$	**d**	$6(4z+7v+9)$
e	$2(3b-4)$	**f**	$4(2c-3)$	**g**	$6(5d-1)$	**h**	$8(3e-6+2f)$
i	$3(a+2f)$	**j**	$5(3b+4g)$	**k**	$7(6c-7h)$	**l**	$9(5+3h-4i)$

6 This is part of Bethan's homework. Bethan has made a mistake in every answer. Explain what Bethan has done wrong. Work out the correct answers.

Question

Multiply out the brackets.

a $4(x + 4)$ b $2(6x - 3)$

c $3(2 - 5x)$ d $6(2 - x)$

Solution

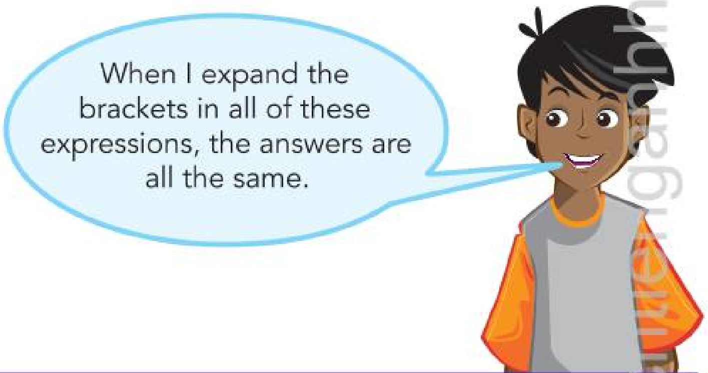
a $4(x + 4) = 4x + 8$ b $2(6x - 3) = 12x - 3$

c $3(2 - 5x) = 6 + 15x$ d $6(2 - x) = 12 - 6x = 6x$

Look again at Question **6**. What method did you use to answer this question? Do you think this was the best method or would you use a different method if you had to answer the question again?

7 Arun looks at these four expressions.

$2(12x + 15)$	$3(10 + 8x)$
$4(6x + 26)$	$6(5 + 4x)$



Is Arun correct? Explain your answer and show your working.

Think like a mathematician

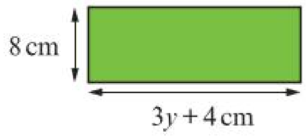
8 Discuss with a partner the answers to these questions.

a When you expand the brackets in the expressions $3(4b + 5)$ and $3(5 + 4b)$, do you get the same answer?

b When you expand the brackets in the expressions $2(5c - 1)$ and $2(1 - 5c)$, do you get the same answer?

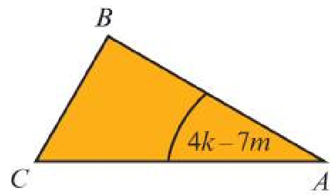
Give evidence to justify your answers.

9 The diagram shows a rectangle.
The width of the rectangle is 8 cm.
The length of the rectangle is $3y + 4$ cm.
Write an expression, in its simplest form, for the:



a area of the rectangle
b perimeter of the rectangle

10 In the triangle ABC, the angle at A is $(4k - 7m)°$.
The angle at B is two times the size of the angle at A.
Write an expression, in its simplest form, for the size of the angle at B.



11 This is how Rachel expands and simplifies the expression $5(2x + 4) - 12$.

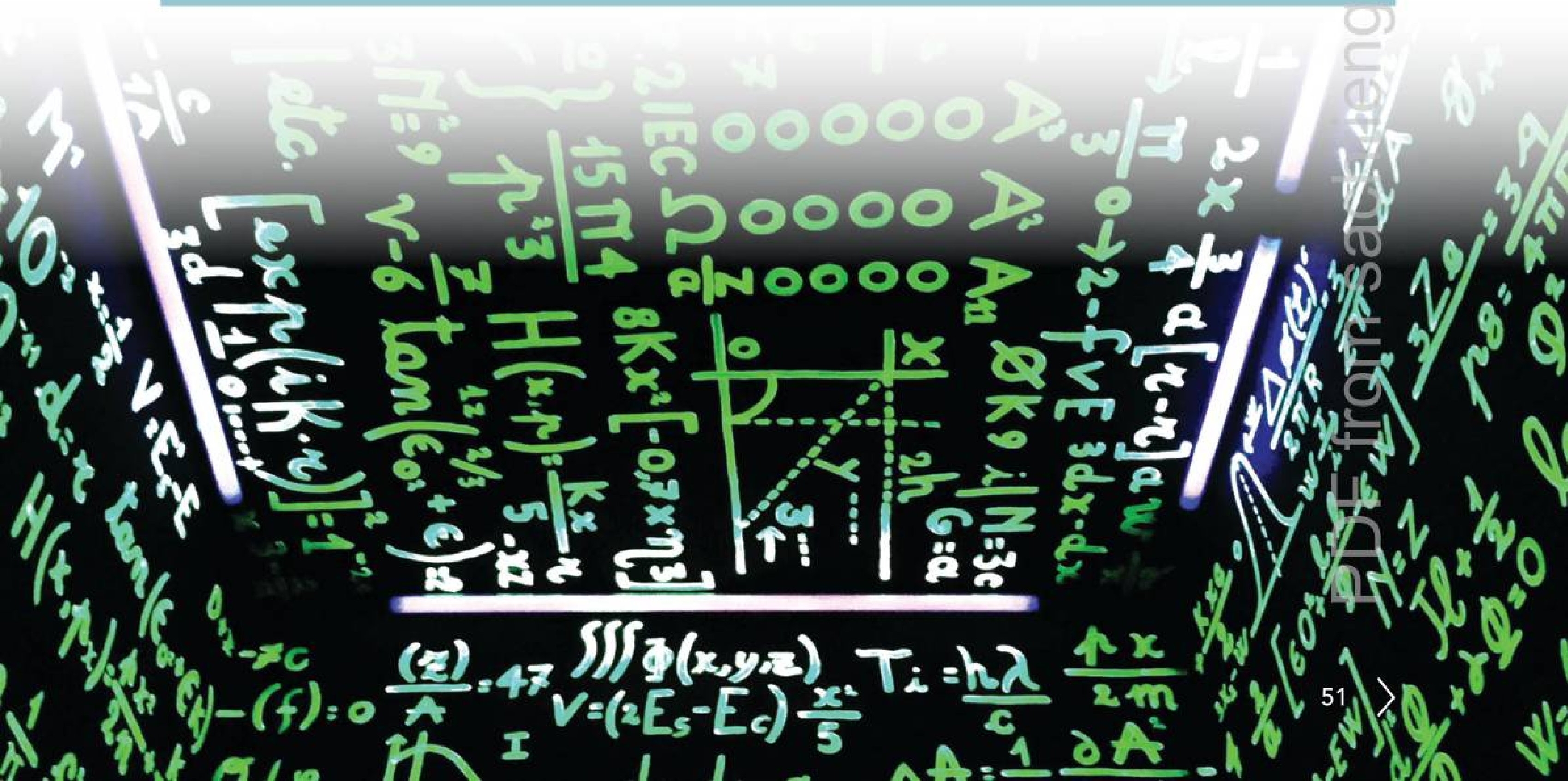
$$\begin{aligned} 5(2x+4) - 12 &= 5 \times 2x + 5 \times 4 - 12 \\ &= 10x + 20 - 12 \\ &= 10x + 8 \end{aligned}$$

Expand and simplify:

a $4(x + 7) - 1$ **b** $7(x + 3) + 5x$ **c** $12 + 3(2x - 3)$

Summary checklist

☐ I can multiply out a bracket.



2.5 Constructing and solving equations

In this section you will …

- write and use equations.

Key words

solve
solution

To **solve** an equation, you need to find the value of the unknown letter.

Consider the equation: $x + 5 = 12$

Subtract 5 from both sides of the equation: $x + 5 - 5 = 12 - 5$

You have found the **solution** to the equation: $x = 7$

Worked example 2.5

Solve these equations and check your answers.

a $x - 3 = 12$ **b** $2y + 4 = 16$

Answer

a $x - 3 + 3 = 12 + 3$ — Add 3 to both sides.

$x = 15$ — Work out the value of x, then substitute this value back into the equation to check that the answer is correct.

Check: $15 - 3 = 12$ ✓

b $2y = 16 - 4$ — Subtract 4 from both sides.

$2y = 12$ — Simplify the right-hand side.

$y = \frac{12}{2}$ — Divide both sides by 2.

$y = 6$ — Work out the value of y, then substitute this value back into the equation to check that the answer is correct.

Check: $2 \times 6 + 4 = 12 + 4 = 16$ ✓

Exercise 2.5

1 Copy and complete the workings to solve these equations.
Check your answers are correct.

a $x + 6 = 10$
$x + 6 - 6 = 10 - \square$
$x = \square$

b $x - 6 = 10$
$x - 6 + 6 = 10 + \square$
$x = \square$

c $2x = 10$
$\frac{2x}{2} = \frac{10}{\square}$
$x = \square$

2 Solve each of these equations and check your answers.

a $x+4=11$ **b** $x+3=6$ **c** $2+x=15$ **d** $7+x=19$

e $x-4=9$ **f** $x-2=8$ **g** $x-12=14$ **h** $x-18=30$

i $3x=12$ **j** $5x=30$ **k** $7x=70$ **l** $12x=72$

3 Dayita uses this method to solve an equation when the unknown is on the right-hand side of the equation.

Solve the equation: $12 = y + 3$
Write this as: $y + 3 = 12$
Solve as normal: $y + 3 - 3 = 12 - 3$
$y = 9$

Use Dayita's method to solve these equations.

a $15=y+3$ **b** $9=y+2$ **c** $13=y-5$

d $25=y-3$ **e** $24=8y$ **f** $42=6y$

4 Write an equation for each of the following. Then solve each equation to find the value of the unknown number.

a I think of a number and add 3. The answer is 18.

b I think of a number and subtract 4. The answer is 10.

c I think of a number and multiply it by 4. The answer is 24.

Tip

In part **a**, $n+3=18$.

5 Zara is considering the equation $4n=24$.

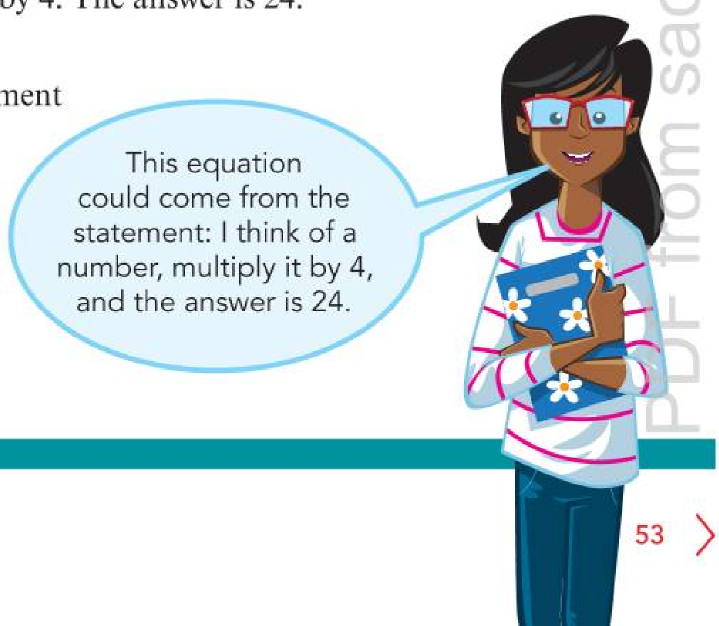
a Write an 'I think of a number' statement for each of these equations.

i $n-8=3$

ii $n+5=12$

iii $8n=96$

b Solve the equations in part **a**.



6 This is part of Steffan's homework.

Question

Solve these equations.

a $x + 7 = -2$ b $x - 6 = -2$ c $5x = -35$

Solution

a $x + 7 = -2$
$x = -2 - 7$
$x = -5$

b $x - 6 = -2$
$x = -2 - 6$
$x = -8$

c $5x = -35$
$x = \frac{-35}{5}$
$x = 7$

Mark Steffan's homework. If he has made any mistakes, explain the mistakes and work out the correct answers.

7 Solve each of these equations and check your answers.

a $2a + 3 = 13$ **b** $4a + 1 = 17$ **c** $3a - 2 = 13$

d $4 = 2c - 8$ **e** $14 = 3c + 2$ **f** $29 = 4c - 3$

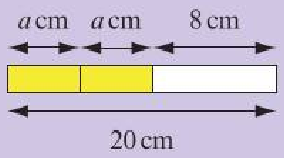
Think like a mathematician

8 In pairs or groups, discuss how you could answer this question.

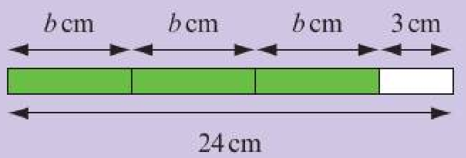
The total length of each set of bricks is shown.

a Write an equation involving the lengths of the bricks.

i



ii



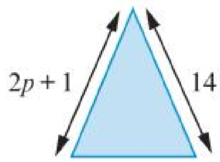
b Solve your equations to find the lengths of the bricks.

c Compare your equations and answers with those of other groups and discuss any differences. How could you improve your work?

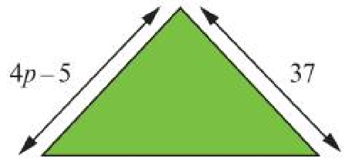
9 The diagrams show the lengths of the equal sides of these isosceles triangles.
For each triangle:

i Write an equation. **ii** Solve your equation to find the value of p.

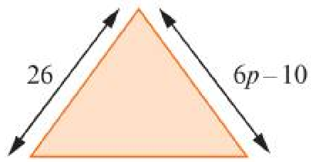
a



b



c



10 a Write an equation for each of these situations.

- **i** Paul has some DVDs. He sells three of the DVDs. He has 26 DVDs left.
- **ii** Yaz has some books. She buys five more books. She now has 18 books.
- **iii** On Monday, Nial goes for a bike ride. On Tuesday he rides twice as far as he did on Monday. On Tuesday he rides 48 km.
- **iv** Jana's age is three more than twice the age of Jin. Jana is 35 years old.

b Solve your equations in part **a** to answer the following questions.

- **i** How many DVDs does Paul have to start with?
- **ii** How many books does Yaz have to start with?
- **iii** How far does Nial ride on Monday?
- **iv** How old is Jin?

Activity 2.3

Work in pairs for this activity.
On a piece of paper, write down four situations like the ones in Question **10**.
Swap your paper with another pair of learners.
Write down an equation for each of the situations you have been given.
Solve each of the equations.
Swap back your pieces of paper and mark each other's work.
Discuss any mistakes that have been made.

11 Kenji has the following cards.

$4m + 4$	$2m - 6$	$6m + 2$	$=$	32	44	20

He chooses one blue card, the red card and one yellow card to make an equation.
Which blue and yellow card should he choose to give him the equation with:

a the largest solution for m?

b the smallest solution for m?

Explain your decisions and show that your answers are correct.

Summary checklist

- [ ] I can understand and solve equations.
- [ ] I can write equations and solve them.

2.6 Inequalities

In this section you will …

- use letters to represent a range of numbers.

Key words

advantages
disadvantages
inequality
inequality symbols
integer
negative infinity
open interval
positive infinity

In this unit so far, you have used a letter to represent an unknown number.

You can also use a letter and an **inequality** to represent an **open interval**.

The **inequality symbols** used to represent open intervals are:

< meaning 'is less than'

> meaning 'is greater than'.

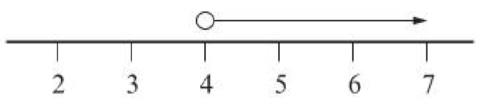
Example: The inequality $x > 4$ means that x can be any number greater than 4.

The inequality $y < 7$ means that y can be any number less than 7.

Each inequality represents an open interval. In the first interval, x cannot be equal to 4. In the second interval, y cannot be equal to 7.

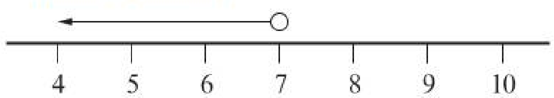
You can show an open inequality on a number line like this:

$x > 4$



The arrow pointing to the right shows that x can be any number above 4 and continues towards **positive infinity**.

$y < 7$



The arrow pointing to the left shows that y can be any number below 7 and continues towards **negative infinity**.

Tip

You use an open circle for < and > inequalities.

Worked example 2.6

a **i** Show the inequality $x > 5$ on a number line.

ii What is the smallest **integer** that x could be?

iii List the integer values that x could be.

Remember that an integer is a whole number.

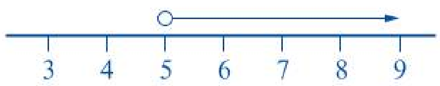
b **i** Show the inequality $y < 2$ on a number line.

ii What is the largest integer that y could be?

iii List the integer values that y could be.

Answer

a **i**



You use an open circle for the > sign, and start the line at the number 5.

The line goes to the right because it is greater than.

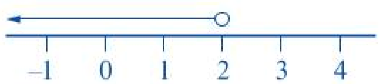
ii 6

x is greater than 5, so 6 is the smallest number.

iii 6, 7, 8, 9, 10, …

You cannot list all of the integers because the list goes on forever, so list the first five and write '…' (called ellipses) to show that the list goes on forever.

b **i**



You use an open circle for the < sign, and start the line at the number 2.

The line goes to the left because it is less than.

ii 1

y is less than 2, so 1 is the largest number.

iii 1, 0, −1, −2, −3, …

You cannot list all of the integers because the list goes on forever, so list the first five and write '…' to show that the list goes on forever.

Exercise 2.6

1 Write in words what each of these inequalities means. The first one has been done for you.

a $x < 10$ x is less than 10

b $x > 10$

c $x < -4$

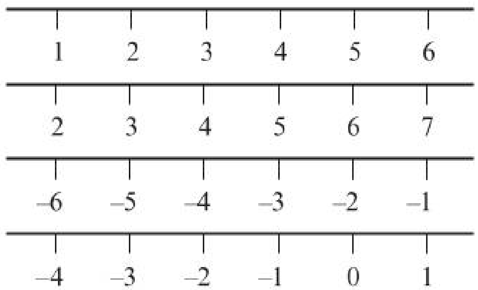
d $x > -4$

2 Write these statements as inequalities. The first one has been done for you.

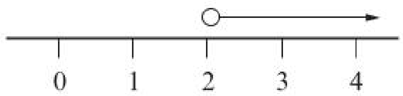
- **a** y is greater than 8 $\quad y > 8$
- **b** n is greater than −1
- **c** p is less than zero
- **d** q is less than −2

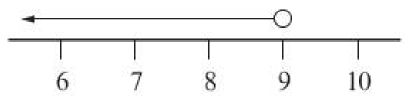
3 Copy each number line and show each inequality on the number line.

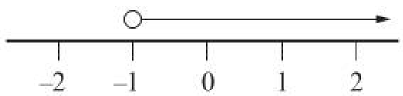
- **a** $x > 3$
- **b** $x < 5$
- **c** $x > -3$
- **d** $x < -1$

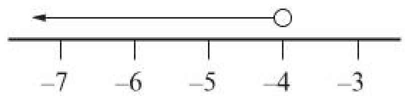


4 Write down the inequality that each of these number lines shows. Use the letter x.

a


b


c


d


5 This is part of Katya's homework.

> Question
> Given the inequality $x > 7$, write down:
> i the smallest integer that x could be
> ii a list of the integer values that x could be.
> Solution
> i The smallest integer is 7.
> ii x could be 7, 8, 9, 10, 11, …

Tip

In the solution to part **ii**, the three dots (called ellipses) after the number 11 show that the list goes on forever.

- **a** Explain the mistakes that Katya has made and write down the correct solutions.
- **b** Discuss your answers to part **a** with a partner.

Make sure you have corrected all of Katya's mistakes.

6 For each of these inequalities:

a $y > 4$ b $y > -7$ c $y > 2.5$

i Write down the smallest integer that y could be.

ii Write a list of the integer values that y could be.

7 For each of these inequalities:

a $n < -6$ b $n < 12$ c $n < 4.5$

i Write down the largest integer that n could be.

ii Write a list of the integer values that n could be.

Think like a mathematician

8 Refer back to questions **6** and **7**. Discuss the answers to these questions with a partner.

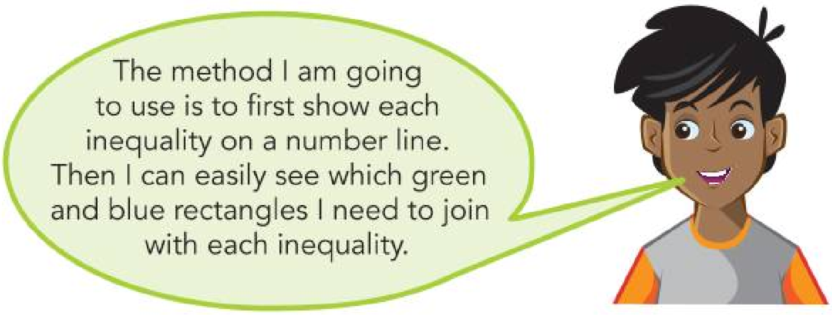
a In Question **6**, why can't you write down the largest integer that y could be?

b In Question **7**, why can't you write down the smallest integer that n could be?

9 Arun is considering the following problem.

Match each inequality on the left to the correct largest or smallest integer in the middle, and to the correct list of integers on the right.
The first one has been done for you.

Inequality	Largest or smallest integer	List of integers
a $x > 5.2$	A The largest integer is −4.	i 5, 4, 3, 2, 1, ...
b $y < 5.2$	B The smallest integer is 0.	ii 6, 7, 8, 9, 10, ...
c $m < -3.5$	C The smallest integer is 6.	iii 0, −1, −2, −3, −4, ...
d $p > -3.5$	D The smallest integer is −3.	iv −4, −5, −6, −7, −8, ...
e $k < 0.9$	E The largest integer is 5.	v 0, 1, 2, 3, 4, ...
f $h > -0.9$	F Largest integer is 0.	vi −3, −2, −1, 0, 1, ...



g Use Arun's method to answer the question.

h Critique Arun's method by describing the advantages and disadvantages of his method.

i Can you improve his method or suggest a better method?

Summary checklist

- [ ] I can understand open inequalities.
- [ ] I can draw open inequalities.

Check your progress

1 Nimrah thinks of a number, n.

Write an expression for the number Nimrah gets each time when she

- **a** multiplies the number by 4
- **b** subtracts 6 from the number
- **c** adds 12 to the number
- **d** multiplies the number by 3, then adds 5.

2 Work out the value of each expression.

- **a** $2a+3$ when $a=8$
- **b** $p-q$ when $p=13$ and $q=7$
- **c** $\frac{b}{3}-5$ when $b=27$

3 Loli lives with three friends. They share the gas bill equally between the four of them.

- **a** Write a formula to work out the amount they each pay, in:
  - **i** words
  - **ii** letters
- **b** Use your formula in part **a ii** to work out the amount they each pay when the gas bill is $96.

4 Simplify these expressions.

- **a** $n+n+n$
- **b** $3c+5c$
- **c** $9x^2-x^2$
- **d** $3xy+5yz-2xy+3zy$

5 Expand the brackets.

- **a** $3(x+2)$
- **b** $6(3-w)$
- **c** $4(3x+2)$
- **d** $3(7-4v+6w)$

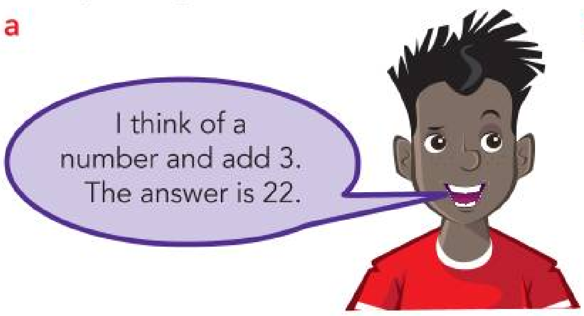
6 Solve each of these equations and check your answers.

- **a** $n+3=8$
- **b** $m-4=12$
- **c** $3p=24$
- **d** $2h+6=24$

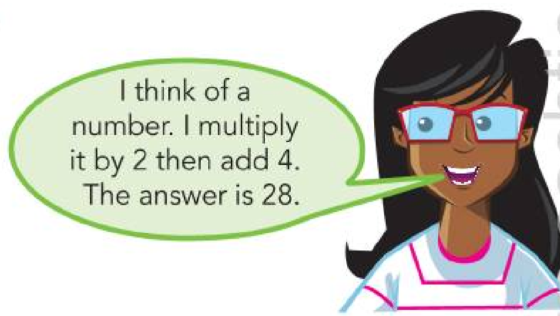
7 Marcus and Zara have set some puzzles. Write an equation for each puzzle.

Solve your equations to find the values of the unknown numbers.

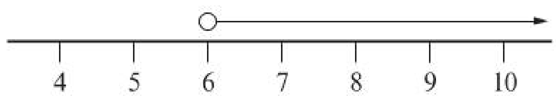
a



b



8 Write down the inequality shown on this number line.



3 Place value and rounding

Getting started

1 Work out:

a 2×10　　b 4×100　　c 7×1000

d 13×10　　e 35×100　　f 81×1000

2 Choose the correct answer, **A**, **B** or **C**, for the following.

		A	B	C
a	$3.4 \times 10 =$	340	0.34	34
b	$2.5 \times 100 =$	250	25	2500
c	$0.57 \times 1000 =$	57	570	5700
d	$1.84 \times 10 =$	184	0.184	18.4
e	$63.2 \times 100 =$	6320	632	63 200
f	$8.95 \times 1000 =$	89 500	895	8950

3 Write T (true) or F (false) for each of these equations. If you write false, work out the correct answer.

a $8 \div 10 = 0.8$　　b $12 \div 100 = 1.2$　　c $830 \div 1000 = 0.83$

d $34.6 \div 10 = 0.346$　　e $17.1 \div 100 = 0.171$　　f $4250 \div 1000 = 0.425$

4 Copy this maths wall diagram and write the missing numbers in the red bricks.
Follow the arrows forwards, then backwards.

→

$1.5 \times 100 =$	____	$\div 10 =$	____	$\times 1000 =$	____	$\div 100 =$	
____	$= 100 \times$	____	$= 10 \times$	____	$= 1000 \div$	____	

←

5 Round each of these numbers to the nearest whole number.

a 6.7　　b 4.2　　c 17.8　　d 145.1

e 12.43　　f 88.88　　g 253.61　　h 124.09

You use numbers every day, but you do not always need the numbers to be exact. Numbers are often rounded. This is because it is easier to work with rounded numbers and to compare them.

You can usually round numbers when exact accuracy isn't important.

For example, look at these two newspaper articles:

Real Madrid take the top spot!

On Saturday afternoon 74 836 football fans saw Real Madrid beat FC Barcelona by two goals to take the top spot in La Liga (Spain's top football league).

Real Madrid Take The Top Spot!

On Saturday afternoon 75 000 football fans saw Real Madrid beat FC Barcelona by two goals to take the top spot in La Liga (Spain's top football league).

The article on the left gives the accurate number of football fans, that is, 74 836. The article on the right rounds the number to 75 000. This means the second article is easier to read, and it isn't really important to the story whether 74 836 or 75 000 football fans watched the game.

For market research, companies usually use accurate numbers in their calculations. They will generally round their answers to give final figures that are easier to compare.

For example, a telephone company may look at the **population** figures of different countries and the numbers of phones used in those countries. The company can use these figures to help it decide how to increase sales of phones or to decide whether more network coverage is needed.

Look at this information about Bangladesh and Hong Kong:

Bangladesh

Population: 158 570 535

Number of phones: 74 192 350

Number of phones per person: 0.467 882...

Hong Kong

Population: 7 122 508

Number of phones: 13 264 896

Number of phones per person: 1.862 391...

Bangladesh has a much bigger population than Hong Kong. However, when you compare the number of phones per person, you can see that there are more phones per person in Hong Kong than in Bangladesh.

You can round both the **decimal numbers** that have been calculated. You can then say that there are approximately 0.5 phones per person in Bangladesh compared with approximately 2 phones per person in Hong Kong.

3.1 Multiplying and dividing by powers of 10

In this section you will ...

- multiply and divide whole numbers by powers of 10
- multiply and divide decimals by powers of 10.

Key words

classify

index (plural indices)

powers of 10

power

You can write all the numbers 10, 100, 1000, 10 000, … as **powers of 10**. The **power** of 10 is written as an **index**. This is the number of 10s that you multiply together to get the final number. It is also the same as the number of zeros that follow the digit 1.

Look at this pattern of numbers:

$10 = 10^1$	10 is ten to the power of 1 or simply 10.
$100 = 10 \times 10 = 10^2$	100 is ten to the power of 2 or '10 squared'.
$1000 = 10 \times 10 \times 10 = 10^3$	1000 is ten to the power of 3 or '10 cubed'.
$10\,000 = 10 \times 10 \times 10 \times 10 = 10^4$	10 000 is ten to the power of 4.

Tip

As this pattern continues, the numbers get bigger and bigger.

Worked example 3.1

a Write 10^6 in:

i numbers **ii** words

b Write the number 100 000 as a power of 10.

Continued

Answer

a **i** 1 000 000	$10^6 = 10 \times 10 \times 10 \times 10 \times 10 \times 10$ You write this as a 1 followed by six zeros.
ii one million	1 000 000 in words is 'one million'.
b 10^5	There are five zeros after the 1 in 100 000, so $100\,000 = 10^5$.

When you multiply a number by a power of 10, you move the digits in the number to the left in the **place value** table.

When you divide a number by a power of 10, you move the digits in the number to the right in the place value table.

Worked example 3.2

Work out the answer to each of the following.

a 23×10^4 **b** 3.581×10^6 **c** $45\,000 \div 10^5$

Answer

a $23 \times 10^4 = 23 \times 10\,000$ $= 230\,000$	10^4 is the same as 10 000. The digits 2 and 3 move four places to the left in the place value table. The four empty spaces are filled with zeros.
b $3.581 \times 10^6 = 3.581 \times 1\,000\,000$ $= 3\,581\,000$	10^6 is the same as 1 000 000. The digits 3, 5, 8 and 1 move six places to the left in the place value table. The three empty spaces are filled with zeros.
c $45\,000 \div 10^5 = 45\,000 \div 100\,000$ $= 0.45$	10^5 is the same as 100 000. The digits 4 and 5 move five places to the right in the place value table. The one empty space before the decimal point is filled with a zero.

Exercise 3.1

1 Write the following in:

i numbers ii words

a 10^3 b 10^5 c 10^7 d 10^1

2 Write each number as a power of 10.

a 100 b 100 000 000 c 10 000 d 10 000 000 000

3 Copy and complete the working for each of these.

a $3 \times 10^4 = 3 \times 10\,000$
$= \square$

b $5 \times 10^6 = 5 \times 1\,000\,000$
$= \square$

c $45 \times 10^5 = 45 \times 100\,000$
$= \square$

d $291 \times 10^3 = 291 \times 1000$
$= \square$

4 Marcus says: 'When I multiply a whole number by 10^4, I just have to add four zeros to the end of the number. When I multiply a whole number by 10^5, I just have to add five zeros to the end of the number. This works for any positive whole number power.'

Is Marcus correct? Discuss in pairs or groups.

5 Work out:

a 23×10^2 b 768×10^4 c 9×10^6

6 Copy and complete the working for each of these.

a $4.2 \times 10^2 = 4.2 \times 100$
$= \square$

b $6.5 \times 10^4 = 6.5 \times 10\,000$
$= \square$

c $12.7 \times 10^3 = 12.7 \times 1000$
$= \square$

d $2.87 \times 10^6 = 2.87 \times 1\,000\,000$
$= \square$

Think like a mathematician

7 Sofia and Maya use different methods to work out 4.56×10^5.
This is what they write:

Sofia

Question
4.56×10^5

Answer
There are five zeros in 10^5, so move the decimal point five places to the right.

= 4 . 5 . 6 . . . .

There are three empty spaces, so fill them with zeros.
So, $4.56 \times 10^5 = 456\,000$.

Maya

Question
4.56×10^5

Answer
The number 4.56 has two decimal places.
The number 10^5 has five zeros.
$5 - 2 = 3$
This means I must add three zeros to the end of the number and cross out the decimal point.
So, $4.56 \times 10^5 = 456\,000$.

a Do you understand how both methods work?
b Critique the two methods. Explain the advantages and disadvantages of each method.
c Which method do you prefer?
d Why doesn't Marcus' method work for this type of question?
Discuss in pairs or groups.

8 Work out:
a 4.7×10^4 b 91.5×10^3 c 0.33×10^7

9 Copy these calculations and fill in the missing numbers.
a $1.5 \times 10^3 = \square$ b $32.1 \times \square = 3210$
c $\square \times 10^5 = 612\,000$ d $124.63 \times 10^{\square} = 124\,630\,000$

10 Copy and complete the working for each of the following.
a $8000 \div 10^3 = 8000 \div 1000$
$= \square$
b $80\,500\,000 \div 10^5 = 80\,500\,000 \div 100\,000$
$= \square$

11 Marcus says: 'When I divide a whole number by 10^4, I just have to cross four zeros off the end of the number. When I divide a whole number by 10^5, I just have to cross five zeros off the end of the number. This works for any positive whole number power.'

Is Marcus correct? Discuss in pairs or groups.

12 Work out:

a $80\,000 \div 10^4$ **b** $510\,000 \div 10^3$ **c** $8\,460\,000\,000 \div 10^5$

Think like a mathematician

13 Sofia and Maya use different methods to work out $2\,350\,000 \div 10^5$.

This is what they write:

Sofia

Question

$2\,350\,000 \div 10^5$

Answer

There are five zeros in 10^5, so move the decimal point five places to the left.

= 23 . 5 . 0 . 0 . 0 . 0 .

The decimal point stops between the 3 and the 5.

So, $2\,350\,000 \div 10^5 = 23.5$.

Maya

Question

$2\,350\,000 \div 10^5$

Answer

The number 2 350 000 has four zeros.

The number 10^5 has five zeros.

$5 - 4 = 1$, so there must be one digit (non-zero) after the decimal point.

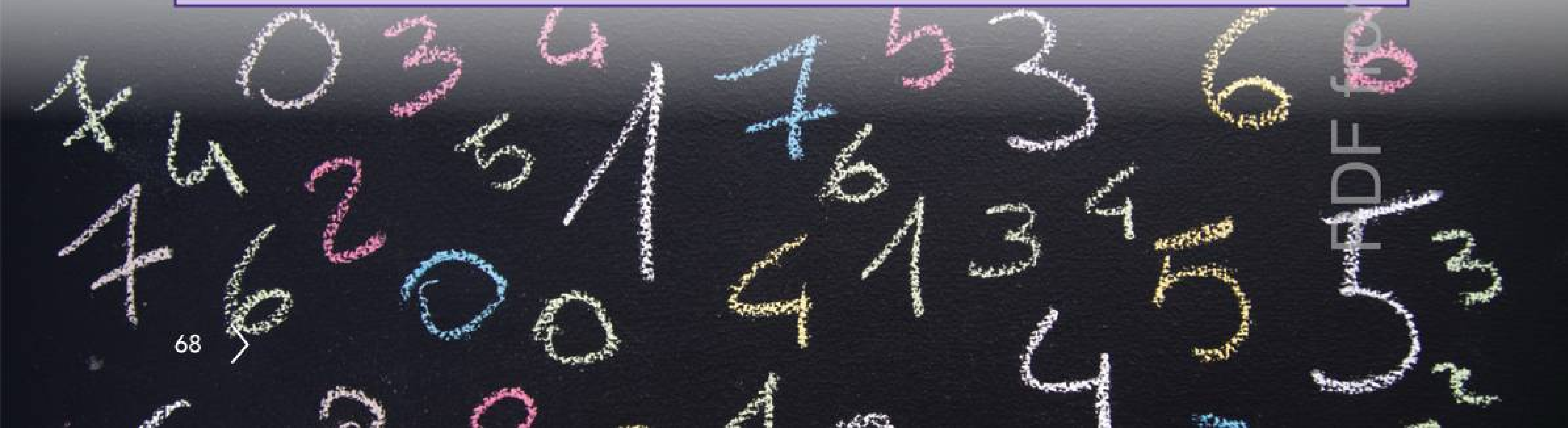
So $2\,350\,000 \div 10^5 = 23.5$.

a Do you understand how both methods work?

b Critique the two methods by explaining the advantages and disadvantages of each method.

c Which method do you prefer?

d Can you think of a better method to use for this type of question? Discuss in pairs or groups.



14 Work out:

a	$230 \div 10$	**b**	$230 \div 10^2$	**c**	$230 \div 10^3$	**d**	$230 \div 10^4$
e	$65 \div 10$	**f**	$65 \div 10^2$	**g**	$65 \div 10^3$	**h**	$65 \div 10^4$
i	$9 \div 10$	**j**	$9 \div 10^2$	**k**	$9 \div 10^3$	**l**	$9 \div 10^4$

15 Choose the correct answer, **A**, **B** or **C**, for each of the following.

a	$670\,000 \div 10^5$	**A**	670	**B**	6.7	**C**	0.67
b	$9520 \div 10^4$	**A**	0.952	**B**	9.52	**C**	0.0952
c	$18\,500\,000 \div 10^6$	**A**	185	**B**	1.85	**C**	18.5

16 These formulae show how to convert between some metric units of length.

number of mm = number of cm × 10

number of mm = number of m × 10^3

number of mm = number of km × 10^6

Tip

Look back at Section 2.2 for a reminder on how to use formulae.

Use the formulae to work out the missing numbers in these conversions.

a	☐ mm = 8 cm	**b**	☐ mm = 15 cm
c	☐ mm = 7 m	**d**	☐ mm = 3.4 m
e	☐ mm = 9 km	**f**	☐ mm = 0.6 km
g	☐ mm = 12.4 cm	**h**	☐ mm = 32.25 km

17 **a** Convert 8 000 000 mm to km.

b Write a formula that will convert a length in mm to a length in km.

c Use your formula from part **b** to work out the missing numbers in these conversions.

i ☐ km = 90 000 000 mm

ii ☐ km = 15 600 000 mm

iii ☐ km = 770 000 mm

Tip

Start by converting mm to cm, then cm to m, and finally m to km.

18 **a** Classify these number cards into groups of the same value. There should be one card left over.

7.8×10^3 | $7\,800\,000\,000 \div 10^7$ | 0.0078×10^4

780×10 | $780 \div 10$ | $78\,000\,000 \div 10^4$ | 78×10

$0.000\,78 \times 10^6$ | $780 \div 10^2$ | $78\,000 \div 10^3$

b Write two new cards that have the same value as the card that is left over.

Explain to a partner the methods you would use to work out the answers to these questions.

a 0.39×10^5 **b** $36\,800\,000 \div 10^6$

Explain why you like to use these methods.

Does your partner use the same methods?

If your partner uses different methods, do you understand their methods?

Summary checklist

- [ ] I can multiply and divide whole numbers by powers of 10.
- [ ] I can multiply and divide decimal numbers by powers of 10.

3.2 Rounding

In this section you will ...

- round numbers to a given number of decimal places.

Key words

round

degree of accuracy

When you are asked to round a number, you will be told how accurate your answer should be. This is called the **degree of accuracy**.

For any degree of accuracy, the method is always the same.

Look at the digit in the position of the required degree of accuracy.

What you do to this digit depends on the value of the digit to the right of it.

- If the value of the digit to its right is 5 or more, increase the original digit by 1.
- If the value of the digit to the right is less than 5, leave the original digit as it is.

Worked example 3.3

Round the number 5.376 398 to the given degree of accuracy.

a one decimal place **b** three decimal places **c** five decimal places

Answer

a 5.376 398 = 5.4 (to 1 d.p.)

The digit in the <u>first</u> decimal place is 3.
The digit to the right of the 3 is 7.
7 is more than 5, so round the 3 up to 4.
The letters 'd.p.' stand for 'decimal place'.

b 5.376 398 = 5.376 (to 3 d.p.)

The digit in the <u>third</u> decimal place is 6.
The digit to the right of the 6 is 3.
3 is less than 5, so the 6 stays the same.

c 5.376 398 = 5.376 40 (to 5 d.p.)

The digit in the <u>fifth</u> decimal place is 9.
The digit to the right of the 9 is 8.
8 is more than 5, so round the 9 up to 10.
This has the effect of rounding the fourth and fifth digits (i.e. 39) up to 40. Notice that you <u>must</u> write down the zero at the end, as the number <u>must</u> have five decimal places.

Exercise 3.2

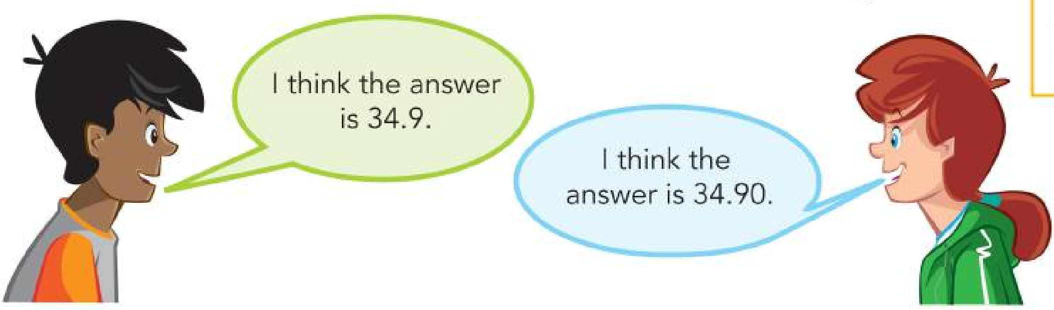
Tip

Remember that all of your answers in question **1** must have exactly two digits after the decimal point.

1 Round each of these numbers to two decimal places (2 d.p.). The first one has been done for you.

a 5.673 = 5.67 (to 2 d.p.) **b** 8.421 **c** 39.555
d 0.487 **e** 138.2229 **f** 0.069 01

2 Arun and Sofia round the number 34.8972 to two decimal places.



a Who is correct? Explain how they got this answer.
b Explain the mistake that the other learner has made.

Think like a mathematician

3 A pedometer measures the distance that you walk.
Liam has a pedometer. It shows that he has walked 9.55 km.
This distance is given to two decimal places.
What distances might Liam have actually walked?
Discuss in pairs or in groups.

4 Round each of these numbers to three decimal places (3 d.p.).

a 12.8943 b 127.99652 c 0.20053 d 9.349612

5 Fina explains her method to round 17.825684 to four decimal places (4 d.p.).

First, I draw a line after the digit in the fourth decimal place.	17.8256|84
Then I circle the digit in the fifth decimal place.	17.8256|(8)4
The digit I have circled is 8, so I must increase the digit before the line by 1, so the 6 becomes a 7.	17.8257 (4 d.p.)

a Do you like this method that Fina uses?
b What are the advantages and disadvantages of this method?
c Can you think of a better/easier method to use? If you can, then write down an explanation of your method.
d Explain how you would use Fina's method to round another number to six decimal places.

6 Choose the correct answer, **A**, **B** or **C**, for each of the following.
Round each of these numbers to four decimal places (4 d.p.).

a 5.662198 A 5.6621 B 5.6622 C 5.6623
b 197.020549 A 197.0206 B 197.0215 C 197.0205
c 0.0089732 A 0.0090 B 0.009 C 0.0089

7 Round each of these numbers to the given degree of accuracy.

a 126.99231 (4 d.p.) b 0.7785 (1 d.p.) c 782.02972 (3 d.p.)
d 3.141592654 (7 d.p.) e 3.9975 (2 d.p.) f 99.9961 (1 d.p.)

8 Zara is looking at this question:

Match the original number on the left to the rounded number in the middle, to the degree of accuracy on the right. The first one has been done for you.

Original number	Rounded number	Degree of accuracy
A 32.7819045	**a** 32.873	**i** 1 d.p
B 32.8729045	**b** 32.789105	**ii** 2 d.p
C 32.7189045	**c** 32.7819	**iii** 3 d.p
D 32.7891045	**d** 32.81790	**iv** 4 d.p
E 32.8792045	**e** 32.7	**v** 5 d.p
F 32.8179045	**f** 32.88	**vi** 6 d.p

Zara makes the following statement.

> The method I am going to use is to start with an original number and round it to 1 d.p. If my answer is in the rounded number list, I will join it with a line. If not, I'll round it to 2 d.p. and see if this number is in the list. If not, I'll continue rounding to more decimal places until I find the answer.

a Use Zara's method to answer the question.

b Critique Zara's method by explaining the advantages and disadvantages of her method.

c Can you improve her method or suggest a better method?

9 Work out the answers to these questions on a calculator.

Round each of your answers to the given degree of accuracy.

a $9 \div 7$ (2 d.p.)

b $4 + \frac{8}{15}$ (4 d.p.)

c $6 - \sqrt{22}$ (3 d.p.)

Activity 3.1

You will write a question for a partner to answer.

On a piece of paper write down a question of your own that is similar to Question **7**.

Make sure your question has parts **a** to **f** and that each part uses six different numbers.

Also, make sure your question asks for the numbers to be rounded to different degrees of accuracy.

Before you ask your partner to answer your question, write the answers on a different piece of paper.

Swap questions with your partner. Answer their question, then swap back and mark each other's work. Discuss any mistakes that have been made.

10 Sofia, Marcus and Arun go out for lunch. The total bill for lunch is $46.48.
They decide to share the bill equally between the three of them.
They use a calculator to work out how much each of them should pay. This is what they say:

- **a** Work out how much Sofia, Marcus and Arun think they should each pay.
- **b** Who do you think has made the best rounding decision? Explain your answer.
- **c** Can you think of a better way to round the answer to help decide how much they should each pay?

Summary checklist

☐ I can round numbers to a given number of decimal places.

Check your progress

1 Write 10^4 in:

a numbers   b words

2 Write both of these numbers as a power of 10.

a 1000   b 1 000 000

3 Work out:

a 4×10^4 b 12×10^6

c 8.9×10^5 d 4.66×10^3

4 Work out:

a $7000 \div 10^3$ b $3\,400\,000 \div 10^4$

c $140 \div 10^2$ d $31\,200 \div 10^5$

5 Round each of these numbers to the given degree of accuracy.

a 78.931 (2 decimal places)   b 0.667 39 (4 decimal places)

c 154.829 09 (3 decimal places)   d 6.505 049 93 (6 decimal places)

4 Decimals

Getting started

1 Write the correct inequality, < or >, between each pair of decimal numbers.

The first one has been done for you.

a $4.2 > 3.9$ **b** 6.4 □ 8.1 **c** 12.5 □ 11.9

d 3.7 □ 3.2 **e** 0.5 □ 0.4 **f** 32.7 □ 32.8

2 Here are four decimal number cards.

15.6	15.3	15.9	15.0

Write the numbers in increasing value, from smallest to largest.

3 Write T (true) or F (false) for each of the following.

a $4.32 > 5.12$ **b** $8.91 < 9.91$ **c** $12.03 > 11.95$

d $1.45 < 1.35$ **e** $0.72 > 0.79$ **f** $10.85 < 10.99$

4 Work out the following, showing your working.

a $4.56 + 8.35$ **b** $9.7 + 4.48$

c $16.78 - 14.93$ **d** $7.87 - 3.9$

5 Match each yellow question card with its correct blue answer card.

Yellow question cards: 5×5.42 | 8×3.3 | 4×6.9 | 12×2.4 | 6×4.25

Blue answer cards: 27.1 | 25.5 | 26.4 | 27.6 | 28.8

6 Work out:

a $6.3 \div 3$ **b** $4.9 \div 7$

c $18.66 \div 6$ **d** $8.25 \div 3$

Tip

Remember that:

< means 'less than'

\> means 'greater than'.

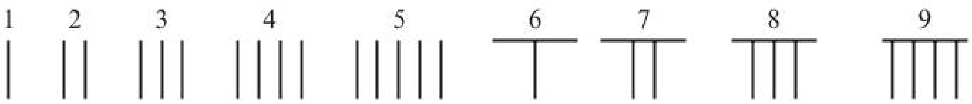
Tip

Set out these calculations with the numbers aligned under each other. So, for part **a**, write the calculation as:

$$\begin{array}{r} 4.65 \\ +\ 8.35 \\ \hline \end{array}$$

The decimal system is a number system based on 10. You can write all the numbers using just the ten digits: 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9.

The world's earliest decimal system used lines to represent numbers, so their digits 1 to 9 looked similar to this:



Before the symbol for zero (0) was invented, people used a blank space to represent zero.

Many countries in the world use a decimal system for their currency, whereby each unit of currency is based on a multiple of 10.

For example:

USA, 1 dollar = 100 cents ($1 = 100 c)
UK, 1 pound = 100 pence (£1 = 100 p)
Europe, 1 euro = 100 cents (€1 = 100 c)

Gambia, 1 dalasi = 100 bututs
China, 1 yuan = 100 fen
Thailand, 1 baht = 100 satang

When you travel to different countries, you need to use different currencies. It is easier to understand new currencies if they are based, like that of your own country's, on the decimal system.

4.1 Ordering decimals

In this section you will …

- compare and order decimals.

Key words

compare
decimal number
fourth
hundredths
order
tenths
whole-number part

To order decimal numbers, you write them from the smallest to the largest.

Different whole-number parts

First, compare the **whole-number part** of the numbers.

Consider these three decimal numbers: 8.9, 14.639, 6.45

If you highlight just the whole-number parts, you get: 8.9, 14.639, 6.45

You can see that 14 is the biggest of the whole numbers and 6 is the smallest.

So, in **order of size**, the numbers are: 6.45, 8.9, 14.639

Same whole-number parts

When you have to put in order numbers with the same whole-number part, you must first compare the **tenths**, then the **hundredths**, and so on.

Consider these three decimal numbers: 2.82, 2.6, 2.816

They all have the same whole number of 2. 2.82, 2.6, 2.816

If you highlight just the tenths, you get: 2.82, 2.6, 2.816

You can see that 2.6 is the smallest, but the other two numbers both have 8 tenths, so highlight the hundredths. 2.6, 2.82, 2.816

You can see that 2.816 is smaller than 2.82.

So, in order of size, the numbers are: 2.6, 2.816, 2.82

Tip

Write the 2.6 at the start because you know it is the smallest number.

Worked example 4.1

For each set, write the decimal numbers in order of size.

a 6.8, 4.23, 7.811, 0.77 **b** 4.66, 4.6, 4.08

Continued

Answer

a 0.77, 4.23, 6.8, 7.811

All these numbers have a different whole-number part, so you don't need to compare the **decimal part**. Simply write the numbers in order of their whole-number parts, which are 0, 4, 6 and 7.

b 4.08, 4.6, 4.66

All these numbers have the same whole-number part, that is, 4. Start by comparing the tenths. 4.08 comes first as it has the smallest number of tenths (0 tenths). 4.6 and 4.66 have the same number of tenths (6 tenths), so compare the hundredths. 4.6 is the same as 4.60, so it has 0 hundredths. 4.6 comes before 4.66, which has 6 hundredths.

Exercise 4.1

1 For each pair, write down which is the smallest decimal number.

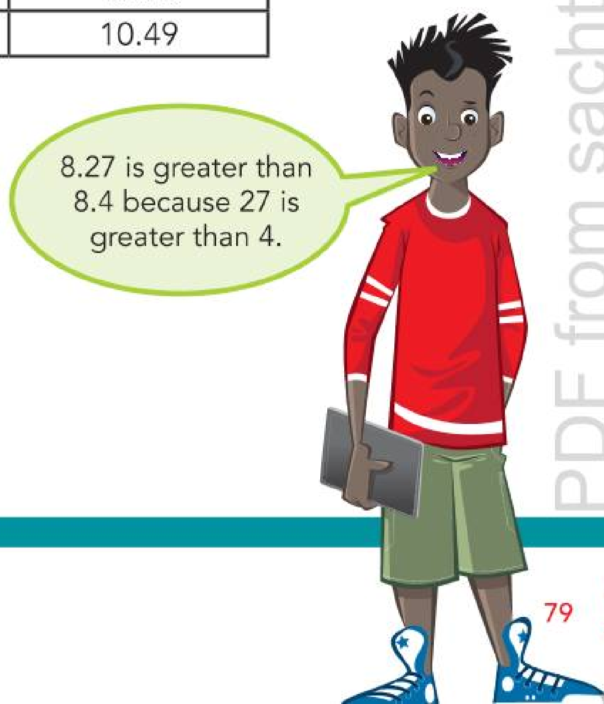
a 13.5, 9.99 **b** 4.32, 3.67 **c** 12.56, 21.652
d 127.06, 246.9 **e** 0.67, 0.72 **f** 3.4, 3.21
g 18.54, 18.45 **h** 0.05, 0.043 **i** 0.09, 0.1

2 The table shows six of the fastest times run by women in the 100 m race.

Name	Country	Year	Time (seconds)
Kerron Stewart	Jamaica	2009	10.75
Marion Jones	USA	1998	10.65
Merlene Ottey	Jamaica	1996	10.74
Carmelita Jeter	USA	2009	10.64
Shelly-Ann Fraser	Jamaica	2009	10.73
Florence Griffith-Joyner	USA	1988	10.49

a Write the times in order of size.
b Which woman has the **fourth** fastest time?

3 Marcus is comparing the numbers 8.27 and 8.4.
Is Marcus correct? Explain your answer.



4 Greg uses the inequalities < and > to show that one number is smaller than or is larger than another number.

> *4.07 is smaller then 4.15, so 4.07 < 4.15.*
> *2.167 is bigger than 2.163, so 2.167 > 2.163.*

Write the correct sign, < or >, between each pair of numbers.

a	6.03 □ 6.24	**b**	9.35 □ 9.41
c	0.49 □ 0.51	**d**	18.05 □ 18.02
e	9.2 □ 9.01	**f**	2.19 □ 2.205
g	0.072 □ 0.06	**h**	29.882 □ 29.88

Tip

The symbol < means 'is smaller than'.

The symbol > means 'is bigger than'.

5 Lin uses the symbol = to show that one number is equal to another. She uses the symbol ≠ to show that one number is not equal to another.

> *8.9 has the same value as 8.90, so 8.9 = 8.90.*
> *0.183 does not have the same value as 0.18, so 0.183 ≠ 0.18.*

Write the correct sign, = or ≠, between each pair of numbers.

a	4.2 □ 4.20	**b**	3.75 □ 3.57
c	0.340 □ 0.304	**d**	9.58 □ 9.580
e	128.00 □ 128	**f**	0.0034 □ 0.034

Tip

The symbol ≠ means 'is not equal to'.

6 Ulrika uses a different method to order decimals. Her method is shown here.

> *Question*
> *Write the decimal numbers 4.23, 4.6 and 4.179 in order of size.*
> *Solution*
> *4.179 has the most decimal places, so give all the other numbers three decimal places by adding zeros at the end:*
> *4.230, 4.600, 4.179*
> *Now compare 230, 600 and 179: 179 is the smallest, then 230, and 600 is the biggest.*
> *In order of size, the numbers are: 4.179, 4.23, 4.6*

For each set, use Ulrika's method to write the decimal numbers in order of size.

a 2.7, 2.15, 2.009
b 3.45, 3.342, 3.2
c 17.05, 17.1, 17.125, 17.42
d 0.71, 0.52, 0.77, 0.59
e 5.212, 5.2, 5.219, 5.199
f 9.08, 9.7, 9.901, 9.03, 9.99
g Critique Ulrika's method by explaining the advantages and disadvantages of her method. Can you improve Ulrika's method?

In pairs or groups, discuss the different methods you can use to order decimals.

Look back at the methods used in the worked example and also in Question **6**.

Individually, decide which is your favourite method and explain to your partner or group why you prefer this method.

7 Write these amounts in order of size.
Use the tip box to help you.

a 38.1 cL, 300 mL, 0.385 L
b 725 mm, 7.3 cm, 0.705 m
c 5.12 kg, 530 g, 0.0058 t, 519 000 mg
d 461.5 cm, 0.0046 km, 0.45 m, 4450 mm

Tip

Remember:
10 mm = 1 cm
100 cm = 1 m
1000 m = 1 km
10 mL = 1 cL
100 cL = 1 L
1000 mL = 1 L
1000 mg = 1 g
1000 g = 1 kg
1000 kg = 1 t

8 Brad puts these decimal number cards in order of size.
There is a mark covering part of the number on the middle card.

3.07	3…	3.083

a Write down three possible numbers that could be on the middle card.
b How many different numbers with three decimal places do you think could be on the middle card?
c Show how you can convince others that your answer to part **b** is correct.

Summary checklist

☐ I can compare and order decimal numbers.

4.2 Adding and subtracting decimals

In this section you will …

- add and subtract decimals.

Key words

decimal part

mentally/using a mental method

written method

When you add and subtract decimal numbers **mentally**, there are different methods you can use.

- When you are adding, you can separate the numbers into their whole-number part and their decimal part. You can then separately add the whole-number parts and add the decimal parts, and finally add the whole-number answer to the decimal answer.
- When you are subtracting, you can separate the number you are subtracting into its whole-number part and its decimal part. Subtract the whole-number part first and then subtract the decimal part second.
- If one of the numbers you are adding or subtracting is close to a whole number, you can round it to the nearest whole number, do the addition or subtraction, then adjust your answer at the end.

Worked example 4.2

Work these out mentally.

a $2.3 + 7.8$ **b** $6.9 + 12.4$ **c** $13.3 - 5.8$

Answer

a $2.3 + 7.8 = 2 + 7 + 0.3 + 0.8$	Separate the numbers into whole-number parts and decimal parts.
$= 9 + 1.1$	Separately, add the whole-number parts and add the decimal parts.
$= 10.1$	Add the whole-number answer to the decimal answer.
b $6.9 + 12.4 = 7 + 12.4 - 0.1$	Round 6.9 up to 7.
$= 19.4 - 0.1$	Add 7 to 12.4.
$= 19.3$	Subtract 0.1.
c $13.3 - 5.8 = 13.3 - 6 + 0.2$	Round 5.8 up to 6.
$= 7.3 + 0.2$	Subtract 6 from 13.3.
$= 7.5$	Add 0.2.

When you use a **written method** to add and subtract decimal numbers, always write the calculation in columns, with the decimal points vertically in line. Then add and subtract as normal, but remember to write the decimal point in your answer.

Worked example 4.3

Work out:

a $27.52 + 4.8$ **b** $43.6 - 5.45$

Answer

a

```
   2 7 . 5 2
 +   4 . 8 0
 -----------
   3 2 . 3 2
 -----------
   1 1
```

First write 4.8 as 4.80. Start with the hundredths column: $2+0=2$

Next add the tenths: $5+8=13$. Write down the 3 and carry the 1.

Now add the units: $7+4+1=12$. Write down the 2 and carry the 1.

Finally, add the tens: $2+1=3$

b

```
  3      1     5   1
   4̸    3  .   6̸   0
 -       5  .   4   5
 --------------------
   3     8  .   1   5
 --------------------
```

First, write 43.6 as 43.60.

Start with the hundredths column. You can't take 5 from 0 $(0-5)$, so borrow from the 6-tenths, then work out $10-5=5$.

Now subtract the tenths: $5-4=1$

Now subtract the units. You can't take 5 from 3 $(3-5)$, so borrow from the 4 tens, then work out $13-5=8$.

Finally, subtract the tens: $3-0=3$

Exercise 4.2

1 Use a mental method to work out the answers to the following.

a $3.5+4.2$ **b** $12.7+4.5$

c $4.9-1.5$ **d** $14.6-6.6$

Tip

Separate the numbers into their whole-number parts and their decimal parts.

Think like a mathematician

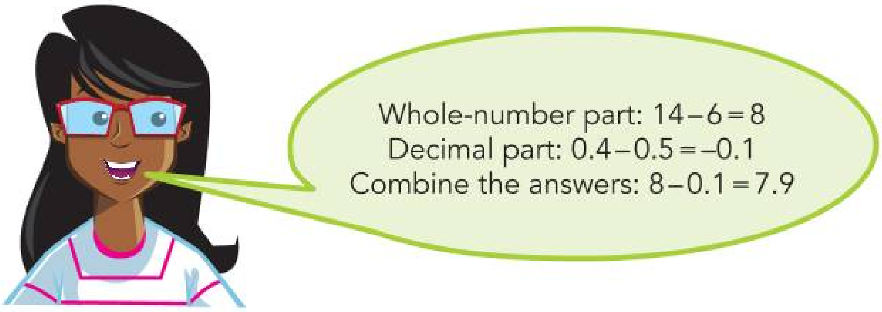
2 What is the easiest way to subtract a decimal from the number 1?

For example, $1-0.3$, $1-0.25$, $1-0.405$ or $1-0.6839$?

3 Match each green question card with its correct pink answer card.

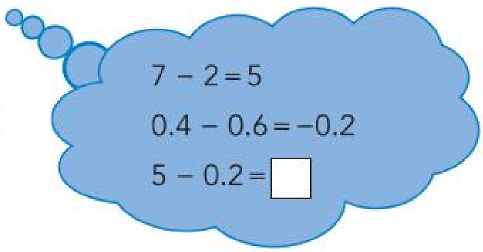
$1-0.36$	$1-0.78$	$1-0.44$	0.716	0.64	0.56
$1-0.284$	$1-0.432$		0.22	0.568	

4 Zara mentally works out $14.4-6.5$ like this:



Use Zara's method to mentally work out the following.
The first one has been started for you.

- **a** $7.4-2.6$
- **b** $8.3-2.9$
- **c** $12.5-9.8$
- **d** $15.1-5.7$

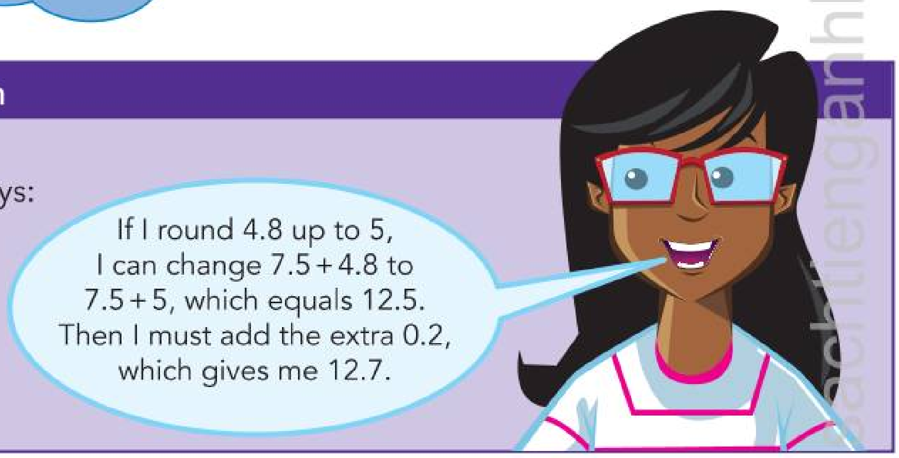


Think like a mathematician

5 Zara works out $7.5+4.8$ mentally like this. She says:

Is Zara correct?

Discuss in pairs or small groups.



6 Mentally work out the following.
Use the method of rounding one of the numbers to a whole number.
Two of them have been started for you.

$4.3 + 8 = 12.3$
$12.3 - 0.1 = \square$

a $4.3 + 7.9$ **b** $8.9 + 9.6$

c $22.8 + 3.3$ **d** $5.4 - 1.9$

$5.4 - 2 = 3.2$
$3.2 + 0.1 = \square$

e $14.9 - 4.4$ **f** $21.1 - 6.7$

7 Use a written method to work out the following.

a
```
  2 5 . 8 1
+   8 . 4
-----------

-----------
```

b
```
  8 . 7 6
- 4 . 1
---------

---------
```

c
```
  3 8 . 9 1
-   9 . 7 8
-----------

-----------
```

8 Bo records his mass at the start and end of every month.
Here are his records for June and July.

	Mass (kg)		Mass (kg)
Start of June	95.45	End of June	91.92
Start of July	91.92	End of July	88.35

a During which month, June or July, did Bo's mass decrease the most?

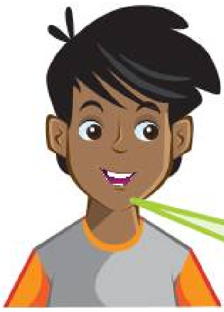
b At the start of August Bo's mass was 88.35 kg. During August his mass decreased by 1.82 kg. What was Bo's mass at the end of August?

9 Use a written method to work out the following.

a $4.76 - -12.52$ **b** $32.6 - -0.742$

Tip
Subtracting a negative is the same as adding.

10 Marcus and Arun use different methods to work out $10 - 4.83$.



- **a** Critique both methods by explaining the advantages and disadvantages of each of their methods.
- **b** Can you improve on their methods?
- **c** Which method do you prefer to use to work out subtractions like these? Explain why you prefer this method.

11 Work out:

a $10 - 6.42$ **b** $20 - 12.83$ **c** $30 - 4.55$ **d** $40 - 16.782$

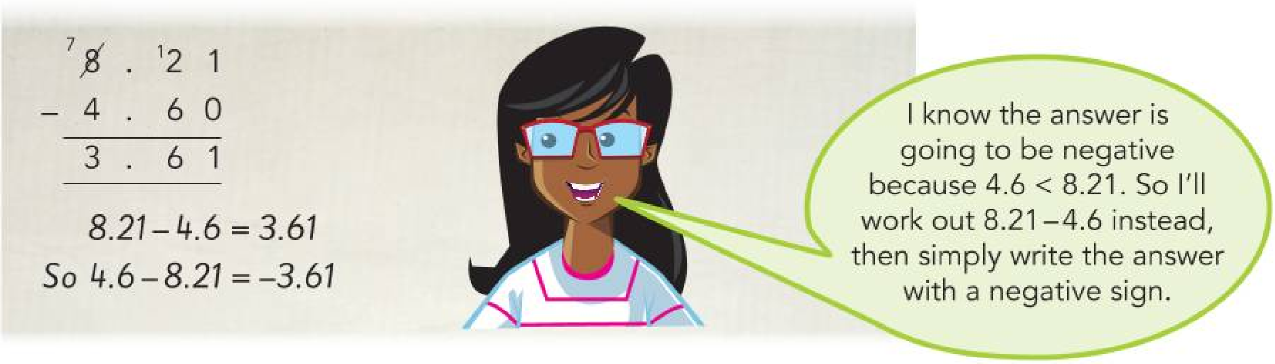
12 At the cinema, Priya spends \$4.75 on a ticket, \$1.75 on sweets and \$0.85 on a drink.

- **a** How much does she spend in total?
- **b** Priya pays with a \$10 note. How much change will she receive?

13 Jed works as a plumber. He has four lengths of pipe that measure 1.8 m, 3.5 m, 2.45 m and 0.85 m.

- **a** What is the total length of the four pipes?
- **b** Jed needs 10 m of pipe in total to finish a job. How much more pipe must he buy?

14 This is how Zara works out $4.6 - 8.21$.



Use Zara's method to work out the following.

a $5.43 - 9.57$ **b** $8.12 - 15.4$

c $-13.8 + 7.92$ **d** $6.582 - (4.5 + 5.061)$

Tip

For part **c**:

$-13.8 + 7.92$
$= 7.92 - 13.8$

15 Use a written method to work out the following.

a $-5.43 - 9.57$ **b** $-8.12 - 15.4$

c $-5.43 - -9.57$ **d** $-8.12 - -15.4$

Summary checklist

- [ ] I can add numbers with different numbers of decimal places.
- [ ] I can subtract numbers with different numbers of decimal places.

4.3 Multiplying decimals

In this section you will …

- multiply decimals by whole numbers.

Key word

fill in

Follow these steps when you multiply a decimal by a whole number:

- First, work out the multiplication without the decimal point.
- Finally, put the decimal point in the answer. There must be the same number of digits after the decimal point in the answer as there were in the question.

Worked example 4.4

a Work these out mentally.

i 0.002×4 **ii** 30×0.06

b Use a written method to work out 476×3.7.

Answer

a **i** $2 \times 4 = 8$ — Do the multiplication without the decimal point.

$0.002 \times 4 = 0.008$ — Put the decimal point back in the answer. There are three digits after the decimal point in the question, so there must be three digits after the decimal point in the answer.

ii $30 \times 6 = 180$ — Do the multiplication without the decimal point.

$30 \times 0.06 = 1.80$ — Put the decimal point back in the answer. There are two digits after the decimal point in the question, so there must be two digits after the decimal point in the answer.

$30 \times 0.06 = 1.8$ — You can write 1.80 as 1.8 because the zero is not needed.

b

$$\begin{array}{r} 476 \\ \times \quad 37 \\ \hline 3332 \\ +\ 14280 \\ \hline 17612 \\ \hline \end{array}$$

Do the multiplication without the decimal point.
Use your favourite method for multiplication.
Work out 476×7.
Work out 476×30.
Add together the answers to the multiplications.

$476 \times 3.7 = 1761.2$ — Put the decimal point back into the answer. There is one digit after the decimal point in the question, so there must be one digit after the decimal point in the answer.

Exercise 4.3

1 Use a mental method to work out the following.

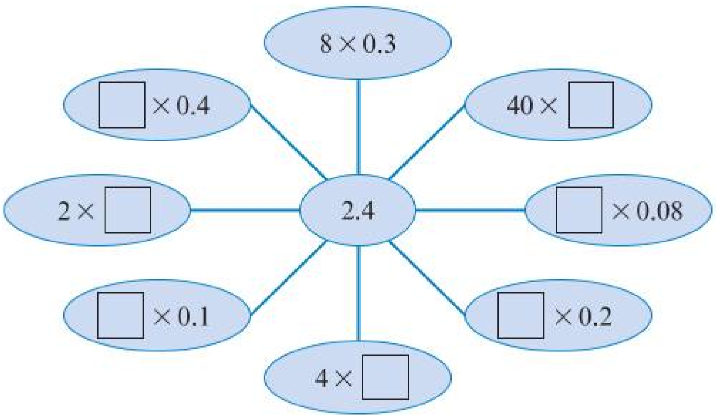
a 0.1×8 **b** 0.5×5 **c** 0.9×2

2 Use a mental method to work out the following. All the answers are shown in the cloud.

a 6×0.02 **b** 4×0.3 **c** 3×0.004 **d** 120×0.1

12 0.12
1.2 0.012

3 Copy this diagram and fill in the missing numbers.
All the calculations give the answer in the centre oval.
All the missing numbers are given in the rectangle.



30
0.6
12
6
1.2
0.06
24

4 Kai works out that $521 \times 53 = 27\,613$.
Use this information to write down the answer to the following.

a 521×5.3 **b** 521×0.53

6 **a** Work out 162×34.

b Use your answer to part **a** to write down the answers to the following.

i 162×3.4 **ii** 162×0.34 **iii** 162×0.034

iv 16.2×34 **v** 1.62×34 **vi** 0.162×34

Think like a mathematician

5 Look again at Question **4**.

a What is the answer to 52.1×53?
What do you notice about this answer and the answer to part **a** of Question **4**?

b Why is the answer to 521×0.53 the same as the answer to 5.21×53?

Activity 4.1

On a piece of paper, write a question of your own that is similar to Question 5.
On a different piece of paper, write down the answers to your question.

Swap questions with a partner and work out the answers to their question.

Swap your papers back and mark each other's work. Discuss any mistakes.

7 Raj uses these methods to work out and check his answer.

Question Work out 0.45×372.

Solution First, work out 45×372.

×	300	70	2
40	12 000	2800	80
5	1500	350	10
Total	13 500	3150	90

$$\begin{array}{r} 13500 \\ 3150 \\ +\quad 90 \\ \hline 16740 \end{array}$$

So $0.45 \times 372 = 167.40$
$= 167.4$

Check Round 0.45 to 0.5.
Round 372 to 400.
$0.5 \times 400 = 200$, which is close to 167.4.

a Write down the advantages and disadvantages of Raj's method.

b Can you improve his method?

Which method do you prefer to use to multiply decimals? Explain why you prefer this method.

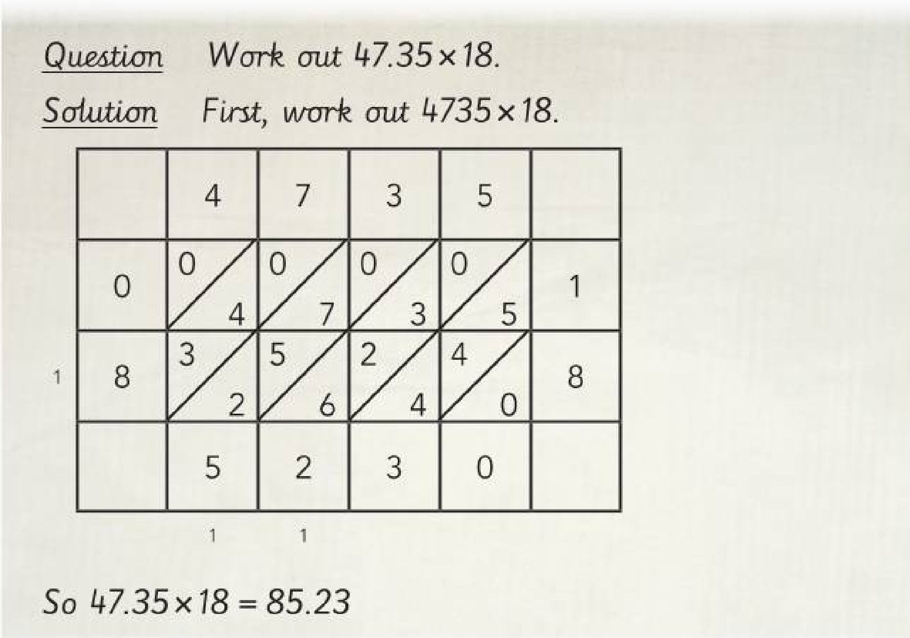
8 Work out these multiplications. Show how to check your answers.

a 3.2×52 **b** 8.1×384 **c** 0.78×41

Tip

For the check for part **c**, work out 0.8×40.

9 This is part of Anna's homework.



Question Work out 47.35 × 18.

Solution First, work out 4735 × 18.

	4	7	3	5	
0	0 / 4	0 / 7	0 / 3	0 / 5	1
1 8	3 / 2	5 / 6	2 / 4	4 / 0	8
	5	2	3	0	
	1	1			

So 47.35 × 18 = 85.23

a Without checking the method and working out the answer, how can you tell that Anna is incorrect?

b Work out the correct answer, showing all your working.

10 In 1 gram of green gold there is 0.23 g of copper.

How many grams of copper are there in 36 g of green gold?

11 Darren exchanges some British pounds (£) to US dollars ($). For every £1 he receives $1.29.

Darren says, 'If I exchange £275, I should receive about $350.'

Is Darren correct? Explain your answer.

12 Samir manages a hotel.

The table shows the cost of items that he buys for the hotel bathrooms.

Item	Cost (each)
bottle of shampoo	$0.26
bottle of shower gel	$0.23
bottle of hand lotion	$0.32
bar of soap	$0.18

Samir buys:

- 350 bottles of shampoo
- 425 bottles of shower gel
- 275 bottles of hand lotion
- 600 bars of soap.

What is the total cost of these items?

Summary checklist

☐ I can multiply decimals by whole numbers.

4.4 Dividing decimals

In this section you will …

- divide decimals by whole numbers.

Key words

estimation
inverse calculation
short division

Follow these steps when you divide a decimal by a whole number:

- Use **short division** (or the method that you prefer).
- Keep the decimal point in the question, and remember to write the decimal point in the answer above the decimal point in the question.

Worked example 4.5

Work out:

a $4.258 \div 2$ **b** $41.481 \div 18$

Answer

a

$$\begin{array}{r} 2 \\ 2\,\overline{)\,4.258} \end{array}$$

First, work out $4 \div 2 = 2$. Write 2 above the 4.

$$\begin{array}{r} 2. \\ 2\,\overline{)\,4.258} \end{array}$$

Now write the decimal point in the answer.

$$\begin{array}{r} 2.12 \\ 2\,\overline{)\,4.25{}^{1}8} \end{array}$$

Then continue the division: $2 \div 2 = 1$. Write 1 above the 2.

$5 \div 2 = 2$ r1. (Note: 'r' means 'remainder'.) Write 2 above the 5 and carry the 1 onto the 8.

$$\begin{array}{r} 2.129 \\ 2\,\overline{)\,4.25{}^{1}8} \end{array}$$

$18 \div 2 = 9$ exactly. Write 9 above the ${}^{1}8$.

Continued

b

$$\begin{array}{r} 2 \\ 18\,\overline{)\,41.{}^{5}481} \end{array}$$

First, work out $41 \div 18 = 2$ r5.

Write 2 above the 1. Carry the 5 onto the 4.

$$\begin{array}{r} 2. \\ 18\,\overline{)\,41.{}^{5}481} \end{array}$$

Now write the decimal point in the answer.

$$\begin{array}{r} 2.30 \\ 18\,\overline{)\,41.{}^{5}48{}^{8}1} \end{array}$$

Then continue the division: $54 \div 18 = 3$ exactly.
Write 3 above the ^{5}4.

$8 \div 18 = 0$ r8. Write the 0 above the 8 and carry the 8 onto the 1.

$$\begin{array}{r} 2.3045 \\ 18\,\overline{)\,41.{}^{5}48{}^{8}1{}^{9}0} \end{array}$$

$81 \div 18 = 4$ r 9. Write 4 above ^{8}1 and carry the 9 onto a new zero.

$90 \div 18 = 5$ exactly.

Exercise 4.4

1 Copy and complete these divisions.

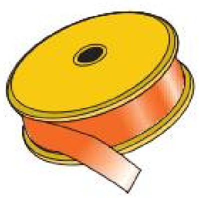
a $\begin{array}{r} 2.1 \\ 3\,\overline{)\,6.4{}^{1}14} \end{array}$ **b** $\begin{array}{r} 1.8 \\ 5\,\overline{)\,9.{}^{4}3{}^{3}85} \end{array}$ **c** $\begin{array}{r} 0.8 \\ 8\,\overline{)\,6.{}^{6}5{}^{1}28} \end{array}$

d $\begin{array}{r} 1. \\ 4\,\overline{)\,5.{}^{1}232} \end{array}$ **e** $\begin{array}{r} 1. \\ 7\,\overline{)\,7.644} \end{array}$ **f** $\begin{array}{r} 0. \\ 9\,\overline{)\,0.846} \end{array}$

2 Work out:

a $8.654 \div 2$ **b** $8.922 \div 6$ **c** $32.925 \div 5$ **d** $58.912 \div 8$

3 Maggie pays \$9.28 for 8 m of ribbon.
What is the cost of the ribbon, per metre?



4 In a supermarket, five chickens cost \$18.25. What is the cost of one chicken?

5 Six friends have a meal in a restaurant. The total bill is \$145.50.
They share the bill equally between them. How much do they each pay?

6 Copy and complete these divisions.

a $\begin{array}{r|l} & 2\ . \\ \hline 12 & 2\ 7\ .\ {}^{3}8\ 5\ 2 \end{array}$

b $\begin{array}{r|l} & 3\ . \\ \hline 15 & 4\ 6\ .\ {}^{1}8\ 7\ 5 \end{array}$

c $\begin{array}{r|l} & 3\quad . \\ \hline 25 & 7\ 8\ {}^{3}2\ .\ 8\ 2\ 5 \end{array}$

7 Lara works out 112.4 ÷ 16. She writes:

This is my 16 times table.

1	2	3	4	5	6	7	8	9
16	32	48	64	80	96	112	128	144

I can use the table to work out the division like this:

$\begin{array}{r|l} & 7\ .\quad 2\ 5 \\ \hline 16 & 1\ 1\ 2\ .\ 4\ {}^{4}0\ {}^{8}0 \end{array}$

So, 112.4 ÷ 16 = 7.25.

a Explain the mistake that Lara has made.

b Write down the correct answer.

8 Kyle works out 251.55 ÷ 26. He writes:

This is my 26 times table.

1	2	3	4	5	6	7	8	9
26	52	78	104	130	156	182	208	234

I can use the table to work out the division like this:

$\begin{array}{r|l} & 9\ .\ 6\quad 7\ \ r13 \\ \hline 26 & 2\ 5\ 1\ .\ {}^{17}5\ \ {}^{19}5 \end{array}$

So, 251.55 ÷ 26 = 9.67 remainder 13.

a Instead of stopping the division and writing 'remainder 13', what should Kyle have done?

b Work out the correct answer.

Think like a mathematician

9 What calculations could you do to check that the answer to a division is correct?
For example, how can you check that $56.322 \div 9 = 6.258$ is:

a approximately correct? **b** exactly correct?

Discuss in pairs or small groups.

10 **a** Copy and complete the table below, which shows the 14 times table.

1	2	3	4	5	6	7	8	9
14	28	42						

b Use the table to help you to work out $126.392 \div 14$.

c Show how to check that your answer to part **b** is correct.
Use **estimation** and an **inverse calculation**.

> **Tip**
> Your rounded answer to part **b** × 14 should equal approximately 126.

11 Work with a partner to answer this question.

a Mair works out that $235 \times 47 = 11\,045$.
Use this information to work out:

i $11\,045 \div 47$ **ii** $1104.5 \div 47$ **iii** $110.45 \div 47$ **iv** $11.045 \div 47$

b Explain the method you used to work out the answers to parts **a i**, **ii**, **iii** and **iv**.

c Use this method to work out the answers to the following.

i $1104.5 \div 235$ **ii** $110.45 \div 235$ **iii** $11.045 \div 235$

d Check your answers with those of other learners in your class to see if you agree.
If you disagree on any of the answers, discuss where any mistakes have been made.

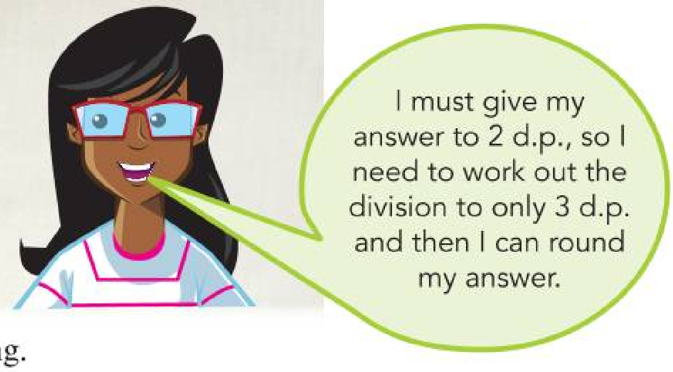
12 This is part of Zara's homework.

Question Work out $8.46 \div 7$.
Give your answer to 2 d.p.

Solution

$$\begin{array}{r|l} & 1\,.\,2\;0\;8\;\ldots \\ \hline 7 & 8\,.\,{}^{1}4\;6\;{}^{6}0\;{}^{4}0 \end{array}$$

$8.46 \div 7 = 1.208\ldots = 1.21$ (2 d.p.)



Use Zara's method to work out the following.
Round each of your answers to the required degree of accuracy.

a $7.62 \div 5$ (1 d.p.) **b** $9.428 \div 7$ (2 d.p.) **c** $8.6 \div 13$ (3 d.p.)

13 Copy and complete these divisions.

a
$$\begin{array}{r|l} & 3\,.\,\square\;8\;2 \\ \hline 2 & \square\,.\,{}^{1}9\;\square\;\square \end{array}$$

b
$$\begin{array}{r|l} & \square\,.\,5\;0\;7 \\ \hline 6 & 9\,.\,\square\;\square\;2 \end{array}$$

c
$$\begin{array}{r|l} & 1\,.\,\square\;\square\;9 \\ \hline \square & 8\,.\,{}^{3}4\;9\;\square \end{array}$$

Tip

You need to work out the division to only one decimal place more than the degree of accuracy you need.

Which questions did you find the easiest? Which questions did you find the hardest?

Are you confident in answering these types of questions?

What can you do to increase your confidence?

Summary checklist

- [ ] I can divide decimals by whole numbers.

4.5 Making decimal calculations easier

In this section you will …

- simplify calculations containing decimals.

Key words

equivalent fraction
place value
partitioning

When you are calculating using decimals, you can often make a calculation easier using a variety of methods, such as:

- using the place value of the decimal
- using the correct order of operations.

Worked example 4.6

Work out:

a 0.06×3500 **b** 8.2×9 **c** $12.56 \div 40$

Answer

a $0.06 = \frac{6}{100} = 6 \div 100$	In this method, you write the place value of the decimal as an **equivalent fraction** and then as a division.
$0.06 \times 3500 = 6 \div 100 \times 3500$	Rewrite the decimal as a division.
$= 6 \times 3500 \div 100$	You can do the × and ÷ in any order.
$= 6 \times 35$	Do 3500 ÷ 100 first, then 6 × 35.
$= 6 \times 30 + 6 \times 5$	Use **partitioning** to make the multiplication easier.
$= 180 + 30$	
$= 210$	
b $8.2 \times 9 = 8.2 \times (10 - 1)$	Swap the 9 for (10 − 1)
$= 8.2 \times 10 - 8.2 \times 1$	Separately, work out 8.2 × 10 and 8.2 × 1
$= 82 - 8.2$	Now subtract 8.2 from 82.
$= 82 - 8 - 0.2$	First subtract the 8
$= 74 - 0.2$	and then subtract the 0.2,
$= 73.8$	which gives an answer of 73.8.

Continued

c $\frac{12.56}{40} = \frac{12.56 \div 10}{40 \div 10}$

$= \frac{1.256}{4}$

$= 0.314$

Dividing by 40 isn't easy. Start by dividing the top and bottom of the fraction by 10.

Now you only have to divide the decimal by 4.

Use your favourite method for division. You should get a final answer of 0.314.

Exercise 4.5

1 Complete the workings to make the following calculations easier. Use the place value method.

a 0.7×180

$= \frac{7}{10} \times 180$

$= 7 \div 10 \times 180$

$= 7 \times 180 \div 10$

$= 7 \times \square$

$= 7 \times \square + 7 \times \square$

$= \square + \square$

$= \square$

b 0.04×7600

$= \frac{4}{100} \times 7600$

$= 4 \div 100 \times 7600$

$= 4 \times 7600 \div 100$

$= 4 \times \square$

$= 4 \times \square + 4 \times \square$

$= \square + \square$

$= \square$

Tip

Remember to use partitioning to make the whole number multiplication easier. For example, $7 \times 18 = 7 \times 10 + 7 \times 8$.

2 Work out the answers to the following. Use the same method as in Question **1**.

a 0.6×410 b 0.9×320 c 0.02×3200 d 0.08×5300

Think like a mathematician

3 Look at the answers to questions **1** and **2**.
Compare the questions to the final multiplication you had to do.
For example:

1a $0.7 \times 180 = 7 \times 18$ **2a** $0.6 \times 410 = 6 \times 41$

1b $0.04 \times 7600 = 4 \times 76$ **2c** $0.02 \times 3200 = 2 \times 32$

Can you see a pattern? Can you write a general rule that you could follow?
Explain how your rule works. Will it always work for any numbers?

4 Alek has \$3450 to invest. He decides to buy some gold, silver and precious stones.

The table shows the amount he will spend on each item.

Item	Amount
gold	$0.6 \times \$3450 = \$$______
silver	$0.3 \times \$3450 = \$$______
precious stones	$0.1 \times \$3450 = \$$______

a Copy and complete the table.

b Show how you can check that your answers are correct.

5 Complete the workings to make these calculations easier.

a 4.6×9
$= 4.6 \times (10 - 1)$
$= 4.6 \times 10 - 4.6 \times 1$
$= \square - \square$
$= \square$

b 7.3×9
$= 7.3 \times (10 - 1)$
$= 7.3 \times 10 - 7.3 \times 1$
$= \square - \square$
$= \square$

6 Work out the answers to the following. Use the same method as in Question **5**.

a 6.8×9 **b** 4.7×9 **c** 12.6×9

Think like a mathematician

7 Discuss this question in pairs or small groups.

This is part of Pedro's classwork.

Question Work out 7.6×8.

Solution Use: $7.6 = 7 + 0.6$
$7 \times 8 = 56$
$0.6 \times 8 = 4.8$
$56 + 4.8 = 60.8$

a Do you understand the method that Pedro has used?

b Do you think this is an easy or difficult method to use? Explain your answer.

Pedro has used this method to multiply a two-digit decimal (7.6) by a one-digit whole number (8).

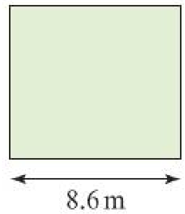
c Do you think it would be easy to use this method to answer questions such as 5.67×7 or 5.6×45? Explain your answer.

8 Use Pedro's method to work out the following.

a 4.2×6 **b** 7.8×5 **c** 6.3×8

9 A square has a side length of 8.6 m.
The formula to work out the perimeter of a square is:

$P = 4L$ where: P is the perimeter
L is the side length



Use the formula to work out the perimeter of the square.

Tip

Look back at Unit 2.2 for a reminder on how to use formulae.

10 Complete the workings to make these divisions easier. Then work out the answer.

a $14.55 \div 30 = \frac{14.55}{30} = \frac{14.55 \div 10}{30 \div 10} = \frac{1.455}{\square}$

b $67.35 \div 50 = \frac{67.35}{50} = \frac{67.35 \div \square}{50 \div \square} = \frac{\square}{\square}$

c $45.85 \div 700 = \frac{45.85}{700} = \frac{45.85 \div 100}{700 \div 100} = \frac{0.4585}{\square}$

d $893.6 \div 200 = \frac{893.6}{200} = \frac{893.6 \div \square}{200 \div \square} = \frac{\square}{\square}$

11 Write an explanation to convince that the answer to $\frac{45.6}{30}$ is the same as the answer to $\frac{4.56}{3}$.

12 Twenty members of a football club go out for dinner at a restaurant. The total cost of the meal is $564.25. The total cost is shared equally between them.

a How much does each member pay? Round your answer to the nearest

i cent **ii** dollar

b Which of your answers in part **a i** or **ii** is the most suitable amount for each member to pay? Explain your answer.

Summary checklist

☐ I can use different methods to make decimal calculations easier.

Check your progress

1 Write these decimals in order of size.
6.481, 6.549, 6.5, 6.45, 6.09

2 Use a mental method to work out:

a $5.4 + 12.9$

b $8.2 - 5.7$

3 Use a written method to work these out:

a
$$\begin{array}{r} 3.297 \\ +\ 1.932 \\ \hline \\ \hline \end{array}$$

b
$$\begin{array}{r} 42.35 \\ -\ 6.7\ \ \\ \hline \\ \hline \end{array}$$

4 Work out:

a $1 - 0.674$

b $10 - 5.78$

5 Use a mental method to work out:

a 2×0.04

b 7×0.003

6 Tuong works out that $638 \times 21 = 13\,398$. Use this information to write down the answer to the following.

a 638×2.1

b 6.38×21

7 Work out 0.53×481.

8 Work out $51.492 \div 7$.

9 a Copy and complete the table, which shows the 13 times table.

1	2	3	4	5	6	7	8	9
13	26	39						

b Use the table to help you work out $238.745 \div 13$.

c Show how to check that your answer to part **b** is correct. Use a reverse calculation.

10 Work out the following. Use the methods you have learned to make solving the questions easier.

a 0.03×2100

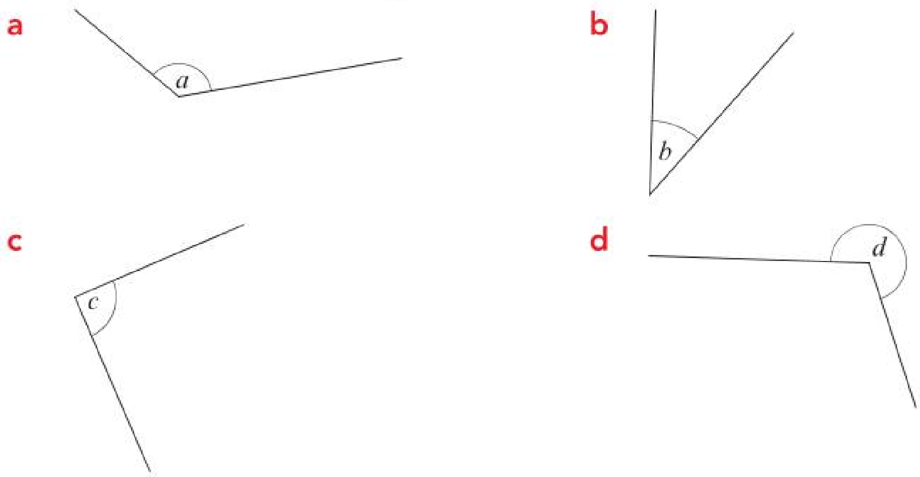
b 8.6×9

c $34.8 \div 30$

5 Angles and constructions

Getting started

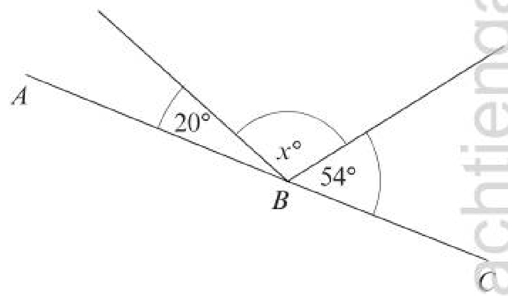
1 Estimate the size of each angle.



2 State whether each of the angles in Question **1** is acute, right, obtuse or reflex.

3 ABC is a straight line.

Calculate the value of x. Show how you worked out your answer.



4 Two angles of a triangle are 50° and 72°.

a Calculate the third angle.

b Explain how you worked out the third angle.

A length is the **distance between two points**.

You can use units, for example, metres, kilometres and millimetres, to measure lengths.

When you change direction, you turn through an angle.

You measure the size of an angle in degrees.

A whole turn is 360 degrees. You write this as 360°.

Why is a whole turn 360?

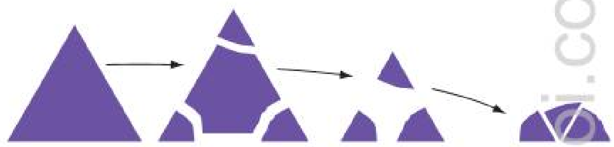
The Babylonians and ancient Egyptians divided a whole turn into 360 parts as long ago as 1500 BCE. This clay tablet excavated in Shush in modern-day Iran shows this.

The Babylonians and ancient Egyptians may have used 360 parts because some calendars at that time divided the year into 360 days.

360 is a useful number because many simple fractions of 360 are whole numbers, including $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$ and $\frac{1}{6}$.

You already know that the sum of the angles on a straight line is 180°.

You also know that the sum of the angles of a triangle is 180°.



In this unit you will discover other useful angle facts and use angles to solve problems.

› 5.1 A sum of 360°

In this section you will ...

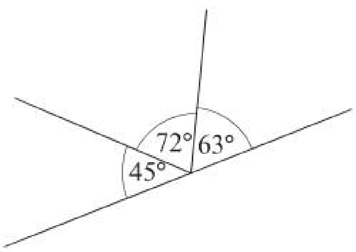
- use the fact that the sum of the angles around a point is 360°
- show and use the fact that the angles of any quadrilateral add up to 360°.

Key words

quadrilateral

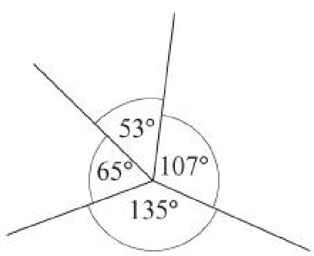
sum

The sum of the angles on a straight line is 180°.



$45° + 72° + 63° = 180°$

A whole turn is 360°. The sum of the angles around a point is 360°.



$65° + 53° + 107° + 135° = 360°$

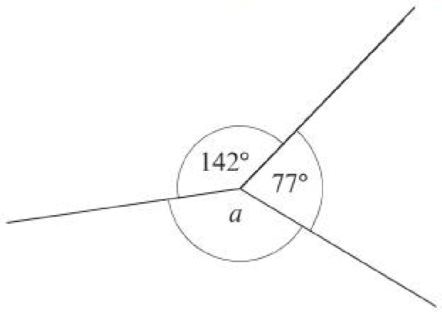
You can apply your algebra skills to find unknown angles, represented by letters.

Tip

See Unit 2 for a reminder on using algebra.

Worked example 5.1

Here are three angles around a point.



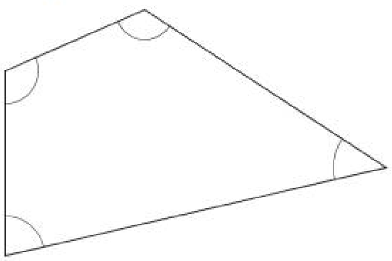
Answer

$142° + 77° = 219°$

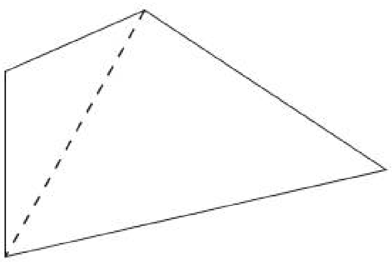
The sum of the three angles is $360°$, so $a = 360° - 219° = 141°$.

The sum of the angles of a triangle is $180°$.

A **quadrilateral** has four straight sides and four angles.



You can draw a straight line to divide the quadrilateral into two triangles.



The six angles of the two triangles make the angles of the quadrilateral.

The sum of the angles of each triangle is $180°$.

The sum of the angles of the quadrilateral is $2 \times 180° = 360°$.

This result is true for <u>any</u> quadrilateral.

You can use the geometrical properties of shapes to calculate missing angles.

Worked example 5.2

Three of the angles of a quadrilateral are each equal to 85°. Work out the fourth angle.

Answer

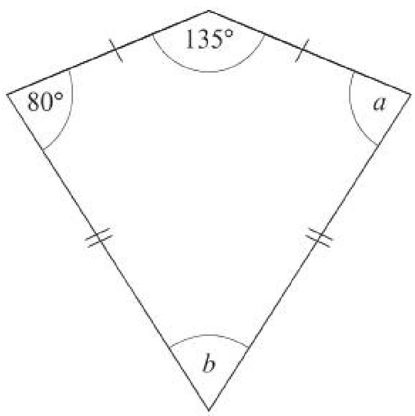
$3 \times 85° = 255°$

The sum of three of the angles is 255°.

All four angles add up to 360°.

The fourth angle is $360° - 255° = 105°$.

Worked example 5.3



This shape is a kite. Calculate the missing angles.

Answer

There is a **vertical line** of symmetry, so angle $a = 80°$.

The four angles add up to 360°.

$135° + 80° + 80° = 295°$, so angle $b = 360° - 295° = 65°$

Exercise 5.1

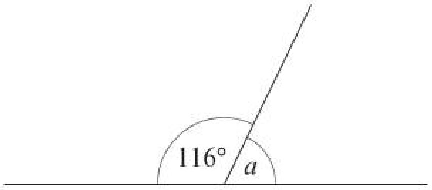
Throughout this exercise, you need to apply your algebra skills to find unknown values, represented by letters.

Tip

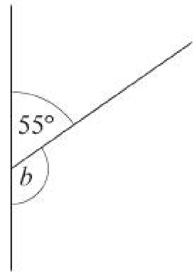
See Unit 2 for a reminder on using algebra.

1 Work out the size of the angle that has a letter.

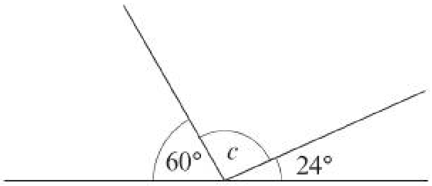
a



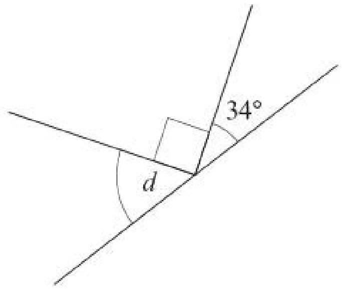
b



c

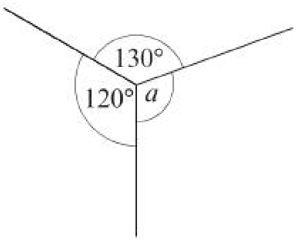


d



2 Calculate the size of each angle that has a letter.

a



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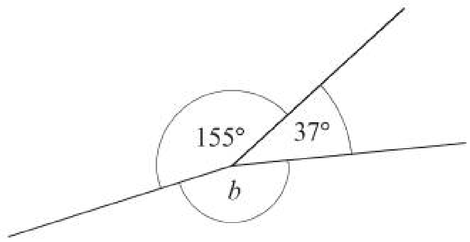
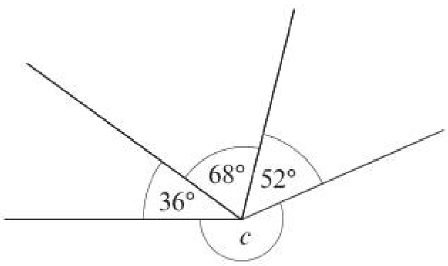
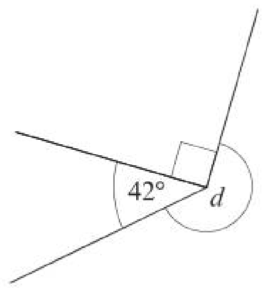


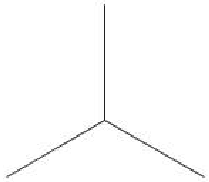
c

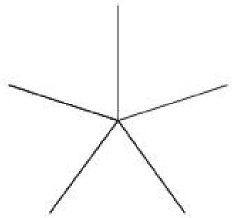


d

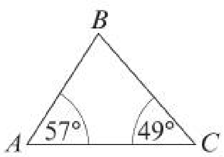


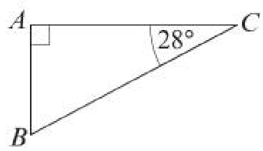
3 The angles in each of these diagrams are all the same size. What is the size of each angle?

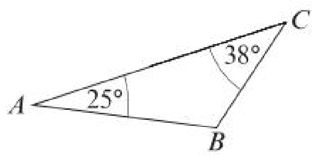
a


b


4 Calculate the size of angle B in each of these triangles.

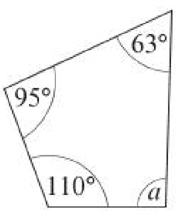
a


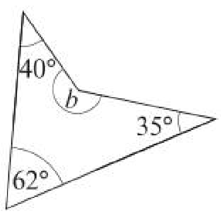
b


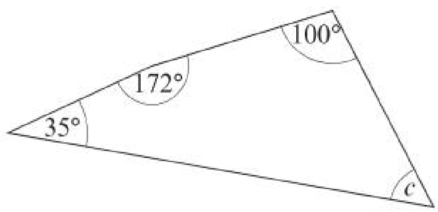
c


5 Three angles of a quadrilateral are 60°, 80° and 110°. Work out the fourth angle.

6 In these quadrilaterals, calculate the size of the angles that have a letter.

a


b


c


7 All the angles of a quadrilateral are equal. What can you say about the quadrilateral?

8 Sofia measures three of the angles of a quadrilateral.

Sofia says:

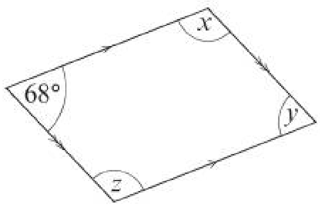
a Show that she has made a mistake.

b Show your answer to part **a** to another learner. Is your answer clear? Could you improve your answer?

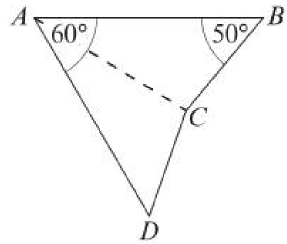
9 One angle of a quadrilateral is 160°. The other angles are all the same size.

Work out the size of the other three angles.

10 This shape is a parallelogram. Work out angles x, y and z.



11 $ABCD$ is a quadrilateral.
Angle $A = 60°$ and angle $B = 50°$.
Calculate angles C and D.



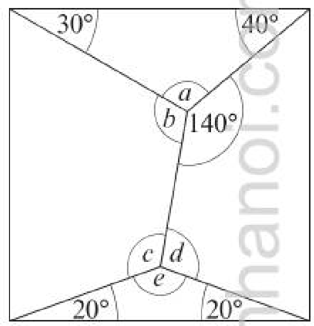
Think like a mathematician

12 All the angles of a quadrilateral are multiples of 30°.

a When all the angles are different, show that there is only one possible set of angles.

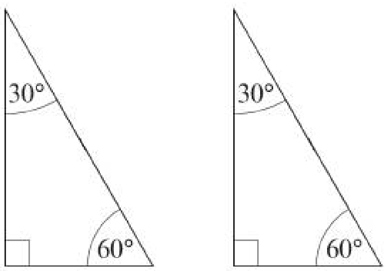
b If one of the angles is 90°, find the other three angles. Show that you have found all possible answers.

13 This is a rectangle.
Work out the angles that have a letter.

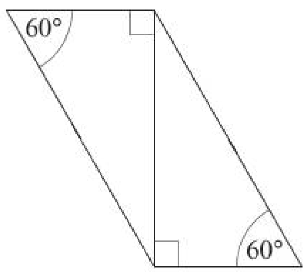


In what order did you find the angles?
Could you find the angles in a different order?

14 Here are two identical triangles.



You can put the triangles together to make a quadrilateral, as shown.



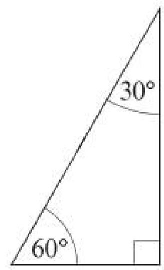
a **i** Find the angles of this quadrilateral.

ii Show that the sum of the angles is 360°.

b Find all the different ways of putting the two triangles together to make a quadrilateral. You can turn the triangle over, as shown, if you prefer.

c **i** Find the angles of your quadrilaterals.

ii Show that the sum is 360° for each quadrilateral.



Compare your answers with a partner's answers. Have you got the same answers? Are your diagrams the same or are they different?

Summary checklist

- [ ] I know that the sum of the angles around a point is 360°. I can use this fact to calculate missing angles.
- [ ] I know that the sum of the angles of a quadrilateral is 360°. I can use this fact to calculate missing angles.

5.2 Intersecting lines

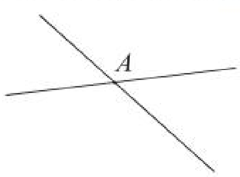
In this section you will …

- recognise the properties of angles on perpendicular lines and intersecting lines
- recognise the properties of angles on parallel lines.

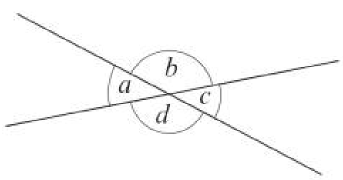
Key words

intersect
opposite angles
perpendicular
parallel
transversal

These two lines **intersect** at A.



When two lines intersect, **opposite angles** are equal.

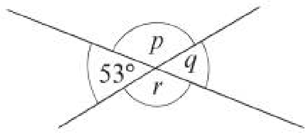


In this diagram, a and c are opposite angles. Angles a and c are equal.
Also b and d are opposite angles. Angles b and d are equal.
You can use your algebra skills to find unknown angles, represented by letters.

> **Tip**
>
> See Unit 2 for a reminder on using algebra.

Worked example 5.4

Work out angles p, q and r.



Answer

$53°$ and p are angles on a straight line.

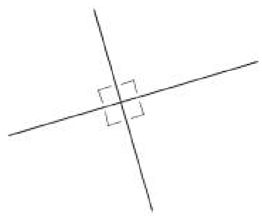
The sum of $53°$ and p is $180°$.

So $p = 180° - 53° = 127°$.

$53°$ and q are opposite angles, so $q = 53°$.

p and r are opposite angles, so $r = 127°$.

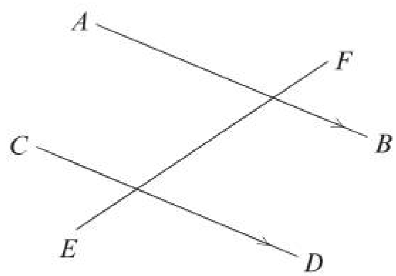
These two lines intersect, as shown. The angle between the two lines is a right angle. They are **perpendicular** lines.



AB and CD are two lines that do not intersect. They are **parallel**.

The arrows show that the lines are parallel.

The line EF crosses the parallel lines. This line is called a **transversal**.

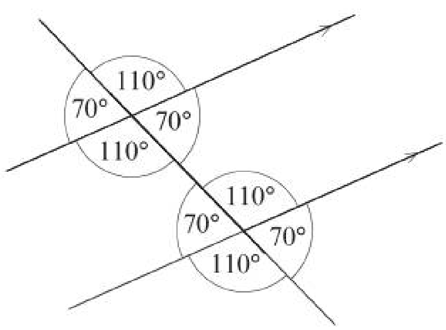


In the previous diagram there are only two different sizes of angle.

The four angles at the top are the same as the four angles at the bottom.

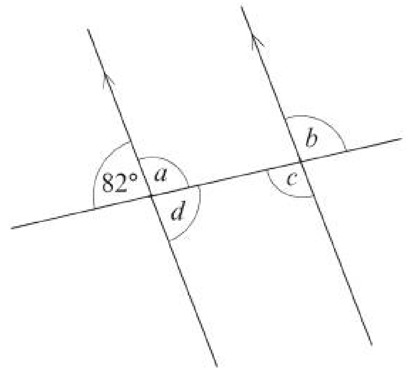
$70° + 110° = 180°$

$70° + 110° + 70° + 110° = 360°$



Worked example 5.5

Work out the unknown angles, a, b, c and d, in this diagram.



Answer

$82°$ and a are angles on a straight line. The sum is $180°$.

$a = 180° - 82° = 98°$

$82°$ and d are opposite angles. They are equal. $d = 82°$

a and b are in the same position. They are equal. $b = 98°$

c and b are opposite angles. They are equal. $c = 98°$

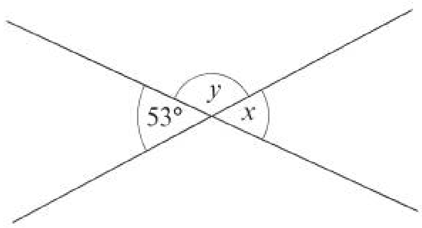
Compare the angles at the two points where the transversal crosses the parallel lines.

Exercise 5.2

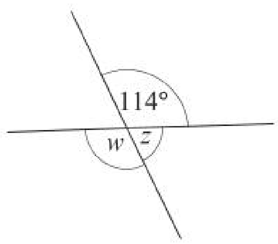
Throughout this exercise, you need to apply your algebra skills to find unknown values, represented by letters.

1 Work out the angles that have a letter.

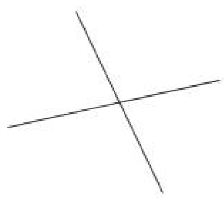
a



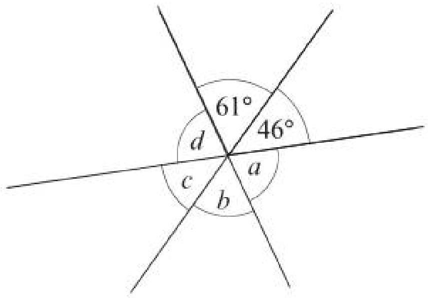
b



2 Two straight lines are shown.
There are four angles. One of the angles is $87°$.
Work out the other three angles.

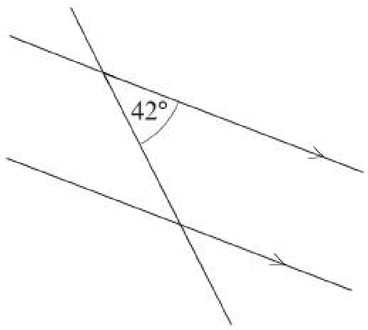


3 Three straight lines meet at a point.



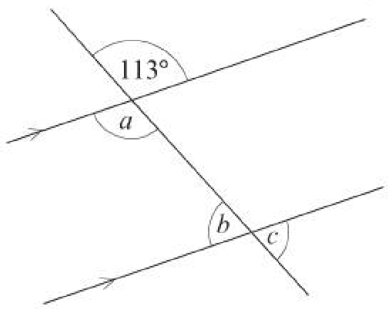
Calculate the values of a, b, c and d. Give reasons for your answers.

4 There are two parallel lines in this diagram. One angle is 42°.

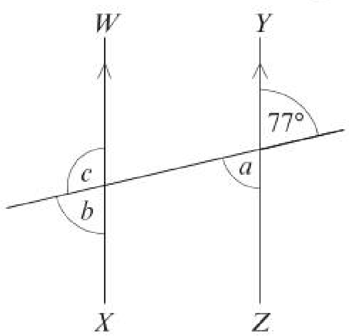


Copy the diagram and write in the size of all the other angles.

5 Work out the unknown angles a, b and c.

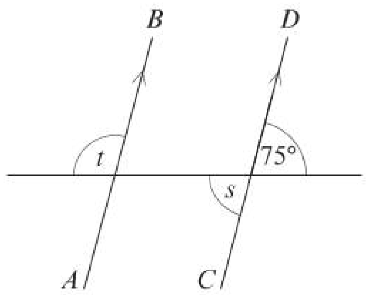


6 Lines WX and YZ are parallel.

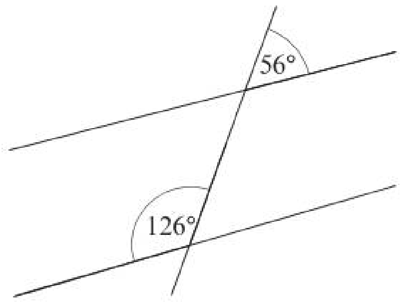


One angle is 77°. Find a, b and c.

7 AB and CD are parallel lines. Calculate s and t.



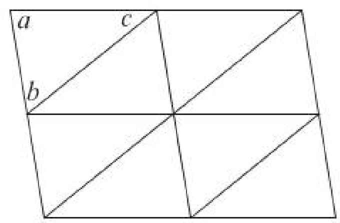
8 Look at the diagram.



a Explain why these two lines cannot be parallel.

b Give your answer to part **a** to a partner to read. Can your answer be improved?

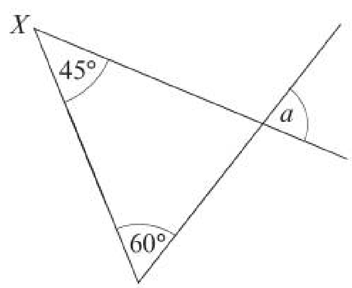
9 This shape is made from eight identical triangles.



a **Sketch** the diagram and **label** the other angles equal to a, b or c.

b Use arrows to mark any parallel lines.

10 The diagram shows angle X is 45°.



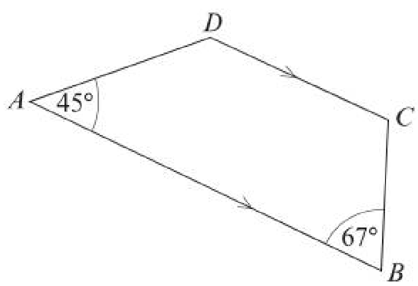
a Calculate a.

b Angle X is increased to 90°. Find the new value of a.

c Angle X is increased to 119°. Find the new value of a.

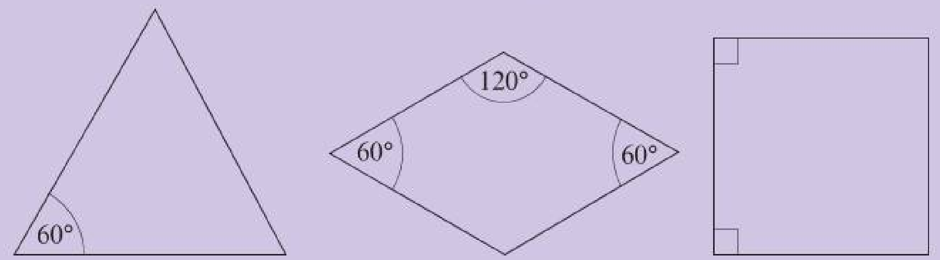
d Can angle X be more than 119°? Give a reason for your answer.

11 This trapezium has a pair of parallel sides. Use this fact to calculate the missing angles.



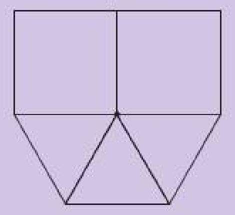
Think like a mathematician

12 These shapes are an equilateral triangle, a rhombus and a square.



All the sides are the same length.

Two squares and three triangles can be placed around a point, as shown.



a How do you know that the shapes fit exactly around a point?

b Find a different way to fit two squares and three triangles around a point.

c Show how to fit only triangles around a point.

d Find all the possible ways of fitting only rhombuses around a point.

Can you be sure you have found all the possible ways in part **d**?

Look back through this exercise. What facts do you need to remember? Make brief notes, with diagrams, to help you remember these facts.

Summary checklist

- [ ] I know the angle properties of perpendicular lines.
- [ ] I know the angle properties of intersecting lines.
- [ ] I know the angle properties of parallel lines and transversal lines.

5.3 Drawing lines and quadrilaterals

In this section you will …

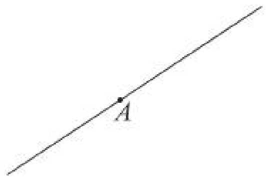
- draw quadrilaterals, perpendicular lines and parallel lines.

Key words

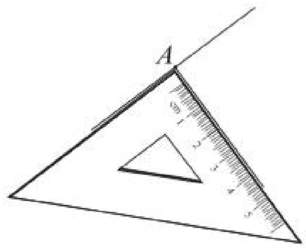
protractor
perpendicular
parallel
quadrilateral
set square

You can use a ruler and a **protractor** or a **set square** to make accurate drawings.

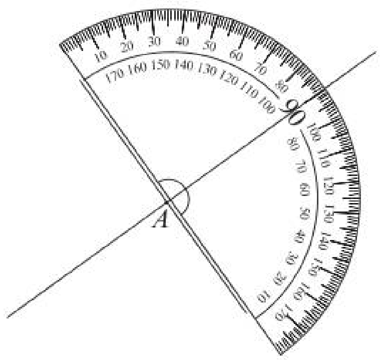
1 Here is a line.



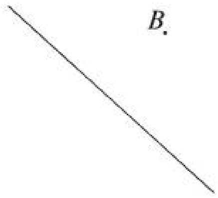
This diagram shows how you can use a set square to draw a second line at A that is **perpendicular** to the first line. Put one edge of the set square on the line. Draw along the other edge.



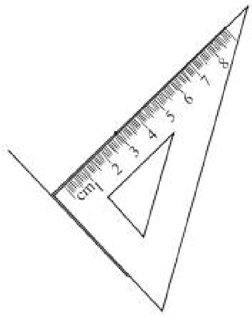
The next diagram shows how you can also use a protractor to draw the same line. Put the centre mark of the flat edge of the protractor at A so that the 90 degree marker is on the line. Draw along the flat edge of the protractor.



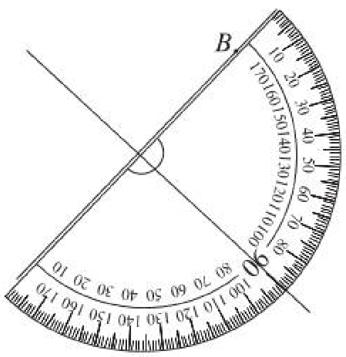
2 Here are a line and a point *B*.



This diagram shows how you can use a set square to draw a second line through *B* that is perpendicular to the first line. Draw a line along the edge of it.



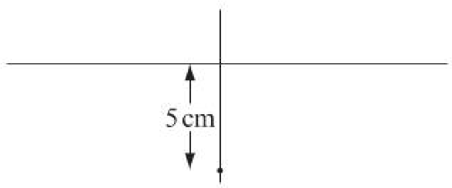
The next diagram shows how you can also use a protractor to draw the same line. Draw a line along the protractor's flat edge, with the original line aligned with the 90° mark.



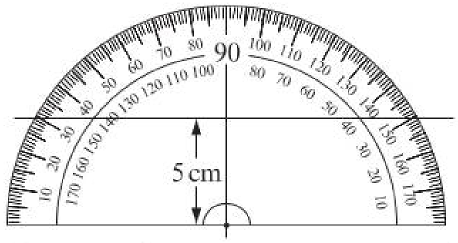
3 Here is a line.

You want to draw a second line that is **parallel** to the first line. The lines must be 5 cm apart.

Draw a perpendicular line with a set square or a protractor. Measure 5 cm.



Use a protractor to draw the parallel line, as shown.



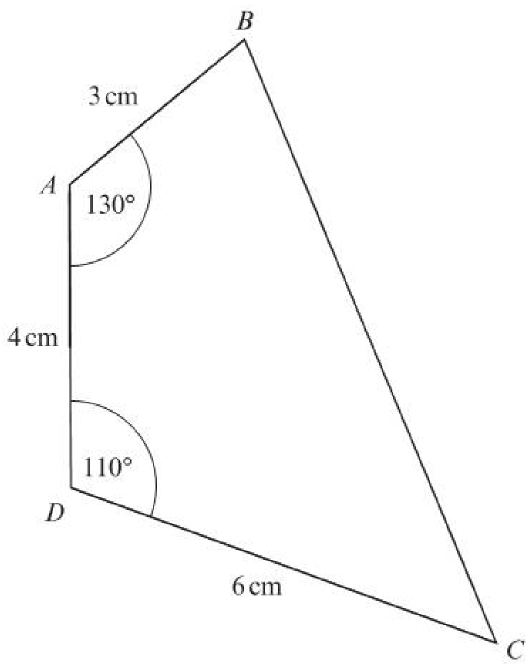
You can also use a set square to draw the same line.

If you want to draw a **quadrilateral**, you need to know some of the sides and angles. You don't need to know all of the sides and angles.

Worked example 5.6 shows you how to draw a quadrilateral using a ruler and a protractor.

Worked example 5.6

This is a sketch of a quadrilateral.

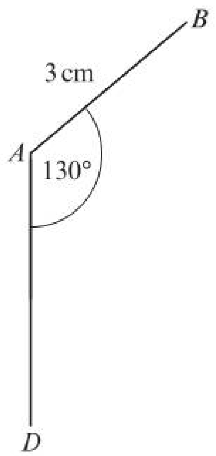


Make an accurate drawing of the quadrilateral.

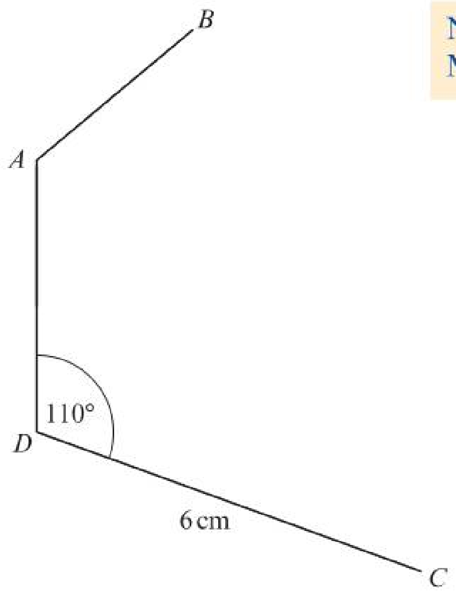
Continued

Answer

Draw a line whose length you know. Choose, for example, AD.

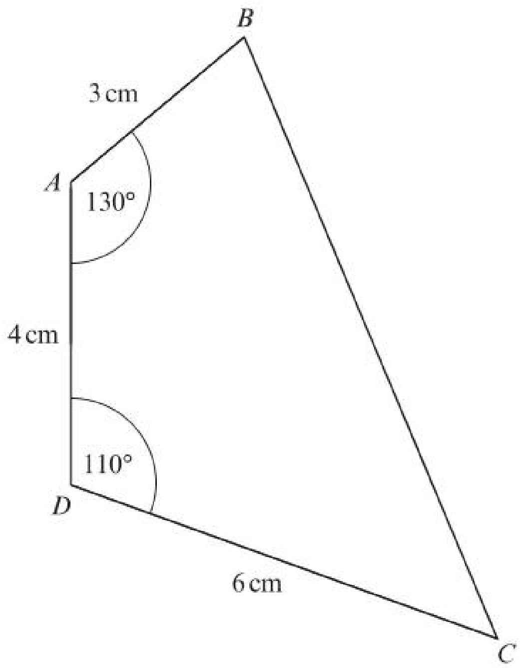


Place the protractor at **vertex** A. Draw a line at an angle of 130°. Measure 3 cm and label the end point of the line B.



Now put the protractor at D. Draw a line at an angle of 110°. Measure 6 cm and label the end point of the line C.

Continued



Now join points *B* and *C*.

You did not need to know the angles at *B* and *C* or the length of *BC*.

If you started by drawing line *CD*, what would be the next step?

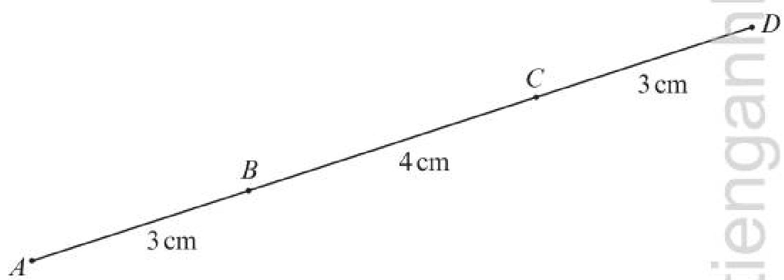
Exercise 5.3

1 **a** Make an accurate drawing of this line.

b Draw a line at *B* that is perpendicular to *AD*.

c Draw a line at *C* that is perpendicular to *AD*.

d Your lines from parts **b** and **c** should be parallel. Are they?

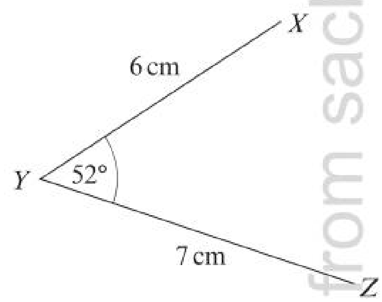


2 **a** Make an accurate drawing of this diagram.

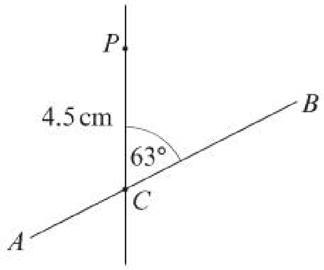
b Draw a perpendicular line from *X* to line *YZ*. Label the intersection as *P*.

c Measure: **i** *XP* **ii** *YP*

d Compare your answers to part **c** with a learner's answers. Do you have the same answers? If not, check your accuracy.

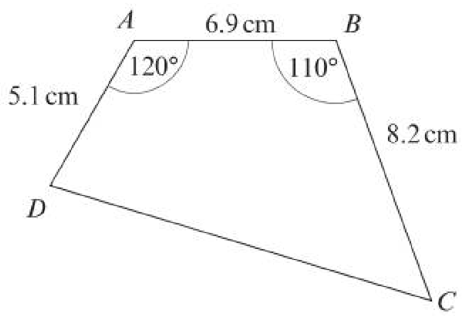


3 **a** Make an accurate drawing of this diagram. The length of PC is 4.5 cm.



b Draw a line through P that is parallel to AB.

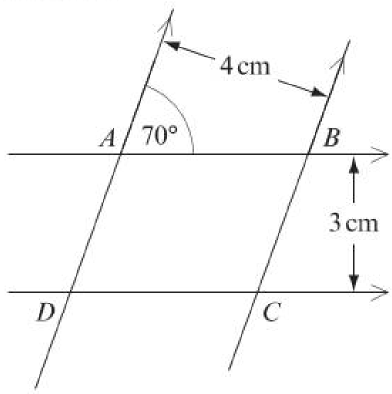
4 **a** Make an accurate drawing of this quadrilateral.



b Measure CD.

c The length of CD should be 13.3 cm. Is your measurement in part **b** close to 13.3 cm? If not, check your drawing.

5 This diagram has two pairs of parallel lines.



a Make an accurate drawing of the diagram.

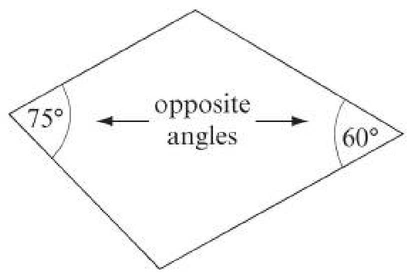
b Draw the line AC and measure the length of this line.

c The length of AC should be 5.4 cm. Is your measurement in part **b** close to 5.4 cm? If not, check your drawing.

6 Three angles of a quadrilateral are 60°, 75° and 130°.

a Calculate the fourth angle of the quadrilateral.

b Draw a quadrilateral with these four angles. The 60° angle must be opposite the 75° angle, as shown in this diagram.



c Draw a different quadrilateral with the same four angles. This time put the 60° angle opposite the 130° angle.

Compare your quadrilaterals with a partner's quadrilaterals. In what way are your diagrams the same? In what way are your diagrams different?

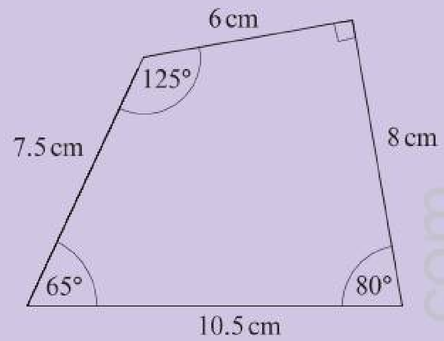
7 Try to draw a quadrilateral where three of the angles are 120°. What happens? Why?

Think like a mathematician

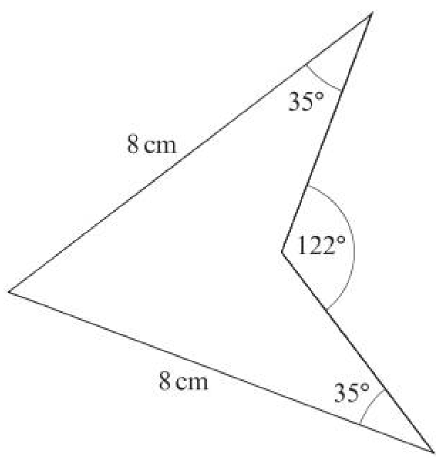
8 a Make an accurate drawing of this quadrilateral.

b You were given the size of every side and angle. Did you need all this information to draw the quadrilateral? What is the least number of measurements you need to draw the quadrilateral accurately?

c Describe the least set of measurements you need in general to draw a quadrilateral accurately.



9 Use the measurements shown to make an accurate drawing of this quadrilateral.



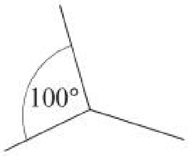
Summary checklist

- ☐ I can draw perpendicular lines.
- ☐ I can draw parallel lines.
- ☐ I can draw quadrilaterals.

Check your progress

1 Here are three angles. Two of the angles are equal.

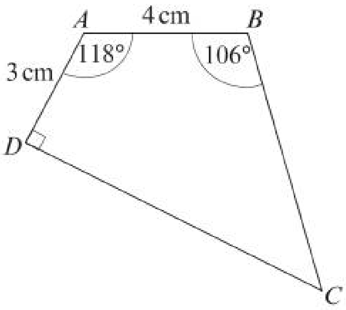
The diagram shows one angle of 100°. Calculate the other two angles. There are two possible answers.



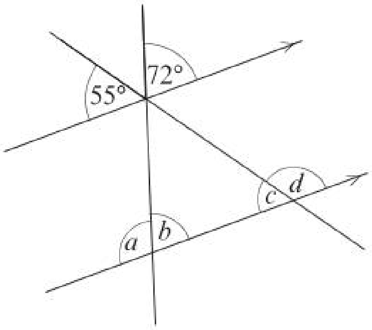
2 **a** Calculate angle *C*.

b Make an accurate drawing of the quadrilateral.

c Measure *CD*.

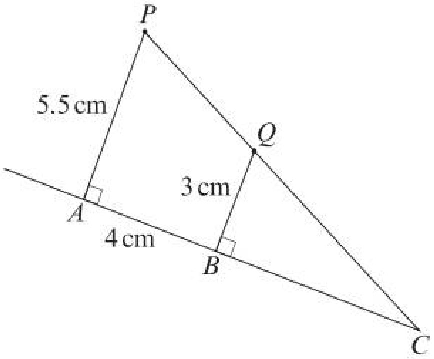


3 Work out the unknown angles *a*, *b*, *c* and *d*.



4 **a** Make an accurate drawing of this diagram.

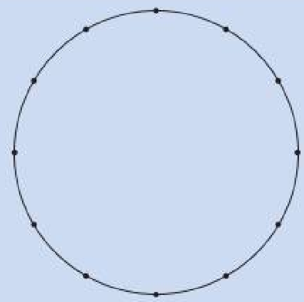
b Measure angle *C*.



Project 2

Clock rectangles

The diagram shows a clock face with 12 equally spaced points around a circle.



Can you find four points that join together to make a rectangle?

How do you know it is a rectangle?

Can you find more than one rectangle?

How many different rectangles is it possible to draw on a clock face?

Once you have explored rectangles on a clock face, here are some more questions you might like to explore.

Consider a clock face that has only eight equally spaced points around the edge. How many rectangles can you find now?

What about a clock face with 18 equally spaced points?

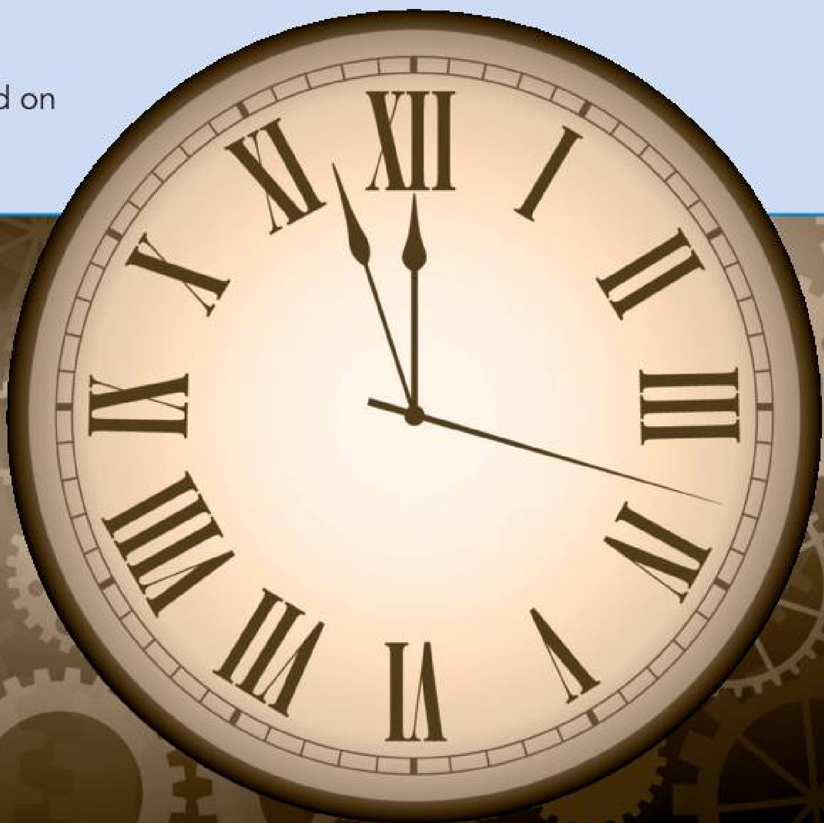
Is there a way to predict the number of rectangles for a clock face with any number of equally spaced points?

What other quadrilaterals can you find on different clock faces?

Which of these is it possible to draw?

- kite
- parallelogram
- rhombus
- trapezium

Do your answers depend on the number of equally spaced points?



6 Collecting data

Getting started

1 a What is a questionnaire?
  b Why would you use a questionnaire?

2 You are planning an investigation of the vehicles using a road.
  a List four **statistical questions** you could ask.
  b List a **prediction** you could make for each question.

3 Here are two predictions:
  - 12-year-old boys are heavier than 12-year-old girls.
  - 12-year-old boys are taller than 12-year-old girls.

  a How could you collect **data** to test this prediction?
  b How would you analyse the data?

4 Here are 20 test marks.

  7 8 8 9 12 14 14 14 15 15 15 16 16 17 18 18 18 18 18 20

  a Which mark is the **mode**?
  b Which mark is the **median**?
  c The sum of the marks is 290. Work out the **mean** mark.

Look at the following three situations.

1. People use a website to make hotel reservations online. The company that manages the website wants to know if people find the site easy to use. The company also wants feedback on the quality of the hotels.

2. A drug company is testing a new drug to help people sleep. There are already drugs available for this. The company wants to know whether people think the new drug is better than existing drugs.

3. Teachers think that drivers are exceeding the speed limit when they drive past a school. This is dangerous for the learners. The teachers want to find out if the drivers are speeding.

In all three cases you need to collect data. You can collect data in different ways.

In the first case, the company could use an online questionnaire.

In the second case, the drug company could interview people using the new drug and ask them about their experience.

In the third case, the teachers could record the speeds of cars as they drive past the school.

In all three cases you have statistical questions. You collect data to answer the questions.

You need to decide:

- what sort of data you want to collect
- how you will collect the data.

6.1 Conducting an investigation

In this section you will ...

- learn how to collect data to investigate statistical questions.

Key words

continuous data
categorical data
data
discrete data
prediction
statistical question

Look at these examples of statistical questions.

1 How many brothers do the learners in your class have?
2 What is the average mass of a baby born in your country?
3 What sports do learners in your school like to watch?

To answer a statistical question you need to collect data.

There are different types of data. The number of brothers you have, the mass of a baby and a sport you watch are all examples of different types of data.

The type of data needed to answer Question **1** is **discrete data**. The values can be only 0, 1, 2, … Discrete data can take particular values only.

The type of data needed to answer Question **2** is **continuous data**.
Masses, lengths and times are all examples of continuous data. They are measurements. They are numbers that can take any value.

The type of data needed to answer Question **3** is **categorical data**.
The data are words, not numbers.

There are several ways to collect data. You can:

- use a questionnaire
- carry out measurements
- make observations
- interview people.

Worked example 6.1

Explain what method you would use to collect data to test each of these predictions. For each case, describe what type of data it is.

a 11-year-old girls can run a distance of 50 metres faster than 11-year-old boys.

b Most teachers in my school wear glasses.

c Plants in the sun grow taller than plants in the shade.

Answer

a Choose some girls and boys to run 50 metres and time each one. This is continuous data.

b Observe each teacher. You can also interview the teachers to ask if they wear glasses for some activities, such as driving or reading. This is categorical data.

c You could do an **experiment**. Plant some seeds in the sun and plant some of the same type of seeds in the shade. When they grow, measure the height of each plant. This is continuous data.

Exercise 6.1

1

discrete	continuous	categorical

Choose the correct word to describe the following.

a the mass of a book **b** the colour of a book

c the number of pages in a book

2 Here are some facts about a person. Write down the type of data for each fact.

a age, in years **b** shoe size

c height **d** time taken to travel to school

e favourite subject

3 Liling is comparing different models of cars. She is collecting data about cars. Give some examples of data about cars that are:

a categorical data

b discrete data

c continuous data

4 Here is a question from a questionnaire. The questionnaire is given to people who stayed at a hotel.

How clean was your room? Circle one number. 1 2 3 4 5

a What is missing from the question?

This table shows some people's replies to this question.

Score	1	2	3	4	5
Frequency	2	4	9	17	21

b How many people replied?

c What was the modal score?

5 Here is a question from a questionnaire.

How many hours of homework do you do? Tick one box.

Between 1 and 2 hours ☐ Between 2 and 3 hours ☐
More than 3 hours ☐

a Write down two things that are wrong with this question.

b Write a better question.

6 You are investigating what people of your age do in their leisure time.

a List some activities that you think should be included.

b Write four questions you would ask in your investigation. Each question should have several tick boxes to choose from that show the possible answers.

c Ask your questions to a partner. Use their replies to help you decide whether you can improve your questions.

7 Work in pairs for this question.

A teacher asks learners to estimate the number of sweets in a jar. She makes two predictions:

- The estimates of the boys will be too big.
- The estimates of the girls will be too small.

a Explain how the teacher can test her predictions.

i What type of data will the teacher need to collect?

ii How can she collect the data?

iii How can she analyse the data?

b Compare your answers to part **a** with the answers of another pair in your class. Can your answer be improved?

8 Adekunle is investigating the number of emails people receive at work. He makes the prediction:

- People get more emails on Mondays than on Fridays.

a How can Adekunle collect data to test his prediction?

b How can he analyse the results?

9 Sofia and Zara throw two dice and add the scores to get the total.

Sofia makes this prediction: Zara makes this prediction:

They throw the two dice 100 times. Their results are shown in the table.

10	7	5	2	10	5	10	9	10	4
7	4	3	7	6	8	7	8	11	9
6	4	10	9	8	6	6	11	8	10
7	7	4	4	5	7	7	7	11	9
9	10	9	9	7	3	8	4	5	10
5	10	8	5	5	6	8	9	3	5
9	5	7	6	8	9	10	7	7	6
9	8	8	3	3	2	4	6	10	9
8	10	5	7	7	9	10	7	10	4
4	2	10	5	4	4	8	9	5	7

a Explain why this is not a good way to record the results.

b Show the frequencies for each number in a suitable table.

c Show the results in a bar chart.

d Is Sofia's prediction correct? Give a reason for your answer.

e Is Zara's prediction correct? Give a reason for your answer.

Think like a mathematician

10 Work in pairs for this question.
A healthy diet includes fruit and vegetables.
Do people your age eat enough fruit and vegetables?
You are going to collect data to investigate this question.

- **a** Write down three predictions to test.
- **b** Explain how you can collect data to test your predictions.
- **c** Describe how you can analyse your data.
- **d** Compare your answers to parts **a**, **b** and **c** with the answers of another pair in your class. Can you improve your answers?

Summary checklist

- [ ] I know the difference between discrete, continuous and categorical data.
- [ ] I can plan an investigation to answer a statistical question.
- [ ] I can decide what data I need to collect.

6.2 Taking a sample

In this section you will …

- learn about taking a sample
- learn about the effect of sample size.

Key words

population
sample
sample size

Here is a prediction: Newborn baby boys are heavier than newborn baby girls.

How could you investigate whether this prediction is true?

It would be very difficult to find the masses of all the babies born. You could find the masses of some of the babies born. This would be a **sample** of the whole population.

The population, in this case, is all newborn babies. The sample is the group of babies you choose.

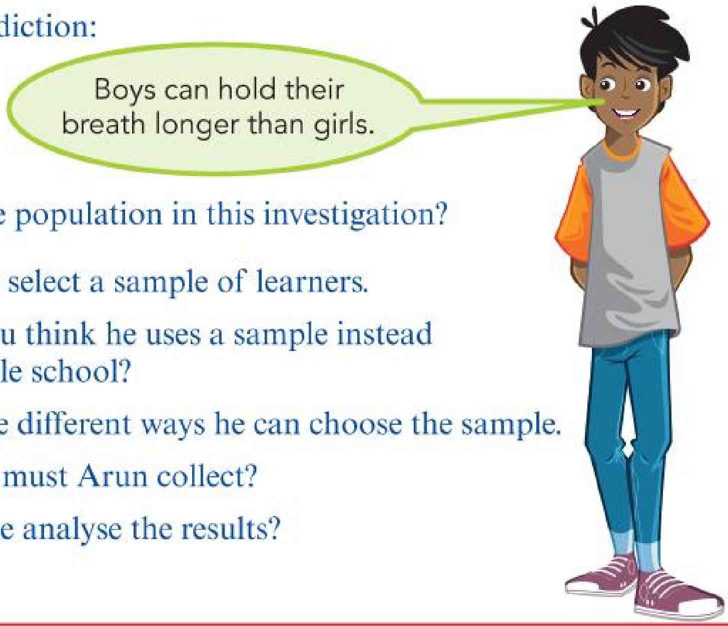
If you can, it is best to get information from the whole population. However, this may take too long or cost too much. In such cases, you can choose a sample. The sample should not be too small or it will not represent the whole population.

In Worked example 6.2 you will see different ways to choose a sample.

Worked example 6.2

Arun is investigating how long learners in his school can hold their breath.

He makes a prediction:



a What is the population in this investigation?

Arun decides to select a sample of learners.

b Why do you think he uses a sample instead of the whole school?

c Explain the different ways he can choose the sample.

d What data must Arun collect?

e How can he analyse the results?

Answer

a The population is all the learners in the school.

b Testing the whole school would take a long time and may not be practical.

c Arun could, for example:

- Put names in a hat and select 40 learners.
- Choose the names from the list of learners in each class.
- Select one or two classes and test all the learners.

d Arun must ask each learner to hold their breath and time it. He must also record whether each learner is a boy or a girl.

e Arun can analyse the boys' data and girls' data separately. He can draw a chart and find an average for the boys' data and an average for the girls' data. He can then compare the averages.

Exercise 6.2

1 Wei is investigating at her school how many hours of homework the learners in her year do each evening.

She predicts that most learners do more than 2 hours each evening.

a How can she collect data to test this prediction?

Wei decides to question a sample of learners.

b Give a reason why it is easier to use a sample than the whole year group.

c What data does she need to collect?

Here are the results of a question given to 25 learners.

How many hours of homework did you do last night?

Homework	Less than 1 hour	Between 1 and 2 hours	Between 2 and 3 hours	More than 3 hours
Frequency	3	6	11	5

d Show the results in a suitable chart.

e What can you say about Wei's prediction?

2 Sofia is investigating birthdays of young people.

She predicts that birthdays in autumn are more common than birthdays in other seasons.

a Why is it not possible to collect data from the whole population?

Sofia decides to use the learners in her school as a sample.

b What data does she need? How can she collect the data?

c She starts to write down the birthday month of each learner in a list like this:

March, October, December, April, …

Explain why this is not a good way to record the data. Suggest a better way.

Sofia displays her results in a table, as shown.

Season	Spring	Summer	Autumn	Winter
Frequency	200	170	230	220

d What is the size of the sample?

e What can you say about Sofia's prediction?

3 A company investigates the success of a telephone helpline.

A survey of callers using the telephone helpline are asked the question:

> *Was the service you received helpful? Please circle one number.*
>
> *Not helpful at all   1   2   3   4   5   Very helpful*

a What prediction is this question testing?

b What is an advantage of asking the question in this way?

c The population is all the callers who use the helpline. Why will the survey only be a sample?

The table summarises the scores received in one day.

Score	1	2	3	4	5
Frequency	10	12	6	1	8

d What can you say about your prediction in part **a**? Give a reason for your answer.

4 Dakarai is comparing two books: A and B.

He predicts that book A has longer words than book B.

a What are the two populations here?

Dakarai decides to choose a page from each book as a sample. He will count the length of every word on the page.

b How could he choose the page each time?

c Describe how he can collect the data.

d Describe a chart he can use to display the data.

e Dakarai wants to find the average length of the words on each page. What is the best average to use? Give a reason for your answer.

f How can he use the average to see if his prediction is correct?

g Do you think the sample is large enough to be sure that he has the correct answer to his prediction?

5 Suki has a dice. She predicts that the dice is not fair.

To test her prediction Suki throws the dice 20 times. Here are the results.

Score	1	2	3	4	5	6
Frequency	4	3	1	4	5	3

a What can you say about Suki's prediction?

Suki decides to throw the dice 100 times. Here are the results.

Score	1	2	3	4	5	6
Frequency	17	19	16	14	11	23

b What can you say about Suki's prediction now?

Suki goes on to throw the dice 500 times. Here are the results.

Score	1	2	3	4	5	6
Frequency	93	92	83	74	48	110

c Do these results confirm your conclusion in part **b**? Give a reason for your answer.

d Is there any benefit in Suki doing more **trials**? Give a reason for your answer.

6 You may choose to work with a partner on this question.

Hospital management wants to know what patients think of the emergency service provided by the hospital. The management decides to employ a company to carry out a survey of patients.

a Why is a large sample better than a small sample?

b What are the disadvantages of a large **sample size**?

c Write two survey questions you could ask patients about the amount of time they waited before they were treated.

d For each question, describe how you would analyse the answers.

e Compare your questions with other learners' questions. Can you suggest improvements to your questions or their questions?

Think like a mathematician

7 Work on this question in a group.

The staff at a theatre want to know more about their customers. They want to know:

- how frequently customers come to the theatre
- if the theatre is attracting people of different ages
- what customers enjoy about the theatre
- how the staff could improve their service.

a Write down two predictions you could test.

The staff ask you to write a short questionnaire. The questionnaire must be easy for people to complete. The staff also want you to analyse the results.

b How could you contact people to complete the questionnaire?

c Write three suitable questions.

d Describe how you will analyse the results.

You may have a small sample size or a large sample size.

a What is the advantage of a large sample size?

b What things do you need to think about if you want to use a large sample size?

Summary checklist

- [ ] I understand the effect of the sample size when I am collecting and representing data.

Check your progress

1 Identify whether the following data is categorical, discrete or continuous.

- **a** the mass of rice in a recipe
- **b** the number of people eating a meal
- **c** the colour of a fruit
- **d** the time taken to eat a meal

2 You are describing a person. Give an example of something about a person that is:

a discrete data **b** continuous data **c** categorical data

3 A restaurant manager asks customers to fill in an online questionnaire after they have had a meal.

- **a** Write down two possible predictions the restaurant could test.

Here is one question from the questionnaire:

'Were the staff polite and helpful?'

The customer replies by choosing from one to five stars.

- **b** Why is this a good way to collect the data?
- **c** You have the results of this question from 65 customers. How can you analyse the results?

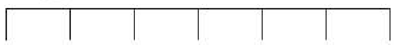
4 A gym manager interviews some members to investigate how they use the gym.

- **a** The manager interviews 10 members. Explain why this sample is too small.
- **b** What are the disadvantages of interviewing everyone who uses the gym?
- **c** How could the manager choose a sample of 50 members to interview?
- **d** State two ways of giving out a questionnaire to the chosen members.

7 Fractions

Getting started

1 **a** Copy the number line.



Write the fractions $\frac{1}{2}$ and $\frac{2}{3}$ in the correct positions on the number line.

b Which is the larger fraction: $\frac{1}{2}$ or $\frac{2}{3}$?

2 Write the correct symbol, < or >, between each pair of fractions.

Remember that < means 'is less than' and > means 'is greater than'.

The first one has been done for you.

a $\frac{5}{8} > \frac{3}{8}$ **b** $\frac{7}{9} \square \frac{8}{9}$ **c** $\frac{1}{3} \square \frac{1}{6}$

d $\frac{3}{5} \square \frac{7}{10}$ **e** $\frac{2}{3} \square \frac{7}{9}$ **f** $\frac{7}{12} \square \frac{3}{4}$

3 Write each of these improper fractions as a mixed number.

a $\frac{5}{3}$ **b** $\frac{7}{5}$ **c** $\frac{11}{9}$ **d** $\frac{15}{4}$

4 Work out the following. Give each answer as a mixed number, in its simplest form.

a $\frac{2}{5} + \frac{7}{10}$ **b** $\frac{5}{6} + \frac{3}{4}$

5 Work out:

a $\frac{1}{2} \times 12$ **b** $\frac{2}{3} \times 15$ **c** $\frac{3}{5} \times 30$

Tip

In part **c**, first write $\frac{1}{3}$ as $\frac{2}{6}$ and then compare with $\frac{1}{6}$.

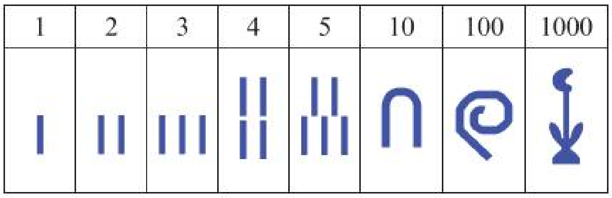
Tip

In an improper fraction, the numerator is bigger than the denominator; e.g. $\frac{3}{2}$. You can write $\frac{3}{2}$ as the mixed number $1\frac{1}{2}$.

The word ‘fraction’ originates from the Latin word fractio, which means ‘breaking into pieces’.

From as early as 1800 BCE, the ancient Egyptians were writing fractions. They used pictures, called hieroglyphs, to write words and numbers.

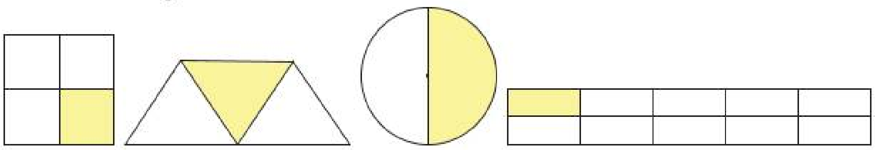
Here are the hieroglyphs the ancient Egyptians used for some numbers.



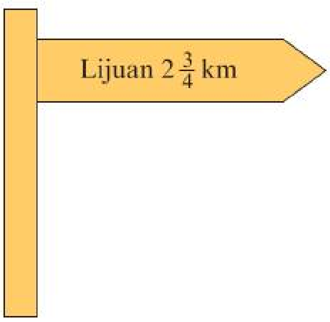
The ancient Egyptians wrote all their fractions with a numerator (number at the top) of 1. To show they were writing a fraction, they drew a mouth picture, which meant ‘part’, above the number.

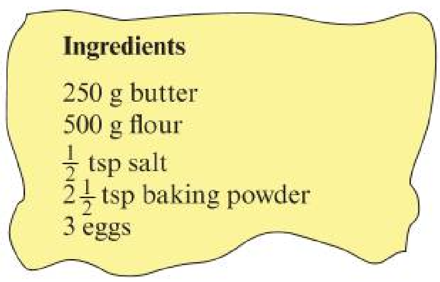
So,  meant $\frac{1}{5}$ and  meant $\frac{1}{100}$.

Can you use Egyptian hieroglyphs to write the fraction shaded in each of these diagrams?



You see fractions often in everyday life, from signs showing distances, to posters in shops and recipes in cookery books.





7.1 Ordering fractions

In this section you will …

- compare and order fractions.

When you write fractions in order of size, you must first **compare** them.
You can compare fractions in two ways:

1 Write them as fractions that have the same denominator.

2 Write them as decimals.

Sometimes when you change a fraction to a decimal, you will get a decimal that goes on forever; for example, $\frac{1}{7} = 0.1428\ldots$

Tip

The three dots (called ellipses) at the end of the decimal show that it goes on forever.

Sometimes there are repeating numbers in the decimal; for example, $\frac{2}{3} = 0.6666\ldots$ and $\frac{21}{37} = 0.567567567\ldots$

These decimals are called **recurring decimals**.

You can write $0.66666\ldots$ as $0.\dot{6}$.

You can write $0.567567567\ldots$ as $0.\dot{5}6\dot{7}$.

Tip

The dots above the 5 and 7 show that the 567 is recurring (or repeating).

Key words

compare
common denominator
denominator
fractional part
improper fraction
mixed number
order of size
recurring decimals
whole-number part

Tip

The dot above the 6 shows that the 6 is recurring (or repeating).

Worked example 7.1

a Write these fractions in order of size, starting with the smallest.

$\frac{3}{2}, \frac{2}{3}$ and $\frac{8}{5}$

b Use decimals to decide which fraction is larger: $2\frac{6}{11}$ or $\frac{23}{9}$?

Continued

Answer

a $\frac{3}{2} = 1\frac{1}{2}, \frac{8}{5} = 1\frac{3}{5}$

First, write any improper fractions as **mixed numbers**.

$\frac{2}{3}, \ldots, \ldots$

$\frac{2}{3}$ is smaller than $1\frac{1}{2}$ and $1\frac{3}{5}$, so write $\frac{2}{3}$ first.

$1\frac{1}{2} = 1\frac{5}{10}, 1\frac{3}{5} = 1\frac{6}{10}$

Now compare the other two fractions by writing them with a **common denominator** of 10.

$\frac{2}{3}, 1\frac{1}{2}, 1\frac{3}{5}$

$1\frac{5}{10} < 1\frac{6}{10}$, so $1\frac{1}{2}$ is less than $1\frac{3}{5}$.

$\frac{2}{3}, \frac{3}{2}, \frac{8}{5}$

Finally, write the answer, using the original fractions given in the question.

b $2\frac{6}{11} = 2\frac{6}{11}, \frac{23}{9} = 2\frac{5}{9}$

First, write any improper fractions as mixed numbers.
As the **whole-number parts** are the same, you now compare the **fractional parts**, $\frac{6}{11}$ and $\frac{5}{9}$.

$\frac{6}{11} = 6 \div 11$

Use division to write $\frac{6}{11}$ as a decimal.

$$\begin{array}{r} 0.\,5\,4\,5\,4\ldots \\ 11\overline{)6.\,^{6}0\,^{5}0\,^{6}0\,^{5}0} \end{array}$$

$\frac{6}{11}$ as a decimal is $0.\dot{5}\dot{4}$.

$\frac{5}{9} = 5 \div 9$

Use division to write $\frac{5}{9}$ as a decimal.

$$\begin{array}{r} 0.\,5\,5\,5\ldots \\ 9\overline{)5.\,^{5}0\,^{5}0\,^{5}0} \end{array}$$

$\frac{5}{9}$ as a decimal is $0.\dot{5}$.

$2\frac{5}{9} > 2\frac{6}{11}$

$2\frac{5}{9}$ is larger than $2\frac{6}{11}$ because 2.55… > 2.54…

$\frac{23}{9} > 2\frac{6}{11}$

Finally, write the answer, using the original fractions given in the question.

Exercise 7.1

Use the common denominator method to answer questions **1** to **5**.

1 This is part of Taylor's homework.

The symbol = shows that a fraction is equal to another.

The symbol ≠ shows that a fraction is not equal to another.

Question Write the correct symbol, = or ≠, between each pair of fractions.

a $\frac{2}{3} \square \frac{10}{15}$ b $\frac{3}{5} \square \frac{13}{20}$

Solution a $\frac{2}{3} = \frac{2 \times 5}{3 \times 5} = \frac{10}{15}$, so $\frac{2}{3} = \frac{10}{15}$.

b $\frac{3}{5} = \frac{3 \times 4}{5 \times 4} = \frac{12}{20}$ and $\frac{12}{20} \neq \frac{13}{20}$, so $\frac{3}{5} \neq \frac{13}{20}$.

Write the correct symbol, = or ≠, between each pair of fractions.

a $\frac{1}{3} \square \frac{2}{9}$ b $\frac{3}{4} \square \frac{12}{16}$ c $\frac{2}{7} \square \frac{9}{35}$

d $\frac{14}{25} \square \frac{3}{5}$ e $\frac{20}{24} \square \frac{5}{6}$ f $\frac{18}{27} \square \frac{7}{9}$

2 Write the correct symbol, < or >, between each pair of fractions. Two of them have been done for you.

a $\frac{21}{5} \square 3\frac{4}{5}$ Working: $\frac{21}{5} = 4\frac{1}{5}$ and $4\frac{1}{5} > 3\frac{4}{5}$.

Answer: $\frac{21}{5} > 3\frac{4}{5}$

b $4\frac{8}{9} \square \frac{46}{9}$ c $\frac{37}{4} \square 9\frac{3}{4}$ d $7\frac{2}{3} \square \frac{22}{3}$

e $\frac{17}{3} \square 5\frac{5}{6}$ Working: $\frac{17}{3} = 5\frac{2}{3}$ $5\frac{4}{6}$ and $5\frac{4}{6} < 5\frac{5}{6}$.

Answer: $\frac{17}{3} < 5\frac{5}{6}$

f $\frac{25}{12} \square 2\frac{1}{4}$ g $3\frac{5}{7} \square \frac{67}{21}$ h $9\frac{3}{4} \square \frac{77}{8}$

Tip

First, change any improper fractions to mixed numbers. When the whole number parts are the same, compare the fractional parts. Use a common denominator if needed.

3 Marcus and Arun compare the methods they use to work out which fraction is larger: $\frac{25}{4}$ or $\frac{63}{10}$?

First, I change both fractions to mixed numbers.

$\frac{25}{4} = 6\frac{1}{4}$ and $\frac{63}{10} = 6\frac{3}{10}$.

Now I compare $\frac{1}{4}$ and $\frac{3}{10}$, using a common denominator of 40.

$\frac{1}{4} = \frac{10}{40}$ and $\frac{3}{10} = \frac{12}{40}$.

$\frac{12}{40} > \frac{10}{40}$, so $\frac{3}{10} > \frac{1}{4}$. This means that $6\frac{3}{10} > 6\frac{1}{4}$, so $\frac{63}{10} > \frac{25}{4}$.

I compare $\frac{25}{4}$ and $\frac{63}{10}$, using a common denominator of 40.

$\frac{25}{4} = \frac{25 \times 10}{4 \times 10} = \frac{250}{40}$ and $\frac{63}{10} = \frac{63 \times 4}{10 \times 4} = \frac{252}{40}$.

$\frac{252}{40} > \frac{250}{40}$, so $\frac{63}{10} > \frac{25}{4}$.

a Critique their methods by explaining the advantages and disadvantages of each method.

b Can you improve either of their methods?

What is your favourite method for comparing fractions? Explain why.

4 Work out which fraction is larger.

a $\frac{47}{6}$ or $\frac{31}{4}$ b $\frac{33}{4}$ or $\frac{42}{5}$ c $\frac{49}{15}$ or $\frac{33}{10}$

Think like a mathematician

5 What method would you use to answer this question?
Put these fraction cards in order of size, starting with the smallest.

$\frac{13}{10}$ $\frac{7}{12}$ $\frac{7}{5}$ $\frac{1}{4}$

6 When you compare fractions by converting them to decimals, how many decimal places do you need to look at?

Use the division method to answer questions **7** to **9**.

7 **a** Copy and complete the workings to write each of these improper fractions as a decimal.

i $\frac{11}{6} = 1\frac{5}{6}$ $\quad 6\overline{)5\,.\,{}^{5}0\,{}^{2}0\,{}^{2}0\;0}$ with quotient $0\,.\,8\;3$ $\quad \frac{5}{6} = \square$ $\quad \frac{11}{6} = 1.8333\ldots$

ii $\frac{19}{11} = 1\frac{8}{11}$ $\quad 11\overline{)8\,.\,{}^{8}0\,{}^{3}0\,{}^{8}0\;0}$ with quotient $0\,.\,7\;2$ $\quad \frac{8}{11} = \square$ $\quad \frac{19}{11} = \square$

iii $\frac{17}{9} = 1\frac{8}{9}$ $\quad 9\overline{)8\,.\,{}^{8}0\,{}^{8}0\;0\;0}$ with quotient $0\,.\,8$ $\quad \frac{8}{9} = \square$ $\quad \frac{17}{9} = \square$

b Write the fractions $\frac{11}{6}$, $\frac{19}{11}$ and $\frac{17}{9}$ in order of size, starting with the smallest.

8 **a** Match each of these fractions to its correct decimal.

$\frac{7}{3}$ $\quad \frac{16}{7}$ $\quad \frac{58}{25}$ $\quad \frac{9}{4}$ $\qquad$ 2.25 $\quad$ 2.28… $\quad$ 2.33… $\quad$ 2.32

b Write the fractions $\frac{7}{3}$, $\frac{16}{7}$, $\frac{58}{25}$ and $\frac{9}{4}$ in order of size, starting with the smallest.

9 Write these fractions in order of size, starting with the smallest.

$3\frac{4}{5}$ $\quad \frac{15}{4}$ $\quad \frac{37}{10}$ $\quad 3\frac{5}{7}$

10 Yasmeen has five improper fraction cards. She puts them in order, starting with the smallest. There are marks on two of the cards.

What fractions could be under the marks?
Give two examples for each card.
Explain how you worked out your answer.

Summary checklist

☐ I can compare and order fractions.

7.2 Adding mixed numbers

In this section you will …

- add mixed numbers.

Key words

- cancel
- collecting like terms
- equivalent fractions
- like terms
- numerator
- proper fraction
- simplest form

You already know that you can only add fractions when the denominators are the same.

When the denominators are different, you must write the fractions as **equivalent fractions** with a common denominator, then add the numerators.

Here is a method for adding mixed numbers, and estimating the answer.

Step ①: Add the whole-number parts. Use this answer as your estimate.

Step ②: Add the fractional parts and **cancel** this answer to its simplest form. If this answer is an improper fraction, write it as a mixed number.

Step ③: Add your answers to steps ① and ②.

Worked example 7.2

Work out:

a $3\frac{1}{7}+5\frac{3}{7}$ **b** $2\frac{1}{4}+3\frac{5}{6}$

Answer

a ① $3+5=8$ — Add the whole-number parts. You can say your estimate is just over 8.

② $\frac{1}{7}+\frac{3}{7}=\frac{4}{7}$ — Add the fractional parts. (In this case, they have a common denominator of 7.)

$\frac{4}{7}$ is a **proper fraction** and is already in its simplest form.

③ $8+\frac{4}{7}=8\frac{4}{7}$ — Add the two parts together to get the final answer.

b ① $2+3=5$ — Add the whole-number parts to give 5. $\frac{5}{6}$ is close to 1, so your estimate is around 6.

② $\frac{1}{4}+\frac{5}{6}=\frac{3}{12}+\frac{10}{12}=\frac{13}{12}$ — Add the fractional parts, using a common denominator of 12.

$\frac{13}{12}=1\frac{1}{12}$ — Check that this fraction is in its simplest form and write it as a mixed number.

③ $5+1\frac{1}{12}=6\frac{1}{12}$ — Add the two parts together to get the final answer.

Exercise 7.2

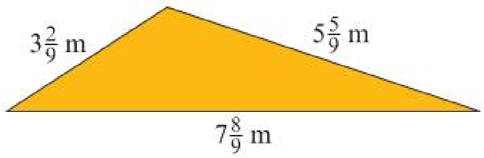
1 Copy and complete these additions. Write down an estimate for each of the additions first.

a $2\frac{4}{9} + 1\frac{4}{9}$ ① $2 + 1 = 3$ ② $\frac{4}{9} + \frac{4}{9} = \frac{\square}{9}$ ③ $3 + \frac{\square}{9} = 3\frac{\square}{9}$

b $7\frac{1}{8} + 3\frac{3}{8}$ ① $7 + 3 = \square$ ② $\frac{1}{8} + \frac{3}{8} = \frac{\square}{8} = \frac{\square}{2}$ ③ $\square + \frac{\square}{2} = \square\frac{\square}{2}$

c $1\frac{5}{7} + 6\frac{4}{7}$ ① $1 + 6 = \square$ ② $\frac{5}{7} + \frac{4}{7} = \frac{\square}{7} = \square\frac{\square}{7}$ ③ $\square + \square\frac{\square}{7} = \square\frac{\square}{7}$

2 The diagram shows the lengths of the three sides of a triangle.



Work out the perimeter of the triangle. Write your answer as a mixed number, in its simplest form.

Tip

The perimeter is the total distance around the edge of the shape.

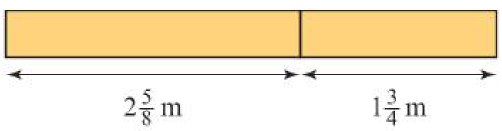
3 Copy and complete these additions. Write down an estimate for each of the additions first.

a $2\frac{1}{2} + 1\frac{1}{4}$ ① $2 + 1 = 3$ ② $\frac{1}{2} + \frac{1}{4} = \frac{2}{4} + \frac{1}{4} = \frac{\square}{4}$ ③ $3 + \frac{\square}{4} = 3\frac{\square}{4}$

b $5\frac{1}{3} + 2\frac{1}{6}$ ① $5 + 2 = \square$ ② $\frac{1}{3} + \frac{1}{6} = \frac{\square}{6} + \frac{1}{6} = \frac{\square}{6} = \frac{\square}{2}$ ③ $\square + \frac{\square}{2} = \square\frac{\square}{2}$

c $1\frac{5}{12} + 3\frac{3}{4}$ ① $1 + 3 = \square$ ② $\frac{5}{12} + \frac{3}{4} = \frac{5}{12} + \frac{\square}{12} = \frac{\square}{12} = \frac{\square}{6} = \square\frac{\square}{6}$ ③ $\square + \square\frac{\square}{6} = \square\frac{\square}{6}$

4 Andrew uses these two pieces of wood to make a shelf.



a What is the total length of the shelf?

b Andrew has a wall that is $4\frac{1}{2}$ m long. Will the shelf fit on this wall? Explain your answer.

5 Hoa drives $12\frac{3}{4}$ km from her home to the doctor's clinic.
She then drives $5\frac{2}{3}$ km from the clinic to her place of work.
What is the total distance that she drives?

6 This is part of Kia's homework. She has made a mistake in her solution.

Question Work out $5\frac{3}{8} + 7\frac{5}{6}$.

Solution ① $5 + 7 = 12$

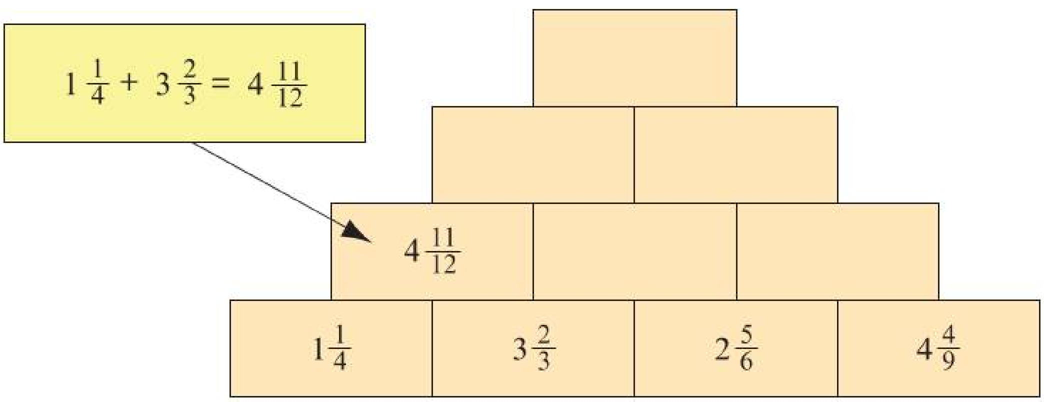
② $\frac{3}{8} + \frac{5}{6} = \frac{9}{24} + \frac{20}{24} = 1\frac{4}{24} = 1\frac{1}{6}$

③ $12 + 1\frac{1}{6} = 13\frac{1}{6}$

a Explain the mistake that Kia has made.

b Work out the correct answer.

7 In this pyramid, you find the mixed number in each block by adding the mixed numbers in the two blocks below it. One addition is shown.
Copy and complete the pyramid.



Tip

To add $\frac{3}{4}$ and $\frac{2}{3}$, use the common denominator 12.

Tip

If you cannot see Kia's mistake, work through the question and then compare your solution with her solution.

8 This is the method Leah uses to collect **like terms** involving mixed numbers.

Simplify this expression by collecting like terms.

$3\frac{7}{8}x + 1\frac{1}{3}y + 4\frac{5}{6}x + 2\frac{1}{5}y$

Adding the x's $3\frac{7}{8} + 4\frac{5}{6}$

$3 + 4 = 7 \quad \frac{7}{8} + \frac{5}{6} = \frac{21}{24} + \frac{20}{24} = \frac{41}{24} = 1\frac{17}{24}$

$7 + 1\frac{17}{24} = 8\frac{17}{24}$

Adding the y's $1\frac{1}{3} + 2\frac{1}{5}$

$1 + 2 = 3 \quad \frac{1}{3} + \frac{1}{5} = \frac{5}{15} + \frac{3}{15} = \frac{8}{15}$

$3 + \frac{8}{15} = 3\frac{8}{15}$

Answer $8\frac{17}{24}x + 3\frac{8}{15}y$

Tip

For a reminder on how to collect like terms, look back at Section 2.3 on algebra.

Use Leah's method to simplify these expressions by collecting like terms.

a $1\frac{3}{4}x + 3\frac{3}{4}x$

b $2\frac{1}{2}y + 2\frac{2}{3}x + 6\frac{3}{5}y$

c $5\frac{7}{8}a + 6\frac{4}{7}b + 2\frac{2}{3}a + 2\frac{1}{2}b$

d $1\frac{1}{5}p + 2\frac{3}{8}q + \frac{2}{3}p + 7\frac{4}{5}q$

Think like a mathematician

9 Work with a partner to answer this question.

Zara is looking at the question $5\frac{2}{3} + 7\frac{7}{8}$.

Without adding any of the fractions, I know that the answer will be between 12 and 14.

a Is Zara correct? Explain your answer.

b Choose two mixed numbers, but don't add them together yet.

Copy and complete this sentence by writing a whole number in each space:

'When I add together my two mixed numbers, the total will be between ☐ and ☐.'

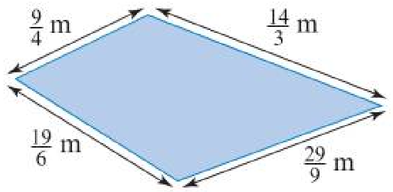
Check that your answer is correct.

Continued

c Think of adding any two mixed numbers.
Write down a general rule for working out between which two whole numbers the total will be.

d How would you change this rule to add three, four or five mixed numbers?

10 Work out the perimeter of this quadrilateral.



Tip

First, change the improper fractions to mixed numbers.

Summary checklist

☐ I can add mixed numbers.

7.3 Multiplying fractions

In this section you will ...

- multiply two proper fractions.

Key word

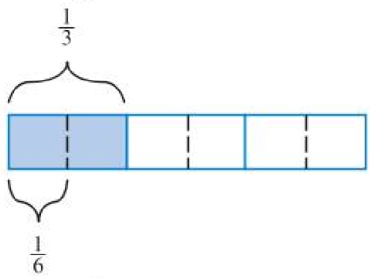
to square

To find a fraction of a fraction, you multiply the fractions together.

The diagram shows a rectangle.

$\frac{1}{3}$ of the rectangle is blue.

You can see from the diagram that $\frac{1}{2}$ of $\frac{1}{3} = \frac{1}{6}$.



This means that $\frac{1}{2}$ of $\frac{1}{3} = \frac{1}{2} \times \frac{1}{3} = \frac{1 \times 1}{2 \times 3} = \frac{1}{6}$.

So, when you multiply fractions, you multiply the numerators together and you multiply the denominators together.

Worked example 7.3

a Work out $\frac{2}{5} \times \frac{7}{8}$. Write your answer in its simplest form.

b In a swimming club, $\frac{4}{5}$ of the members are children. $\frac{1}{3}$ of the children are boys. What fraction of the swimming club members are boys?

Answer

a $\frac{2 \times 7}{5 \times 8} = \frac{14}{40}$

Multiply the numerators together and multiply the denominators together.

$\frac{14}{40} = \frac{7}{20}$

Both 14 and 40 can be divided by 2, so cancel the answer to its simplest form.

b $\frac{1}{3} \times \frac{4}{5} = \frac{4}{15}$

Work out $\frac{1}{3}$ of $\frac{4}{5}$ by multiplying the fractions.

Exercise 7.3

1 Work out the following.

a $\frac{1}{4} \times \frac{1}{2}$ **b** $\frac{3}{4} \times \frac{1}{4}$ **c** $\frac{2}{3} \times \frac{1}{5}$

d $\frac{4}{5} \times \frac{2}{5}$ **e** $\frac{3}{7} \times \frac{3}{4}$ **f** $\frac{7}{9} \times \frac{2}{3}$

2 Work out the following. Write each answer in its simplest form.

a $\frac{3}{4} \times \frac{2}{5}$ **b** $\frac{2}{3} \times \frac{3}{4}$ **c** $\frac{4}{5} \times \frac{3}{8}$

d $\frac{1}{4} \times \frac{8}{9}$ **e** $\frac{3}{10} \times \frac{5}{6}$ **f** $\frac{6}{11} \times \frac{1}{3}$

3 Benji is making a sauce. This is the recipe he uses.

Sauce (serves 4 people)

$\frac{2}{3}$ cup of cashew nuts

$\frac{1}{3}$ cup of water

$\frac{1}{4}$ cup of vinegar

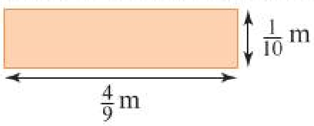
2 tablespoons of honey

$\frac{1}{2}$ teaspoon of salt

Benji makes sauce for two people, so he multiplies all the amounts by $\frac{1}{2}$.
Copy and complete the table, which shows the amount of each ingredient that Benji needs.

Amount for 4 people	Working	Amount for 2 people
$\frac{2}{3}$ cup of cashew nuts	$\frac{1}{2} \times \frac{2}{3} = \square$	$\square$ cup of cashew nuts
$\frac{1}{3}$ cup of water		$\square$ cup of water
$\frac{1}{4}$ cup of vinegar		$\square$ cup of vinegar
2 tablespoons of honey		$\square$ tablespoons of honey
$\frac{1}{2}$ teaspoon of salt		$\square$ teaspoon of salt

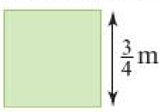
4 Find the area of this rectangle.



Tip

Use the formula:
Area = length × width

5 Work out the area of this square.



Think like a mathematician

6 Look back at Question **5**.
What methods can you use to square a fraction?
For example, what is $\left(\frac{3}{4}\right)^2$?

7 At a hotel, $\frac{5}{9}$ of the staff are employed part-time.

a What fraction of the staff are not employed part-time?

Of the part-time members of staff, $\frac{3}{7}$ are men.

b What fraction of the part-time members of staff are women?

c What fraction of the staff are men employed part-time?

d What fraction of the staff are women employed part-time?

Tip

In part **c**, you need to work out $\frac{3}{7}$ of $\frac{5}{9}$.

8 In a cinema, $\frac{3}{5}$ of the people watching the film are children. $\frac{3}{4}$ of the children are girls.

a What fraction of the people watching the film are girls?

b What fraction of the people watching the film are boys?

Think like a mathematician

9 Work out the answer to $\frac{6}{9} \times \frac{3}{12}$.

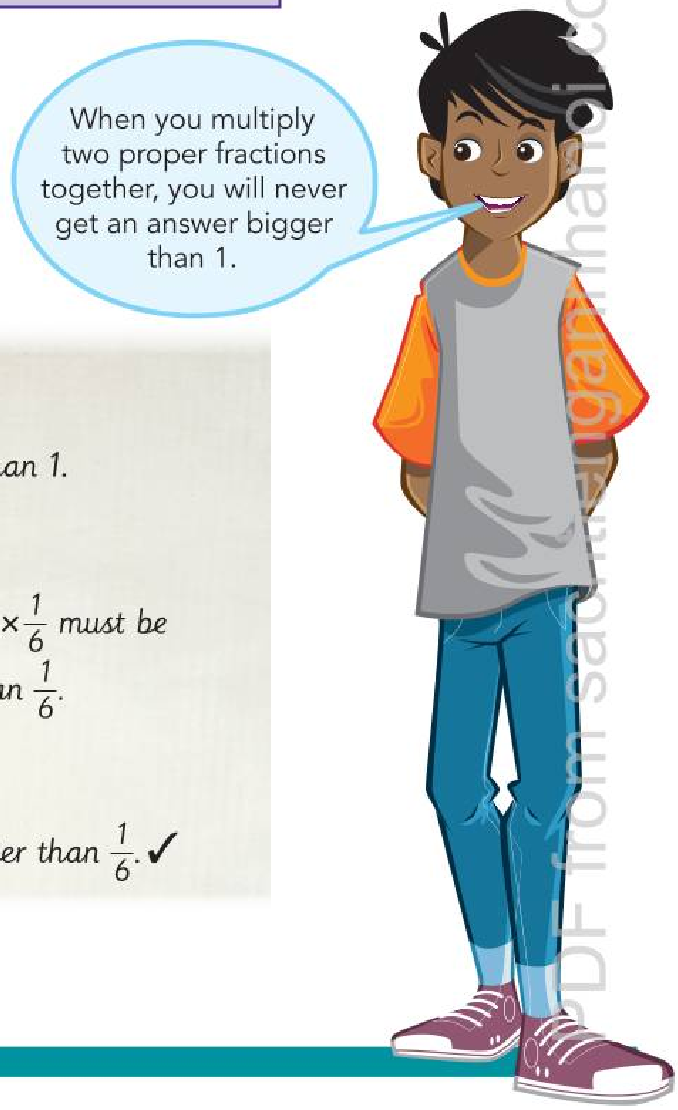
What different methods could you use to work out the answer?

Discuss in pairs or in groups.

Tip

Can you simplify the fractions before you multiply them?

10 Arun says:



Is Arun correct? Explain your answer.

Look back at the questions you have completed in this exercise to help you explain.

11 Samara uses the following method to estimate the answer to a multiplication.

Question: Work out $\frac{3}{4} \times \frac{1}{6}$.

Estimate: $\frac{3}{4}$ is greater than $\frac{1}{2}$, but is less than 1.

$\frac{1}{2}$ of $\frac{1}{6}$ is $\frac{1}{12}$, and $1 \times \frac{1}{6} = \frac{1}{6}$.

So, I know that the answer to $\frac{3}{4} \times \frac{1}{6}$ must be greater than $\frac{1}{12}$ but is smaller than $\frac{1}{6}$.

Accurate: $\frac{3}{4} \times \frac{1}{6} = \frac{3 \times 1}{4 \times 6} = \frac{3}{24} = \frac{1}{8}$

$\frac{1}{8}$ is greater than $\frac{1}{12}$ but is smaller than $\frac{1}{6}$. ✓

For each of the following, use Samara's method to first work out an estimate and then to find the accurate answer.

a $\frac{2}{3} \times \frac{1}{8}$

b $\frac{2}{9} \times \frac{1}{4}$

c $\frac{5}{8} \times \frac{4}{9}$

> **Tip**
>
> Use the fact that $\frac{2}{3}$ is greater than $\frac{1}{2}$, but is less than 1.
>
> Use the fact that $\frac{2}{9}$ is greater than zero, but is less than $\frac{1}{2}$.
>
> Use the fact that $\frac{5}{8}$ is greater wthan $\frac{1}{2}$, but is less than 1.

12 Copy this secret code box.

	E														
$\frac{32}{45}$	$\frac{1}{6}$	$\frac{4}{7}$	$\frac{10}{21}$	$\frac{6}{50}$	$\frac{1}{2}$	$\frac{32}{45}$	$\frac{6}{50}$	$\frac{10}{21}$	$\frac{9}{22}$	$\frac{5}{18}$	$\frac{2}{35}$	$\frac{5}{18}$	$\frac{7}{10}$	$\frac{1}{35}$	$\frac{4}{7}$

E $\frac{1}{4}\times\frac{2}{3}$	U $\frac{1}{5}\times\frac{1}{7}$
L $\frac{2}{3}\times\frac{3}{4}$	I $\frac{1}{5}\times\frac{2}{7}$
H $\frac{3}{4}\times\frac{6}{11}$	S $\frac{4}{9}\times\frac{5}{8}$
F $\frac{4}{5}\times\frac{7}{8}$	A $\frac{3}{5}\times\frac{2}{10}$
N $\frac{6}{7}\times\frac{2}{3}$	T $\frac{5}{7}\times\frac{2}{3}$
M $\frac{8}{9}\times\frac{4}{5}$	

Work out the answer to each of the multiplications in the box on the right.

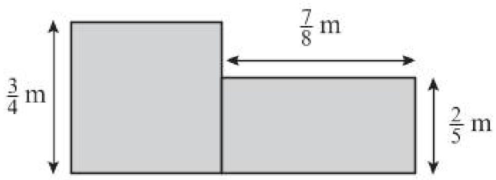
Find the answer in the secret code box, then write the letter from the multiplications box above the answer.

For example, the first multiplication is $\frac{1}{4} \times \frac{2}{3}$.

$\frac{1}{4} \times \frac{2}{3} = \frac{2}{12} = \frac{1}{6}$, so write E above $\frac{1}{6}$ in the secret code box.

What is the secret message?

13 The diagram shows a square joined to a rectangle.



What is the total area of the shape?

Look back at this section on multiplying fractions. What did you find easy? What did you find hard? Are there any parts that you think you need to practise more?

Summary checklist

- [ ] I can multiply two proper fractions.

7.4 Dividing fractions

In this section you will ...

- divide two proper fractions.

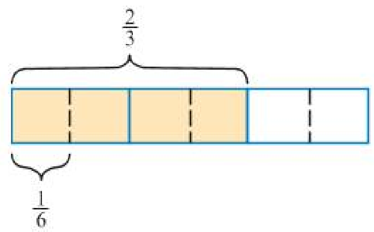
Key words

reciprocal
upside down

The diagram shows a rectangle.

$\frac{2}{3}$ of the rectangle is yellow.

$\frac{1}{6}$ of the rectangle is also shown.



Solving $\frac{2}{3} \div \frac{1}{6}$ is the same as asking 'How many $\frac{1}{6}$ are there in $\frac{2}{3}$?'.

You can see that the answer is 4, so $\frac{2}{3} \div \frac{1}{6} = 4$.

The calculation is $\frac{2}{3} \div \frac{1}{6} = \frac{2}{3} \times \frac{6}{1} = \frac{2 \times 6}{3 \times 1} = \frac{12}{3} = 4$.

Here is a method for dividing a fraction by a fraction.

1 Turn the second fraction **upside down**.
2 Multiply the fractions together, as usual.
3 Write the answer in its simplest form and as a mixed number when possible.

Tip

When you turn a fraction upside down you get a **reciprocal** fraction. For example the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$.

Worked example 7.4

Work out $\frac{2}{5} \div \frac{3}{10}$.

Answer

$\frac{2}{5} \div \frac{3}{10} = \frac{2}{5} \times \frac{10}{3}$ — Turn the second fraction upside down.

$\frac{2 \times 10}{5 \times 3} = \frac{20}{15}$ — Multiply the numerators together, and multiply the denominators together.

$\frac{20}{15} = \frac{4}{3}$ — Both 20 and 15 can be divided by 5, so cancel the fraction to its simplest form.

$\frac{4}{3} = 1\frac{1}{3}$ — $\frac{4}{3}$ is an improper fraction, so write the answer as a mixed number.

Exercise 7.4

1 Copy and complete:

a $\frac{1}{5} \div \frac{3}{4} = \frac{1}{5} \times \frac{4}{3}$

$= \frac{1 \times 4}{5 \times 3}$

$= \frac{\square}{\square}$

b $\frac{2}{3} \div \frac{6}{7} = \frac{2}{3} \times \frac{7}{6}$

$= \frac{2 \times 7}{3 \times 6}$

$= \frac{\square}{\square}$

$= \frac{\square}{\square}$

2 Work out:

a $\frac{1}{4} \div \frac{2}{3}$ **b** $\frac{1}{2} \div \frac{3}{5}$ **c** $\frac{3}{8} \div \frac{4}{7}$

d $\frac{4}{5} \div \frac{1}{9}$ **e** $\frac{3}{5} \div \frac{2}{11}$ **f** $\frac{9}{10} \div \frac{1}{3}$

> **Tip**
>
> In parts **d**, **e** and **f** write your answer as a mixed number.

3 Work out the following. Write each answer in its simplest form and as a mixed number when possible.

a $\frac{3}{4} \div \frac{1}{2}$ **b** $\frac{4}{5} \div \frac{3}{10}$ **c** $\frac{5}{6} \div \frac{2}{3}$

d $\frac{4}{9} \div \frac{1}{3}$ **e** $\frac{6}{7} \div \frac{3}{7}$ **f** $\frac{7}{8} \div \frac{3}{4}$

4 This is part of Sofia's homework. She has made a mistake in her solution.

<u>Question</u>

Work out $\frac{8}{9} \div \frac{4}{5}$.

<u>Solution</u>

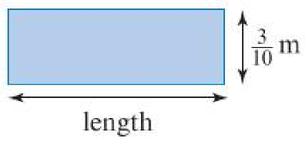
$\frac{8}{9} \div \frac{4}{5} = \frac{9}{8} \times \frac{4}{5} = \frac{36}{40} = \frac{9}{10}$

a Explain Isaac's mistake.

b Work out the correct answer.

5 The area of this rectangle is $\frac{2}{15}$ m².

The width is $\frac{3}{10}$ m.



Work out the length of the rectangle.

Tip

The formula for the area of a rectangle is Area = length × width, o Length = area ÷ width.

6 Cheng is using fraction cards to make correct calculations.

$\frac{7}{12}$ ÷ ? = $\frac{7}{10}$

Which of these four fraction cards is the correct card for the missing fraction in the division?

$\frac{3}{4}$ $\frac{3}{10}$ $\frac{5}{6}$ $\frac{7}{15}$

Think like a mathematician

7 Look again at Question **6**.

- **a** As a class, discuss the different methods that you used to answer the question.
- **b** Critique each method by explaining the advantages and disadvantages of each method.
- **c** Which is the best method that was used? Can you improve this method?

Activity 7.1

On a piece of paper, write four fraction division questions that are similar to the divisions shown in Question **3**.

You must use proper fractions and not mixed numbers.

On a separate piece of paper, work out the answers.

Swap your questions with a partner and answer their questions.

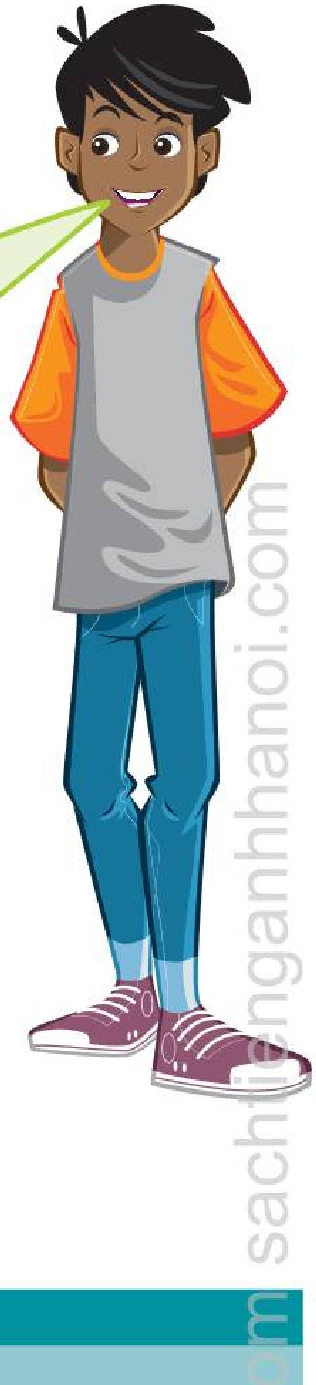
Swap back and mark each other's work. Discuss any mistakes that have been made.

8 Arun is looking for general patterns in the fraction division questions.

He thinks of two ideas.

Are Arun's ideas correct? Explain your answers.

Look back at the questions you have completed in this exercise to help you to explain.



9 Look at this fractions pattern.

Pattern	Working	Answer
$\frac{1}{2}\times\frac{1}{3}$	$\frac{1}{2}\times\frac{1}{3}=\frac{1}{6}$	$\frac{1}{6}$
$\frac{1}{2}\times\frac{1}{3}\div\frac{1}{4}$	$\frac{1}{2}\times\frac{1}{3}=\frac{1}{6}, \frac{1}{6}\div\frac{1}{4}=\frac{1}{6}\times\frac{4}{1}=$	
$\frac{1}{2}\times\frac{1}{3}\div\frac{1}{4}\times\frac{1}{5}$		
$\frac{1}{2}\times\frac{1}{3}\div\frac{1}{4}\times\frac{1}{5}\div\frac{1}{6}$		
$\frac{1}{2}\times\frac{1}{3}\div\frac{1}{4}\times\frac{1}{5}\div\frac{1}{6}\times\frac{1}{7}$		
...		

At which pattern does the answer become greater than 1?

Write down this answer.

Summary checklist

☐ I can divide two proper fractions.

7.5 Making fraction calculations easier

In this section you will …

- simplify calculations containing fractions.

Key word

factor

When you are calculating with fractions, there are methods that you can use to make a calculation easier, such as:

- Break a fraction into parts using **factors**.
- Use equivalent fractions.
- Find factors that are the same in the numerator and the denominator.
- Instead of working out a large fraction of a number, work out the corresponding small fraction and subtract it from the number.

For all calculations, you must always remember to use the correct order of operations.

Worked example 7.5

Work out:

a $\frac{1}{4} \times 600$ **b** $\frac{3}{5} \times 120$ **c** $\frac{7}{8} \times \frac{4}{5}$

Answer

a $\frac{1}{4} = \frac{1}{2} \times \frac{1}{2}$ — Use factors to change the 4 to 2×2. Note that finding $\frac{1}{4}$ of a number is the same as halving the number and then halving the number again.

$600 \div 2 = 300$ — $\frac{1}{2}$ of 600 is 300.

$300 \div 2 = 150$ — $\frac{1}{2}$ of 300 is 150.

b $\frac{3}{5} = \frac{6}{10}$ — Change $\frac{3}{5}$ to $\frac{6}{10}$ because dividing by 10 is easier than dividing by 5.

$120 \div 10 = 12$ — Work out $\frac{1}{10}$ of 120 by dividing 120 by 10.

$12 \times 6 = 72$ — Work out $\frac{6}{10}$ of 120 by multiplying $\frac{1}{10}$, which is 12, by 6.

Continued

c $\frac{7}{8} \times \frac{4}{5} = \frac{7 \times 4}{8 \times 5}$ — Multiply the numerators and denominators, as usual.

$= \frac{7 \times 4}{4 \times 2 \times 5}$ — Replace 8 with 2×4 because there is a 4 in the numerator.

$= \frac{7 \times 4}{2 \times 5 \times 4} = \frac{7 \times 4}{10 \times 4}$ — Rearrange the numerator and denominator to get $\times 4$ at the end.

$= \frac{7}{10} \times \frac{4}{4}$ — Rearrange to get $\times \frac{4}{4}$ at the end.

$= \frac{7}{10} \times 1 = \frac{7}{10}$ — $\frac{4}{4} = 1$, so multiply $\frac{7}{10}$ by 1 to get $\frac{7}{10}$.

Exercise 7.5

1 Copy and complete the workings to make these calculations easier.
Use factors to change the fractions.

a $\frac{1}{2} \times 28$ $\quad 28 \div 2 = \square$

b $\frac{1}{4} \times 520 = \frac{1}{2} \times \frac{1}{2} \times 520$ $\quad 520 \div 2 = 260$
$260 \div 2 = \square$

c $\frac{1}{8} \times 120 = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times 120$ $\quad 120 \div 2 = \square$
$\square \div 2 = \square$
$\square \div 2 = \square$

d $\frac{1}{14} \times 700 = \frac{1}{7} \times \frac{1}{2} \times 700$ $\quad 700 \div 7 = \square$
$\square \div 2 = \square$

Tip

In part **b**, to work out $520 \div 2$, first work out $52 \div 2 = 26$. Then multiply by 10, which gives $26 \times 10 = 260$.

Think like a mathematician

2 In Question **1d**, does it matter if you do $700 \div 7$ or $700 \div 2$ first?
Which is the easier method: $700 \div 7$ then $100 \div 2$ or
$700 \div 2$ then $350 \div 7$?
How can you decide which division to do first?
Discuss with a partner.

3 Work out the following. Use factors to change the fractions, showing all your working.

a $\frac{1}{4} \times 108$ **b** $\frac{1}{6} \times 150$ **c** $\frac{1}{8} \times 280$ **d** $\frac{1}{15} \times 180$

4 Copy and complete the workings to make these calculations easier. Use equivalent fractions.

a $\frac{1}{5} \times 340$ $\frac{1}{5} = \frac{2}{10}$ $340 \div 10 = \square$

$\square \times 2 = \square$

b $\frac{2}{5} \times 160$ $\frac{2}{5} = \frac{4}{10}$ $160 \div 10 = \square$

$\square \times 4 = \square$

Tip

In part **b**, remember that × 4 is the same as × 2 × 2.

5 Work out the following. Use equivalent fractions, showing all your working.

a $\frac{1}{5} \times 270$ **b** $\frac{4}{5} \times 80$ **c** $\frac{3}{5} \times 210$ **d** $\frac{2}{5} \times 320$

Tip

In part **c** you can use partitioning to work out 6 × 21. So, 6 × 21 = 6 × 20 + 6 × 1.

6 Emyr uses the following method to work out $\frac{3}{20} \times 4500$.

I know that $\frac{3}{20} = \frac{3 \times 5}{20 \times 5} = \frac{15}{100}$.

So, to find $\frac{15}{100} \times 4500$

① $4500 \div 100 = 45$

② $15 \times 45 = 15 \times 40 + 15 \times 5$

$= 15 \times 4 \times 10 + 75$

$= 600 + 75$

$= 675$

Use Emyr's method to work out:

a $\frac{1}{20} \times 1100$ **b** $\frac{3}{20} \times 1900$ **c** $\frac{7}{20} \times 900$ **d** $\frac{11}{20} \times 7000$

7 Copy and complete the workings to make these calculations easier.

Use the method of finding factors that are the same in the numerator and the denominator.

a $\frac{5}{6} \times \frac{2}{3} = \frac{5 \times 2}{6 \times 3} = \frac{5 \times 2}{2 \times 3 \times 3} = \frac{5 \times 2}{3 \times 3 \times 2} = \frac{5 \times 2}{\square \times 2} = \frac{5}{\square} \times \frac{2}{2} = \frac{\square}{\square} \times 1 = \frac{\square}{\square}$

b $\frac{3}{5} \times \frac{10}{17} = \frac{3 \times 10}{5 \times 17} = \frac{3 \times 5 \times 2}{5 \times 17} = \frac{3 \times 2 \times 5}{17 \times 5} = \frac{\square \times 5}{17 \times 5} = \frac{\square}{\square} \times \frac{5}{5} = \frac{\square}{\square} \times 1 = \frac{\square}{\square}$

8 Work out the following.

Use the method you used in Question **7**.

Show all your working.

a $\frac{3}{8} \times \frac{5}{6}$ **b** $\frac{13}{14} \times \frac{2}{5}$ **c** $\frac{7}{9} \times \frac{18}{25}$ **d** $\frac{12}{19} \times \frac{2}{3}$

9 This is part of Iqra's homework.

<u>Question</u> Work out $\frac{5}{24} \times \frac{8}{11}$.

<u>Solution</u> $\frac{5}{24} \times \frac{8}{11} = \frac{5 \times 8}{24 \times 11}$

$= \frac{5 \times 8}{8 \times 4 \times 11} = \frac{5 \times 8}{4 \times 11 \times 8}$

$= \frac{5 \times 8}{44 \times 8} = \frac{5}{44} \times \frac{8}{8}$

$= \frac{5}{44} \times 1 = \frac{5}{44}$

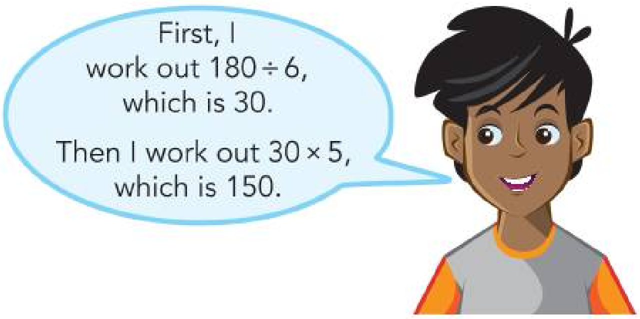
Check every step of her homework.

Is Iqra's homework correct?

Explain your answer.

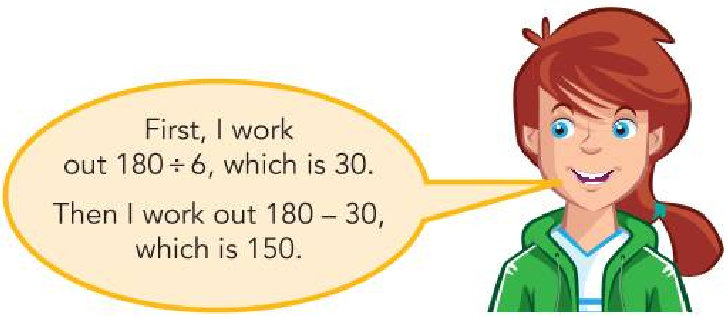
10 This is how Arun works out $\frac{5}{6} \times 180$.

a Explain how Arun's method works.



This is how Sofia works out $\frac{5}{6} \times 180$.

b Explain how Sofia's method works.



c Use both Arun's method and Sofia's method to work out:

i $\frac{6}{7} \times 280$ **ii** $\frac{14}{15} \times 900$

d Whose method did you find it easiest to use to work out the answers to part **c**? Explain why.

e Critique each method by explaining the advantages and disadvantages of each method.

f What do you think is the best method to use to work out questions such as

$\frac{19}{20} \times 1800$, $\frac{24}{25} \times 800$ and $\frac{13}{14} \times 2240$? Explain why.

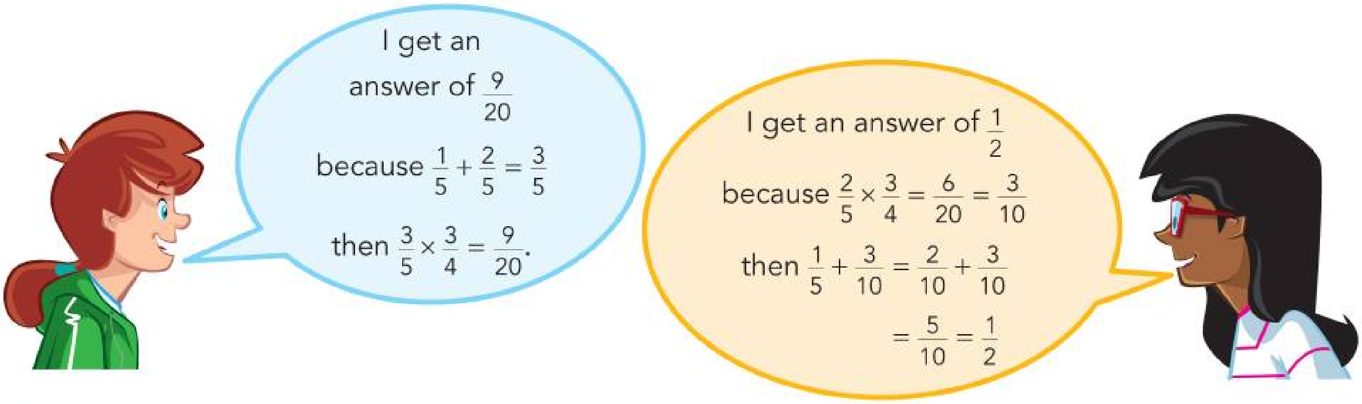
g Use your favourite method to work out the answers to the following.

i $\frac{19}{20} \times 1800$ **ii** $\frac{24}{25} \times 800$ **iii** $\frac{13}{14} \times 2240$

11 Both Sofia and Zara work out the answer to $\frac{1}{5} + \frac{2}{5} \times \frac{3}{4}$.

a Who is correct?

b Explain the mistake that the other person has made.



12 Work out the following. Give each answer in its simplest form, showing all your working.

a $\frac{5}{6} + \frac{7}{9} \times \frac{3}{8}$ **b** $\frac{9}{10} - \frac{2}{3} \times \frac{1}{4}$ **c** $\frac{1}{4} \times \frac{7}{8} + \frac{3}{4} \times \frac{1}{6}$

Summary checklist

☐ I can use different methods to make fraction calculations easier.

Check your progress

1 Write the correct inequality, = or ≠, between each pair of fractions.

a $\frac{6}{4}\square 1\frac{1}{3}$ **b** $3\frac{1}{7}\square \frac{44}{14}$ **c** $5\frac{3}{4}\square \frac{19}{4}$

2 Write the correct symbol, < or >, between each pair of fractions.

a $\frac{3}{7}\square \frac{2}{5}$ **b** $3\frac{1}{2}\square \frac{11}{3}$ **c** $1\frac{17}{20}\square \frac{11}{6}$

3 Work out the following. Give each answer as a mixed number in its simplest form.

a $3\frac{1}{8}+4\frac{5}{8}$ **b** $2\frac{3}{4}+5\frac{3}{5}$

4 Work out the following. Give each answer in its simplest form.

a $\frac{7}{9}\times\frac{3}{5}$ **b** $\frac{5}{6}\div\frac{3}{4}$

5 Work these out, using a method to make the calculation easier. Show all your working.

a $\frac{1}{8}\times 1000$ **b** $\frac{2}{5}\times 420$ **c** $\frac{29}{30}\times 6000$

Project 3

Fraction averages

Here is a set of five fractions:

$\frac{1}{5}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{4}{5}$

The mode of this set of fractions is $\frac{1}{2}$

The median is $\frac{1}{2}$

The mean is $\frac{1}{2}$

Can you find some other sets of five fractions between 0 and 1 for which the mean, median and mode are all $\frac{1}{2}$?

In the set of fractions shown above, the fraction $\frac{1}{2}$ appears three times.

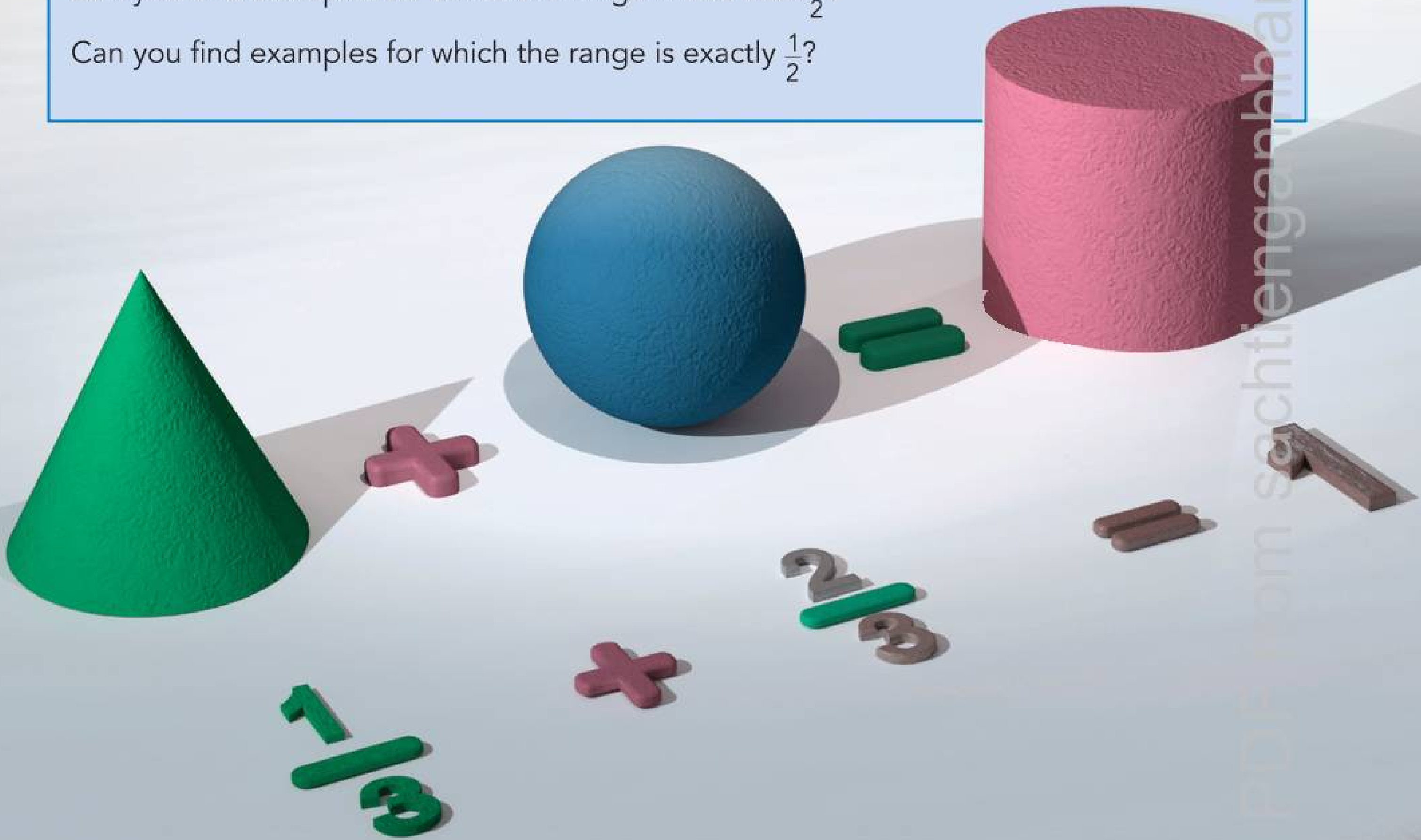
Can you find examples that include $\frac{1}{2}$ only twice?

In the set of fractions shown above, the range of the fractions is $\frac{3}{5}$, which is just a little bit more than $\frac{1}{2}$

Can you find other examples for which the range is more than $\frac{1}{2}$?

Can you find examples for which the range is less than $\frac{1}{2}$?

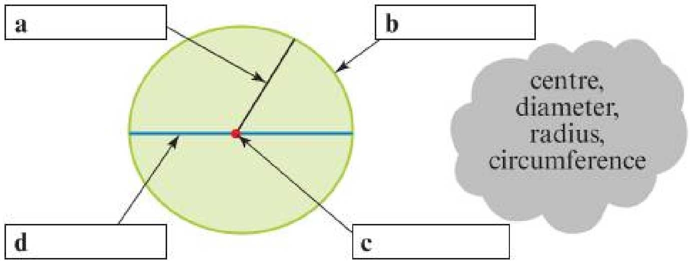
Can you find examples for which the range is exactly $\frac{1}{2}$?



8 Shapes and symmetry

Getting started

1 Label the parts of the circle shown. All the words you need are in the cloud.



2 What is the missing word in these sentences?

a In a **regular polygon**, all the sides are the .... length.

b In an **irregular polygon**, all the sides are not the .... length.

Tip

The same word is missing in both sentences.

3 What are the missing <u>numbers</u> in these descriptions of the properties of a rectangle?

a ☐ pairs of equal length sides

b ☐ pairs of parallel sides

c all angles are ☐°

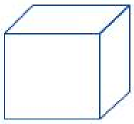
d ☐ lines of symmetry

e **rotational symmetry** of order ☐

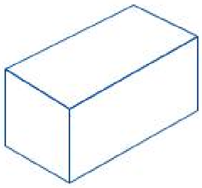
Continued

4 Match each of these 3-dimensional (3D) shapes with its correct name. Choose from the words in the box on the right.

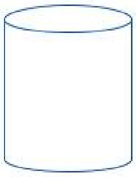
a



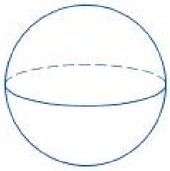
b



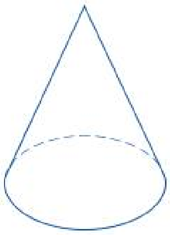
c



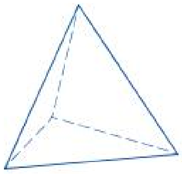
d



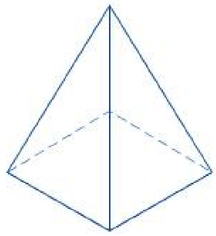
e



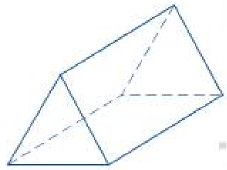
f



g



h



sphere,
tetrahedron,
cone,
cube,
square-based pyramid,
cylinder,
cuboid,
triangular prism

Throughout history, symmetry has been an important part of the design of everyday objects.

Granada is a city in the south of Spain, in the province of Andalucía. The Alhambra Palace in Granada is full of symmetrical designs.

The palace was built in the thirteenth century. It was originally designed as a military area, but it then became the residence of royalty and of the court of Granada.

You can see symmetry everywhere you look, from the design of the gardens and buildings to the tile patterns on the walls.

8.1 Identifying the symmetry of 2D shapes

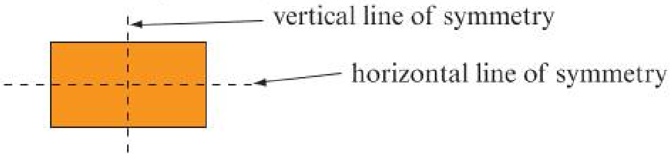
In this section you will …

- identify the reflective symmetry of 2D shapes and patterns
- identify the order of rotational symmetry of 2D shapes and patterns.

Key words

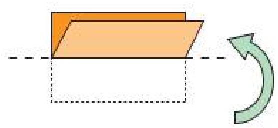
conjecture
full turn
horizontal line
line symmetry
once
rotational symmetry
twice
vertical line

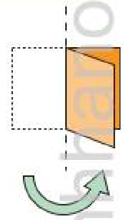
A 2-dimensional (2D) shape might have **line symmetry** and it might have rotational symmetry.



This rectangle has two lines of symmetry. One line of symmetry is **horizontal**. The other line of symmetry is **vertical**.

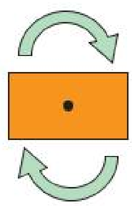
You can use dashed lines to show lines of symmetry on a shape.





When you fold a shape along its lines of symmetry, one half of the shape will fit exactly on top of the other half of the shape.

This rectangle also has rotational symmetry of order 2.



The order of rotational symmetry is the number of times a shape looks the same as it is rotated through one **full turn**.

Worked example 8.1

For each of these shapes, write down:

i the number of lines of symmetry

ii the order of rotational symmetry

a

b

Continued

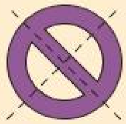
Answer

a **i** two lines of symmetry

ii order 2 rotational symmetry

This shape has two diagonal lines of symmetry.

In one full turn, the shape will look exactly the same twice: **once** after rotating through 180° and once again after 360°.



b **i** no lines of symmetry

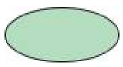
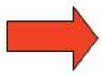
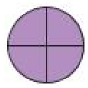
ii order 2 rotational symmetry

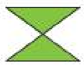
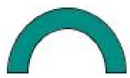
It is not possible to draw any lines of symmetry onto this parallelogram.

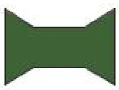
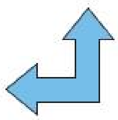
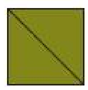
In one full turn, the parallelogram will look exactly the same twice: once after rotating through 180° and once again after 360°.

Exercise 8.1

1 Copy each of these shapes and draw on them the lines of symmetry.

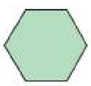
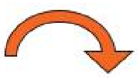
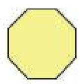
a  b  c  d 

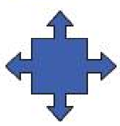
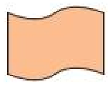
e  f  g  h 

i  j  k  l 

2 For each shape in Question **1**, write down the order of rotational symmetry.

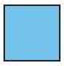
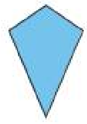
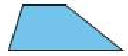
3 Write down the number of lines of symmetry for each of the following shapes.

a  b  c  d 

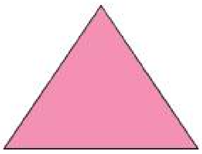
e  f  g  h 

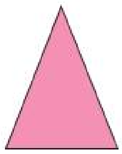
4 For each shape in Question **3**, write down the order of rotational symmetry.

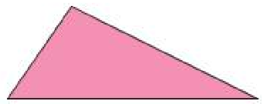
5 Copy and complete the table to show the symmetry properties of these quadrilaterals.

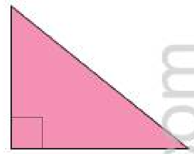
Shape	square 	rectangle 	rhombus 	parallelogram 	kite 	trapezium 	isosceles trapezium 
Number of lines of symmetry							
Order of rotational symmetry							

6 For each triangle shown:

a

equilateral triangle

b

isosceles triangle

c

scalene triangle

d

right-angled isosceles triangle

i Write down the number of lines of symmetry.

ii Write down the order of rotational symmetry.

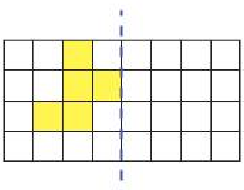
Think like a mathematician

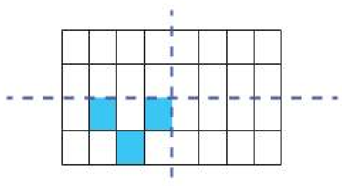
7 **a** How many lines of symmetry does a circle have?

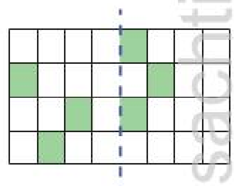
b What is the order of rotation of a circle?

8 In each diagram, the dashed lines are lines of symmetry.

a Copy and complete the pattern in each diagram.

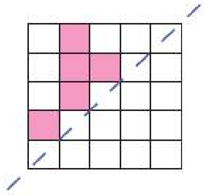
i


ii


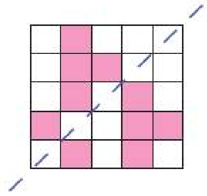
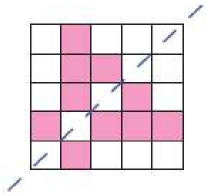
iii


b Write down the order of rotation of each of the patterns given in part **a**.

9 Ali and Ritesh are trying to complete this pattern.

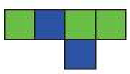
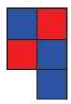
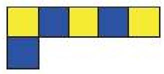
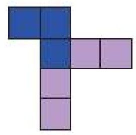


The dashed line is a line of symmetry.
This is what they draw.

Ali:  Ritesh: 

a Who has drawn the correct diagram?
b Explain the mistake that the other person has made.

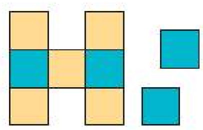
10 Copy these patterns onto squared grid paper.

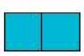
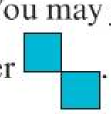
a  **b**  **c**  **d** 

i Add one blue square to each pattern to make a new pattern that has a line of symmetry.
ii Draw the line of symmetry onto each of your patterns.
iii Describe each line of symmetry; that is, is each line a horizontal, vertical or diagonal line of symmetry?

11 Sofia has made this pattern from yellow and blue tiles. She also has two spare blue tiles.

There are eight different ways I can add the two blue tiles to the pattern to make a pattern with only one line of symmetry. There are two different ways I can add the two blue tiles to the pattern to make a pattern with two lines of symmetry. There is only one way I can add the two blue tiles to the pattern to make a pattern with four lines of symmetry.



Show that Sofia's statements are correct. You may join the tiles either side to side  or corner to corner .

Think like a mathematician

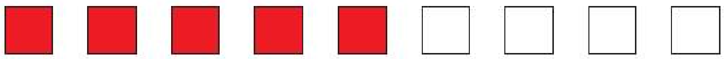
12 In pairs or groups, read through Question **13a**.

a Without drawing any diagrams, conjecture if the pattern of tiles will be a rectangle, a square or neither. How do you know?

b i Where in the pattern must you definitely place one red tile? Explain why.

ii Now draw the patterns.

13 Song has five red tiles and four white tiles.

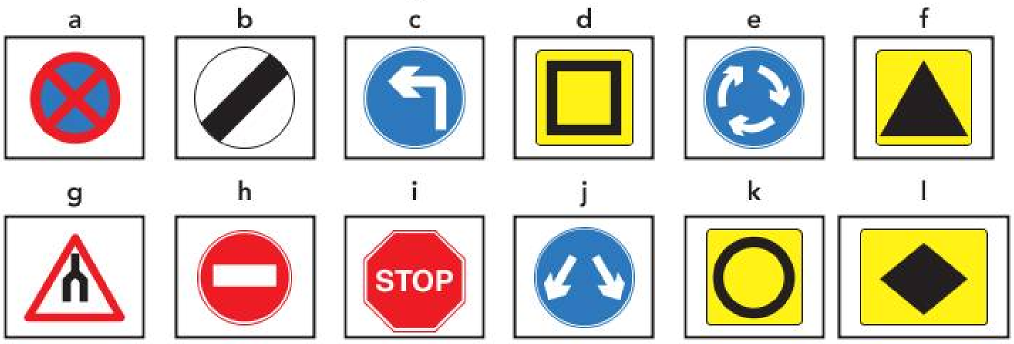


a Draw two different ways that Song could arrange these tiles so that he has a shape with an order of rotational symmetry of 4.

b For each of the patterns you drew in part **a**, how many lines of symmetry do your patterns of tiles have?

Activity 8.1

Here are 12 different road signs.



a Copy and complete this table to characterise these road signs.

	Road sign											
	a	b	c	d	e	f	g	h	i	j	k	l
Number of lines of symmetry												
Order of rotational symmetry												

b Compare your table with a partner's and check each other's work.

If you disagree with any of the answers, discuss why and then agree on the correct answer.

Summary checklist

- [ ] I can identify reflective symmetry of 2D shapes and patterns.
- [ ] I can identify the order of rotational symmetry of 2D shapes and patterns.

8.2 Circles and polygons

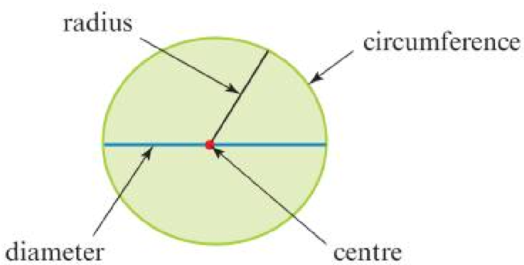
In this section you will ...

- name the parts of a circle
- identify, describe and sketch regular polygons.

Key words

chord
irregular polygon
polygon
regular polygon
sketch
scalene triangle
tangent

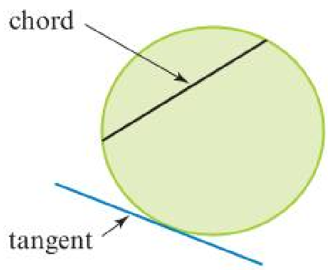
You already know these parts of a circle.



There are two other parts of a circle that you must know.

A **chord** of a circle is a straight line that starts and finishes on the circumference of the circle. If the chord passes through the centre of the circle, it is called the diameter.

A **tangent** to a circle is a straight line that touches the circumference of the circle at only one point.

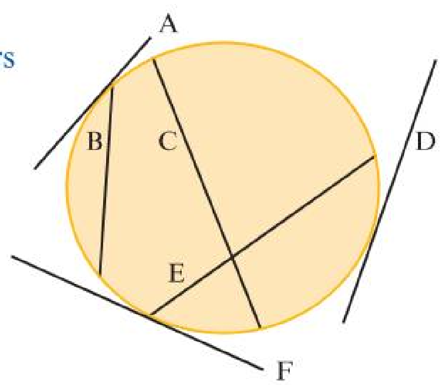


Worked example 8.2

For the diagram shown, write down the letters of the lines that are:

a tangents to the circle

b chords of the circle



Continued

Answer

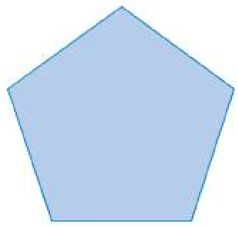
a	A, D, F	These lines touch the circumference of the circle at only one point, so they are tangents to the circle.
b	B, C, E	These lines start and finish on the circumference of the circle, so they are chords of the circle.

You already know the difference between a regular polygon and an irregular polygon.

You must be able to describe the properties of a regular polygon, as well as sketch the polygon.

For example, a regular pentagon has:

- five sides the same length
- five angles the same size
- five lines of symmetry
- rotational symmetry of order 5.



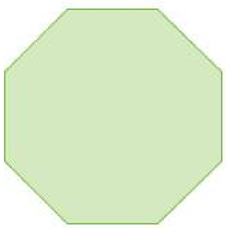
Worked example 8.3

a Sketch a regular octagon.

b Describe the properties of the octagon.

Answer

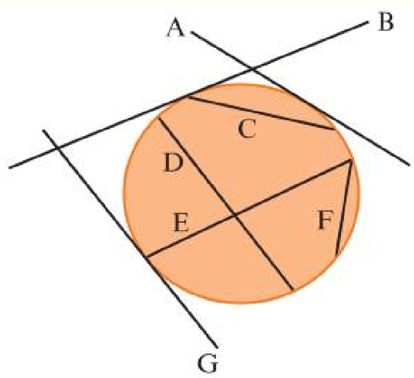
a



b A regular octagon has:

- eight sides the same length
- eight angles the same size
- eight lines of symmetry
- rotational symmetry of order 8.

Exercise 8.2

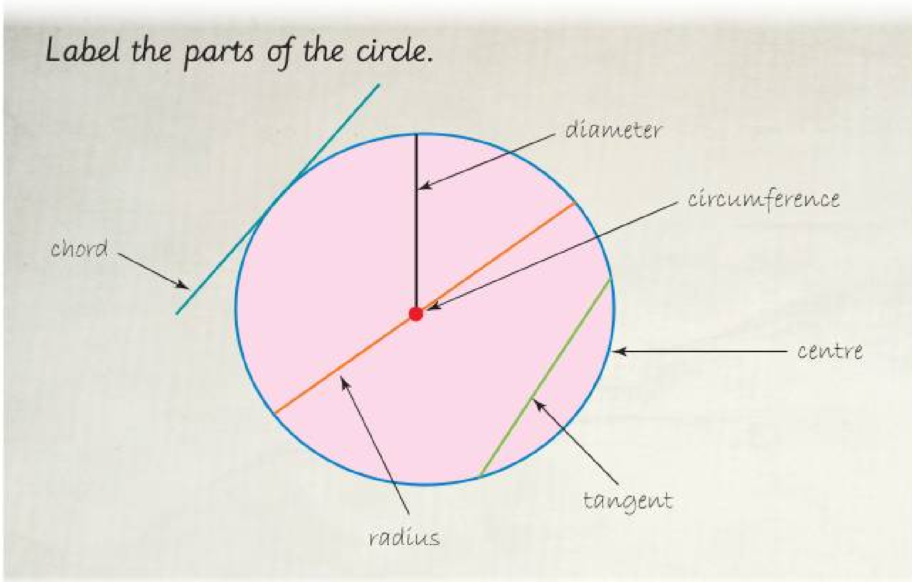


1 For the diagram shown, write down the letters of the lines that are:

a tangents to the circle

b chords of the circle

2 This is part of Omar's homework.



Omar has made a lot of mistakes.

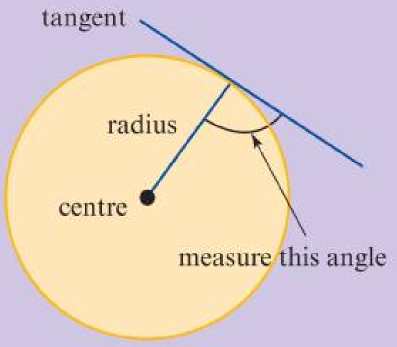
Copy the diagram and correctly label the parts of the circle.

Think like a mathematician

3 **a** Use a pair of compasses to draw four circles of different sizes.

b On each circle draw a tangent to the circle.

c At the point where your tangent meets your circle, draw a radius onto the circle. An example is shown.

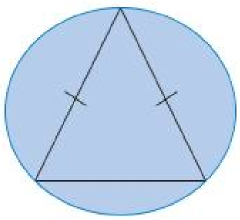


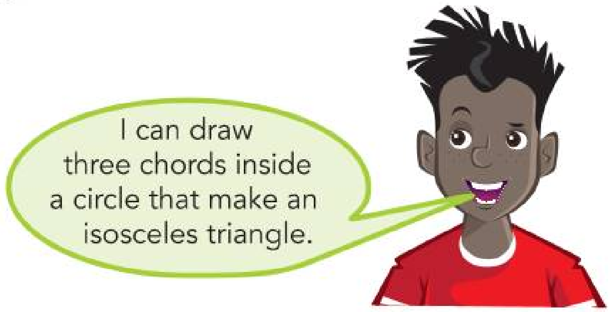
d On each of your circles, measure the angle the radius makes with the tangent.

e What do you notice about all your answers?

Discuss your answers with a partner. Copy and complete this general statement:
The angle between a tangent and a radius is always ____ °.

4 Marcus draws this diagram.





Draw three circles.

a Inside the first circle draw three chords that make a **scalene triangle**.

b Inside the second circle draw four chords that make a rectangle.

c Inside the third circle draw four chords that make a kite.

Think like a mathematician

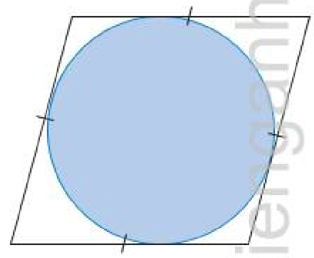
5 a In pairs or groups, discuss the best method to use to draw three chords inside a circle that will make a right-angled triangle.

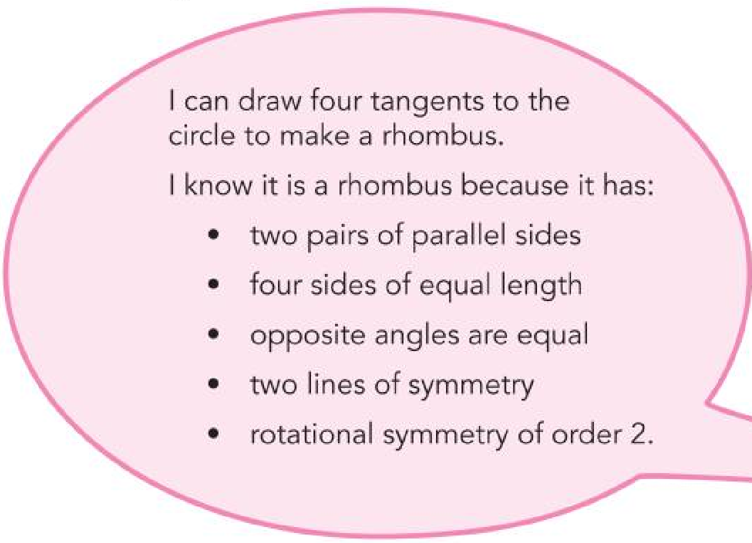
b Individually, draw the diagram from part **a**.

c When all learners in your group have drawn a diagram, compare your diagrams.

d What do you notice about the longest chord of the circle?

6 Zara draws this diagram.

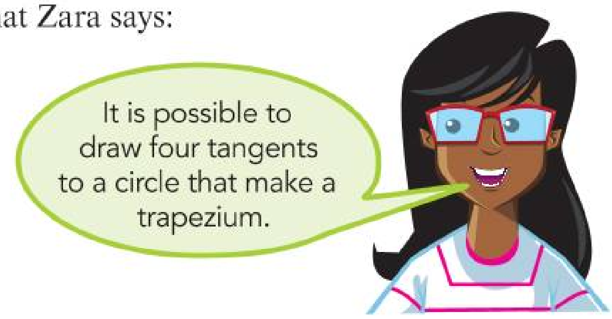




a Draw a circle. Draw four tangents to the circle to make a square.

b Describe the properties that characterise a square.

7 This is what Zara says:



a Is Zara correct? Draw a diagram to help you explain your answer.

b Describe the properties that characterise a trapezium.

8 a Sketch a regular hexagon.

b Describe the properties that characterise a regular hexagon.

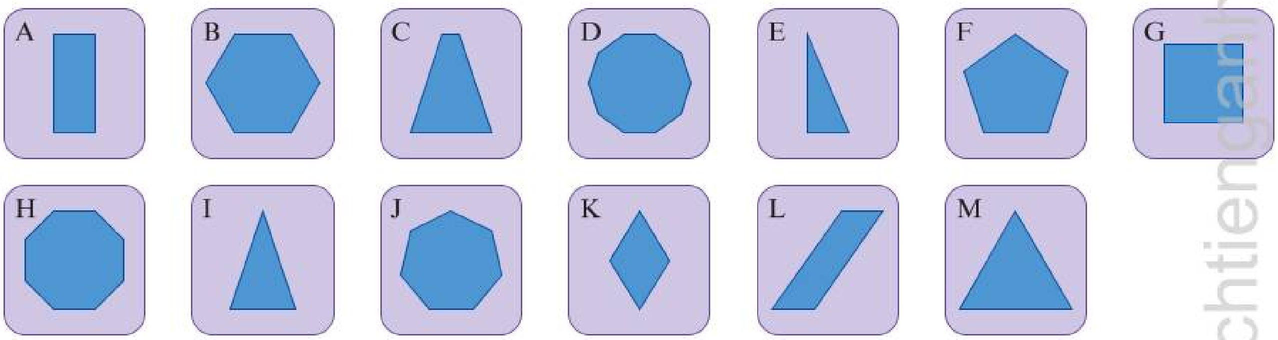
9 The diagram shows a regular decagon.

Copy and complete the properties that characterise a regular decagon.

A regular decagon has:

- ☐ sides the same length
- ☐ angles the same size
- ☐ lines of symmetry
- rotational symmetry of order ☐.

10 Yasiru has these cards. The cards have different shapes on them.



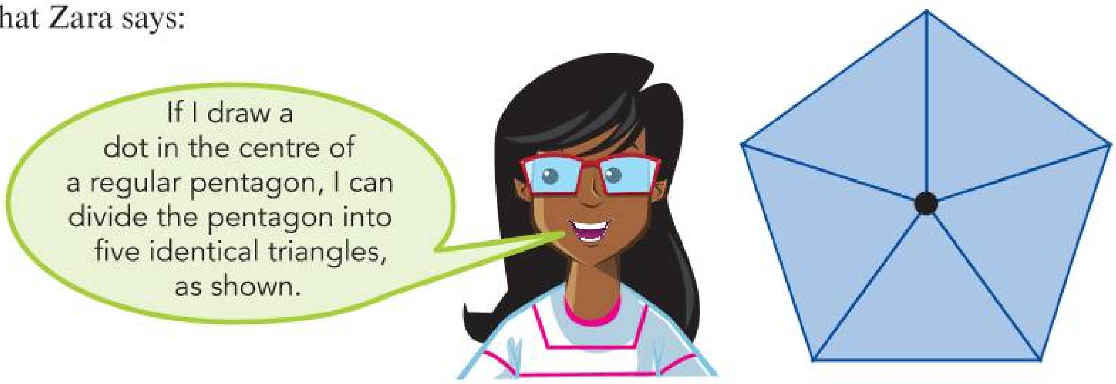
a Classify the cards into groups. You must have at least two groups. You can choose how you organise the shapes, but you must explain why you have put the shapes in these groups.

b Re-classify the cards into different groups. You must have at least two groups. Explain why you have put the shapes into their new groups.

Tip

You can classify using symmetry properties or lengths of sides or number of equal angles, etc.

11 This is what Zara says:



a Sketch a regular hexagon. Use Zara's method to divide the hexagon into identical triangles.

How many identical triangles are there?

b Without drawing any more shapes, copy and complete this table.

Explain how you worked out the answers.

Name of regular polygon	Number of identical triangles inside
pentagon	5
hexagon	
heptagon (7 sides)	
octagon	
nonagon (9 sides)	
decagon	

c To convince that your method is correct, draw one of the other regular polygons and divide the shape into identical triangles. How many identical triangles are there?

Write down the difference between a tangent and a chord.
Explain why the diameter of a circle is also a chord of the circle.
Explain why the diameter of a circle is never a tangent to the circle.

Summary checklist

- [ ] I can identify, describe and sketch regular polygons.
- [ ] I can name the parts of a circle.

› 8.3 Recognising congruent shapes

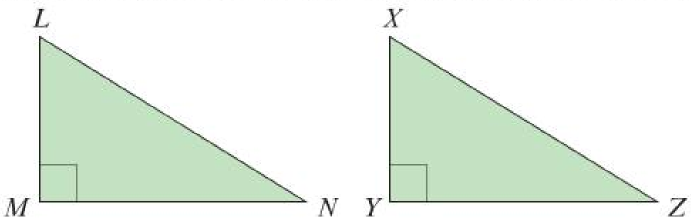
In this section you will …

- identify congruent shapes.

Key words

congruent
corresponding sides
corresponding angles
different orientation

Here are two right-angled triangles, LMN and XYZ.



Can you see that the triangles are identical in shape and size?

Shapes that are identical in shape and size are **congruent**.

The side LM is equal in length to the side XY. So sides LM and XY are **corresponding sides**.

Angle MLN is equal in size to angle YXZ.

So angle MLN and angle YXZ are **corresponding angles**.

In congruent shapes, corresponding sides are equal and corresponding angles are equal.

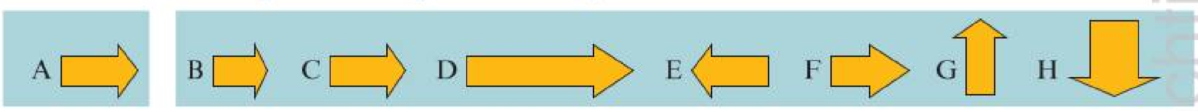
In triangles LMN and XYZ, $LM = XY$, $MN = YZ$ and $LN = XZ$ and $\angle MLN = \angle YXZ$, $\angle LNM = \angle XZY$ and $\angle NML = \angle ZYX$.

Tip

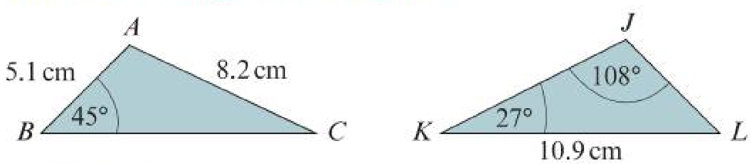
When an angle is described using three letters, the middle letter tells you the angle that is being discussed. So, the angle *LMN* is the angle at the vertex *M*. ∠ *MLN* is a mathematical way of writing 'angle *MLN*'.

Worked example 8.4

a Which of these shapes are congruent to shape A?



b These two triangles are congruent.



i Write down the lengths of the sides BC and JL.

ii What are the sizes of $\angle BAC$ and $\angle KLJ$?

Continued

Answer

a C, E and G

Even though E and G are pointing in different directions to A, they are still identical in shape and size, so they are congruent to A. Shapes B, D, F and H are not congruent to A because B is shorter than A, D is longer than A, F has a longer arrow head than A, and H is wider than A.

The two triangles are congruent but in a **different orientation**.

b **i** $BC = 10.9$ cm — BC and LK are corresponding sides, so $BC = LK$.

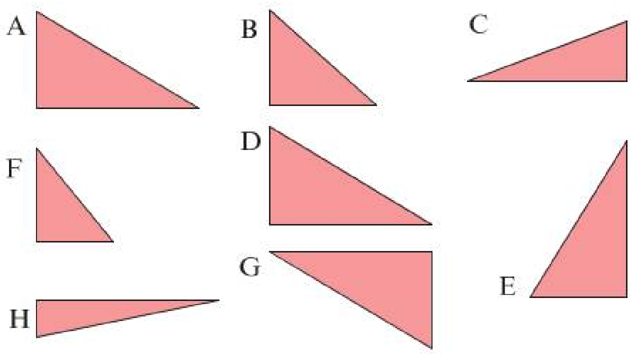
$JL = 5.1$ cm — JL and AB are corresponding sides, so $JL = AB$.

ii $\angle BAC = 108°$ — $\angle BAC$ and $\angle LJK$ are corresponding angles, so $\angle BAC = \angle LJK$.

$\angle KLJ = 45°$ — $\angle KLJ$ and $\angle CBA$ are corresponding angles, so $\angle KLJ = \angle CBA$.

Exercise 8.3

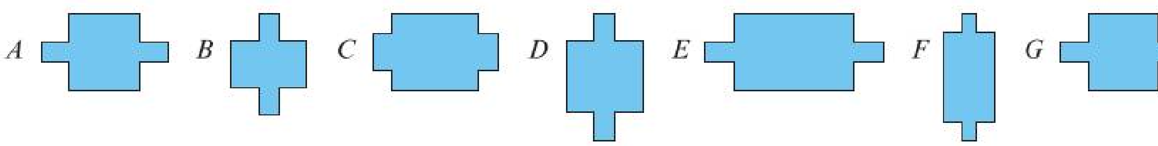
1 Which of these triangles are congruent to triangle A?



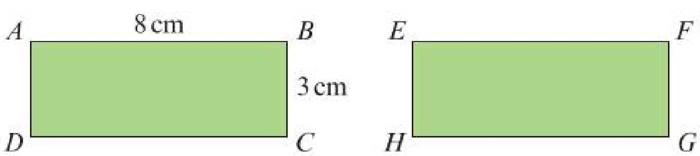
Tip

If you are not sure, use some tracing paper. Trace triangle A, then see which other triangles your tracing fits onto exactly. Remember you can rotate the tracing paper around or flip the tracing paper over.

2 Which of these shapes are congruent to shape A?



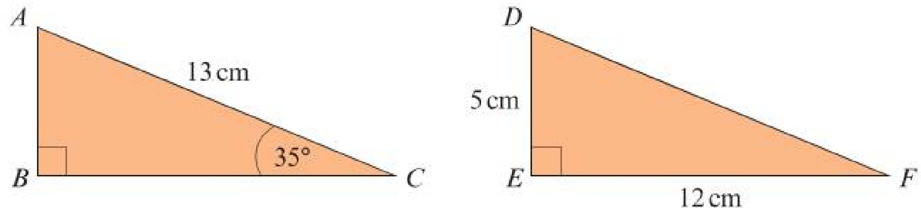
3 Rectangle $ABCD$ is congruent to rectangle $EFGH$.



a Write down the length of the side EF.

b Write down the length of the side FG.

4 Triangle ABC is congruent to triangle DEF.



a Write down the length of the side:

i AB **ii** BC **iii** DF

b Work out the size of $\angle BAC$.

c Write down the size of:

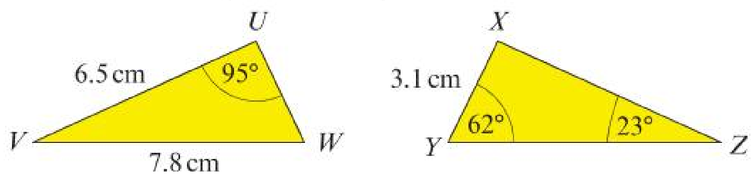
i $\angle EDF$ **ii** $\angle DFE$

d Copy and complete these sentences. The first one has been done for you.

i Side AB corresponds to side DE.
ii Side BC corresponds to side ☐.
iii Side AC corresponds to side ☐.
iv $\angle ABC$ corresponds to $\angle$☐.
v $\angle BAC$ corresponds to $\angle$☐.
vi $\angle ACB$ corresponds to $\angle$☐.

> **Tip**
>
> Remember that the angles in a triangle add up to 180°.

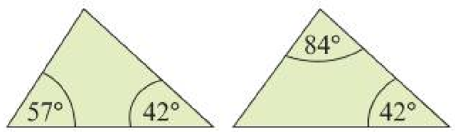
5 These two triangles are congruent.



a Write down the length of the side: **i** UW **ii** XZ **iii** YZ

b Write down the size of: **i** $\angle UVW$ **ii** $\angle UWV$ **iii** $\angle YXZ$

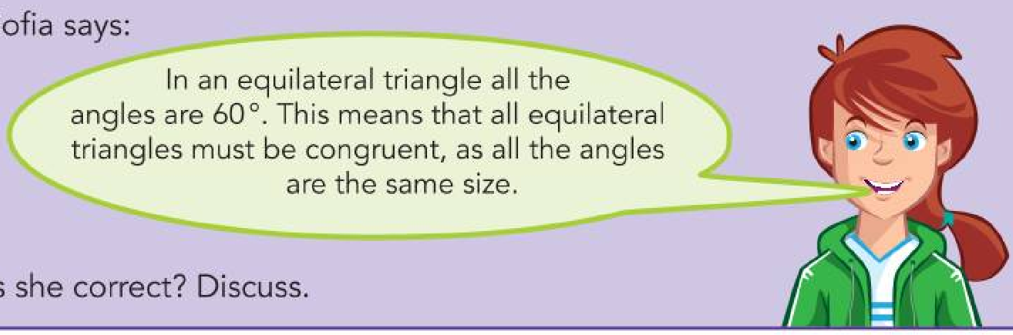
6 Sofia draws these triangles.



Without knowing the lengths of any of the sides, how do you know that Sofia is incorrect?

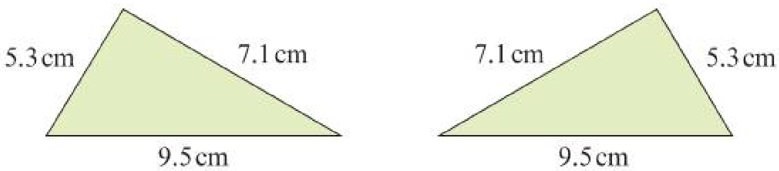
Think like a mathematician

7 Sofia says:

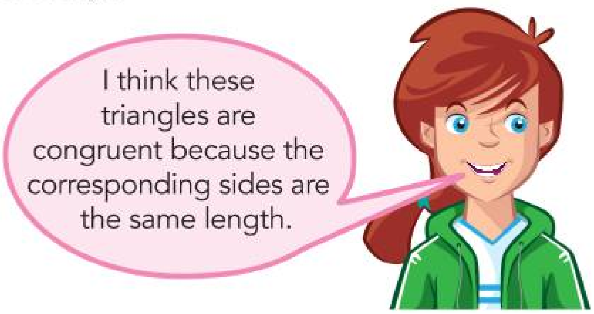


Is she correct? Discuss.

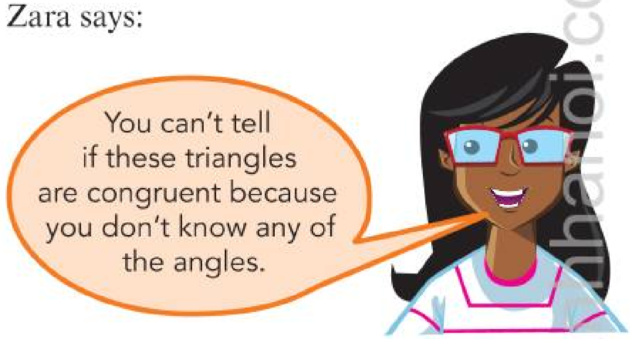
8 Sofia and Zara are looking at these two triangles.



Sofia says:

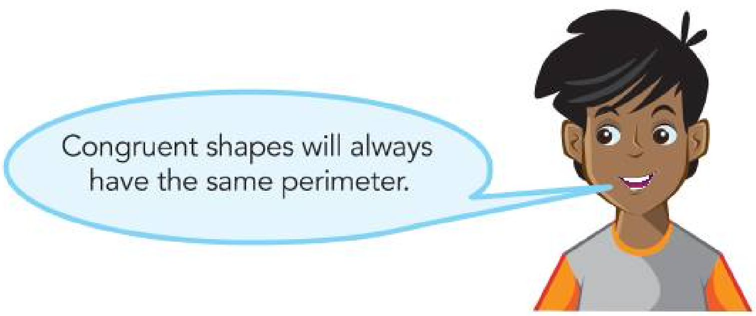


Zara says:



Who is correct, Sofia or Zara? Explain your answer.

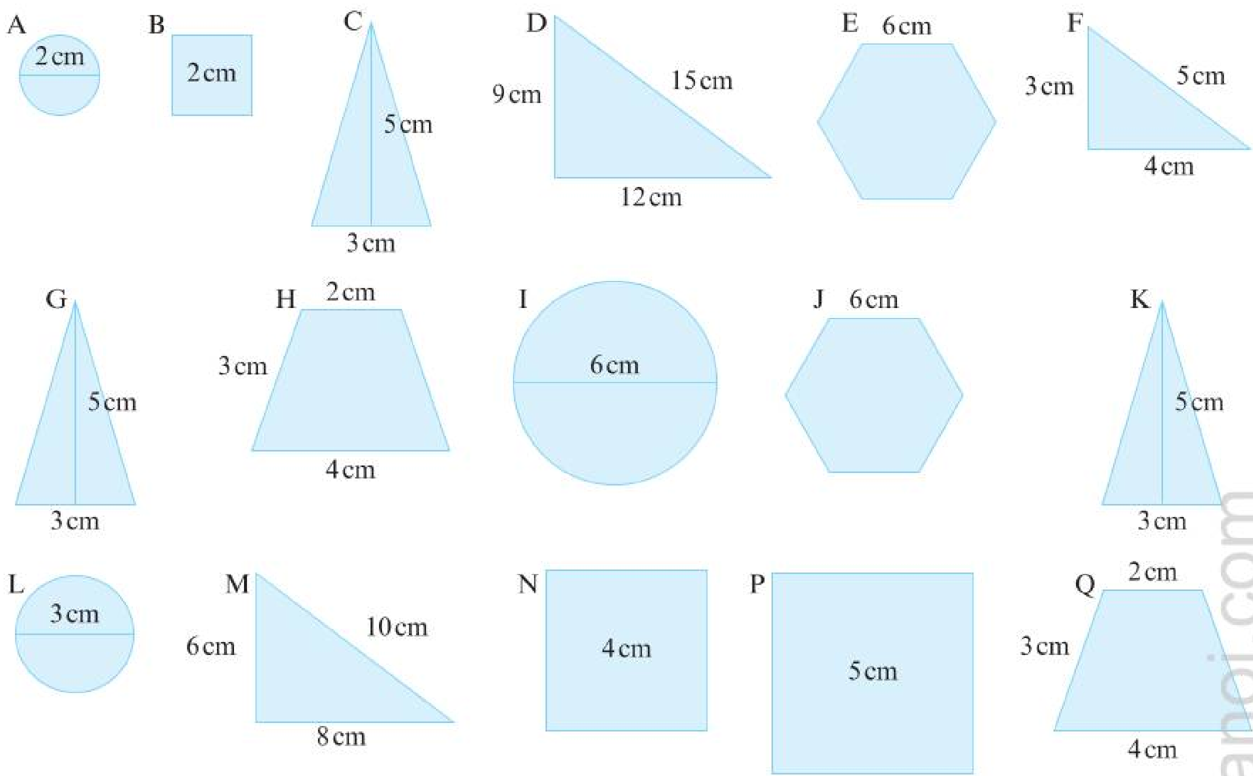
9 Arun is looking at the perimeters of congruent shapes.



a Is Arun correct? Explain your answer.

b What can you say in general about the areas of congruent shapes?

10 Classify these shapes into groups.
Describe the properties that characterise each of your groups.



Summary checklist

- [ ] I can identify congruent shapes.

8.4 3D shapes

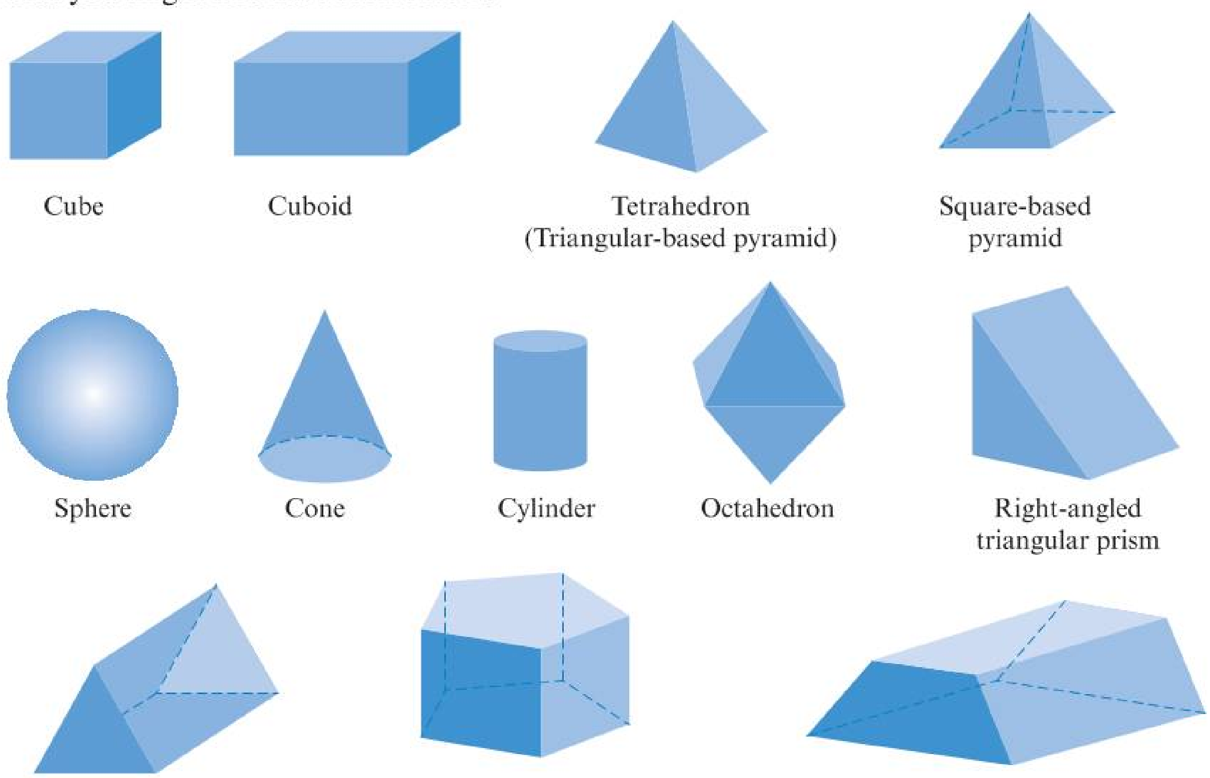
In this section you will …

- identify and describe 3D shapes
- draw front, side and top views of 3D shapes.

Key words

front view
side view
top view
visualise

You already know the names of some 3D shapes.
Here are the 3D shapes you should know, as well as some 3D shapes that you might not have seen before.



Cube; Cuboid; Tetrahedron (Triangular-based pyramid); Square-based pyramid

Sphere; Cone; Cylinder; Octahedron; Right-angled triangular prism

Equilateral triangular prism; Pentagonal prism; Trapezoidal prism

You must be able to identify and describe the properties of a 3D shape.
For example, a cuboid has:

- six faces
- eight vertices
- twelve edges
- all angles are right angles.

Tip

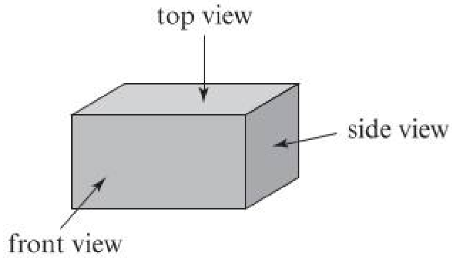
Remember that the vertices are the corners of a shape.

You must also be able to **visualise** and draw what a 3D shape looks like from different directions.

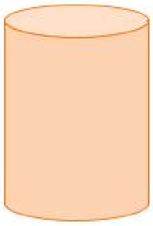
The **top view** is the view from above the shape.

The **front view** is the view from the front of the shape.

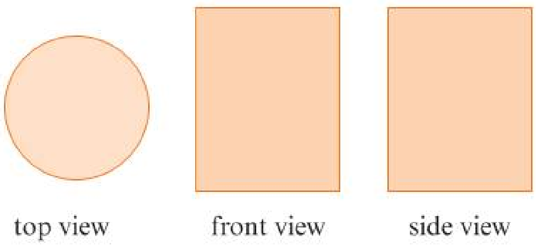
The **side view** is the view from the side of the shape.



Example: Look at this cylinder.



From above you will see a circle, but from the side and the front you will see a rectangle.



> **Tip**
>
> The top view can also be called the plan view or just plan. The side and front views can also be called the side and front elevations.

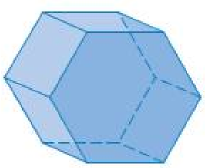
Worked example 8.5

a I have eight faces, where two of my faces are congruent regular hexagons and six of my faces are congruent rectangles. I have twelve vertices and eighteen edges. What shape am I?

b Draw the top view, front view and side view of the shape described in part **a**.

Answer

a hexagonal prism



Two of the faces are hexagons, so these faces must be the front and the back of the shape.

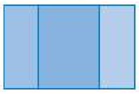
The other six faces are all rectangles, so they must join the front face to the back face.

A hexagon has six sides, so there will be six vertices at the front of the prism and six vertices at the back, giving a total of twelve vertices.

There will be six edges at the front of the prism, six edges at the back of the prism and six horizontal edges joining the front to the back, giving a total of eighteen edges.

Continued

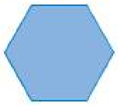
b top view



The sides of the hexagonal prism are all rectangles. When you look at the prism from the top, you will see one complete rectangle.

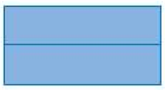
You will also see the rectangles that are on either side of the top one, but they will look smaller because they are at an angle to the top one.

front view



Looking at the front of the shape you will see a hexagon.

side view

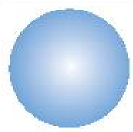
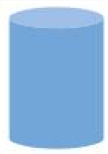
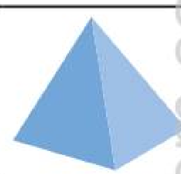
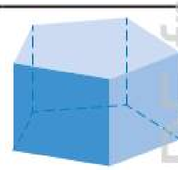


When you look at the prism from the side, you will see two rectangles.

They will look smaller than their actual size because they are at an angle to the vertical.

Exercise 8.4

1 Match the properties (**a** to **d**) of each 3D shape with its name (**A** to **D**) and picture (**i** to **iv**). The first one has been done for you.

Properties	Name of shape	Picture of shape
a I have two faces that are congruent circles. I have one curved surface. I have two edges and no vertices.	**A** tetrahedron	i 
b I have four faces. All of my faces are congruent equilateral triangles. I have six edges and four vertices.	**B** pentagonal prism	ii 
c I have only one curved surface. I have no edges or vertices.	**C** cylinder	iii 
d I have seven faces. Two of my faces are congruent pentagons and the other faces are congruent rectangles. I have 15 edges and ten vertices.	**D** sphere	iv 

2 Write down the properties that characterise a cube.
You must include information about the faces, edges and vertices.

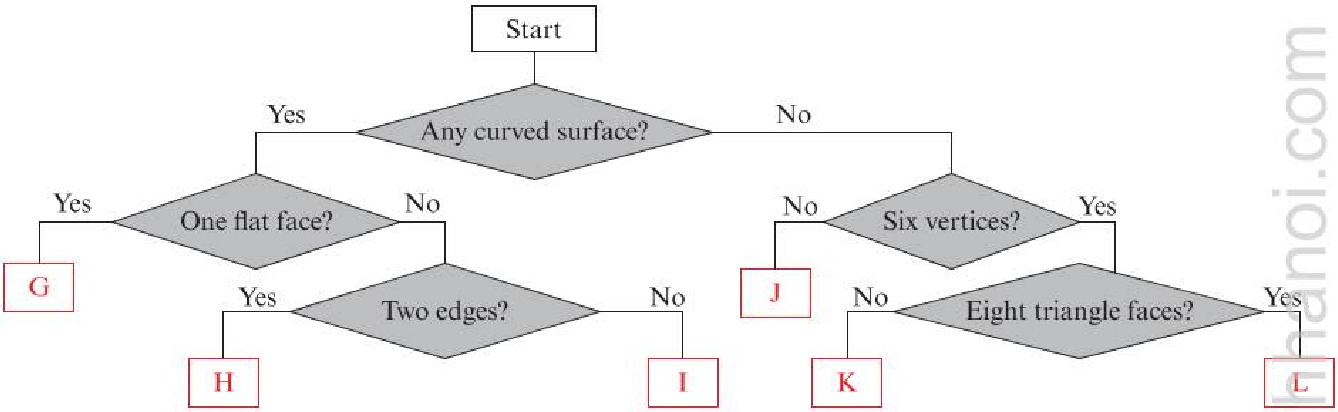
Activity 8.2

a Choose a 3D shape. On a piece of paper write down the properties of that shape.

b Swap your piece of paper with a partner's shape. Try to work out what shape they are describing.

c Check each other's answers. If either of you couldn't work out the shape your partner was describing, then discuss what information was either incorrect or missing.

4 For each of the shapes given, work through this classification flow chart.
Write down the letter you get at the end for each shape.



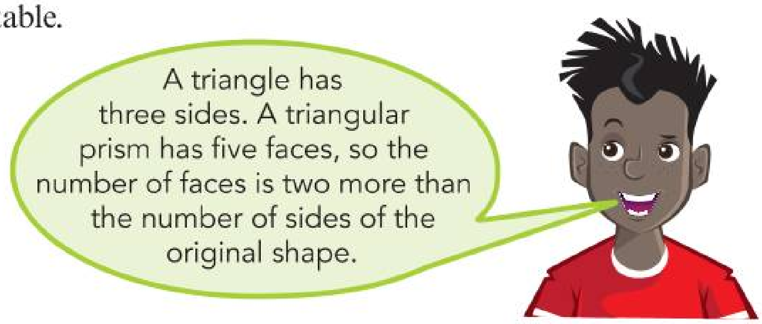
a hexagonal prism **b** cone **c** triangular prism
d sphere **e** octahedron **f** cylinder

5 Marcus makes a table that shows the number of faces, vertices and edges of different prisms.
This is what he has done so far.

Original shape	Number of sides	Shape of prism	Number of faces	Number of vertices	Number of edges
triangle	3	triangular	5	6	9
rectangle	4	rectangular	6	8	12
pentagon	5	pentagonal	7	10	15
hexagon		hexagonal			
heptagon		heptagonal			
octagon		octagonal			

a Copy and complete the table.

b Marcus says:



Is this true for every prism? Explain your answer.

c **i** Look back at the table. Compare the number of sides of the original shapes with the number of vertices of the prisms. What do you notice?

ii Write down a general rule that connects the number of sides of the original shapes with the number of vertices of the prisms.

d **i** Look back at the table. Compare the number of sides of the original shapes with the number of edges of the prisms. What do you notice?

ii Write down a general rule that connects the number of sides of the original shapes with the number of edges of the prisms.

e Use your answer to part **d** to complete this statement.

The number of edges of a prism is always a multiple of _________.

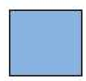
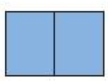
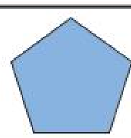
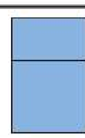
Think like a mathematician

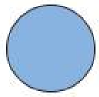
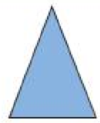
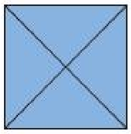
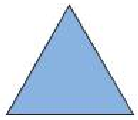
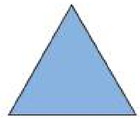
6 Discuss with a partner or in groups the rules that you found in parts **c** and **d** of Question **5**.

Can you explain how these rules work? Would these rules be the same for any prisms? Explain why.

How can you write these rules as formulae, using algebra?

7 The table shows the top view, front view and side view of some 3D shapes. The names of the 3D shapes are missing.

Name of 3D shape	Top view	Front view	Side view
a			
b			

Name of 3D shape	Top view	Front view	Side view
c			
d			

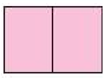
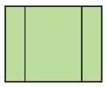
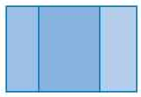
Write the missing names of the 3D shapes.

8 Draw the top view, front view and side view of a right-angled triangular prism.

Think like a mathematician

9 What 3D shapes have the same top view, front view and side view?

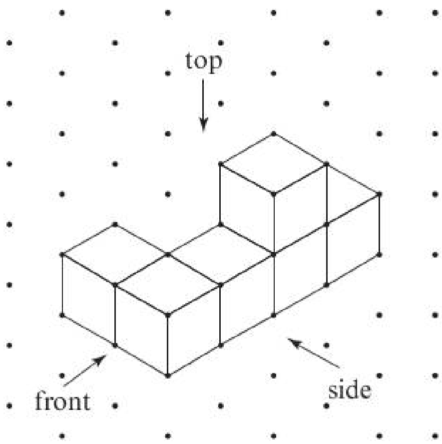
10 These diagrams show the top view of three prisms.

A  B  C 

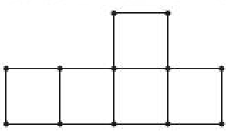
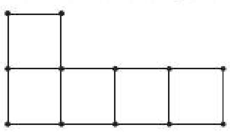
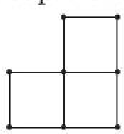
Match each diagram with the correct name of the prism given in the coloured box. Explain how you worked out your answer.

hexagonal prism | pentagonal prism | octagonal prism

11 Emily is drawing 3D shapes on isometric paper.
This is the first shape she draws.

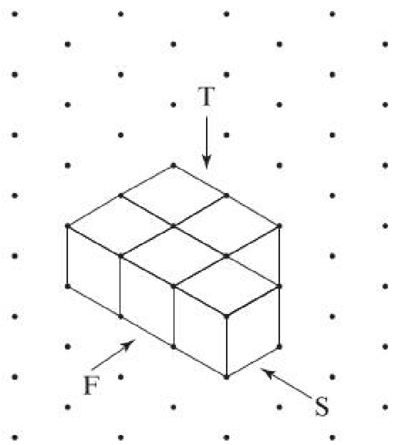


a Write down which of these is the front view, side view and top view.

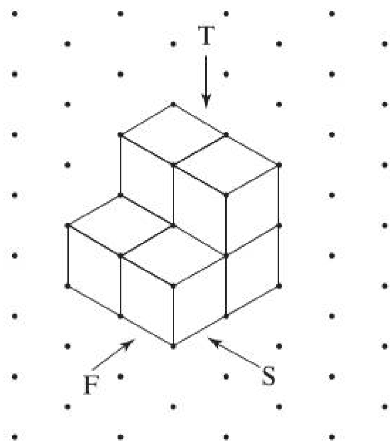
i  ii  iii 

b Draw the front view, side view and top view for each of the shapes **i** to **iv**.
The arrows show the direction that you must look at the shape for the front view (F), side view (S) and top view (T).

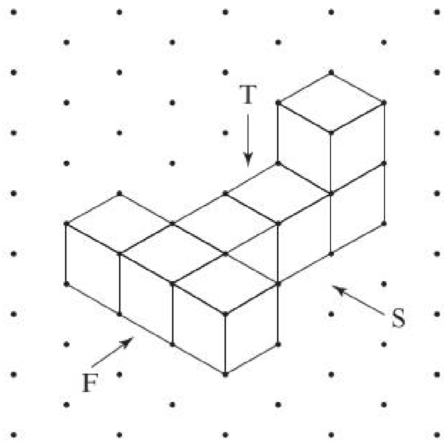
i



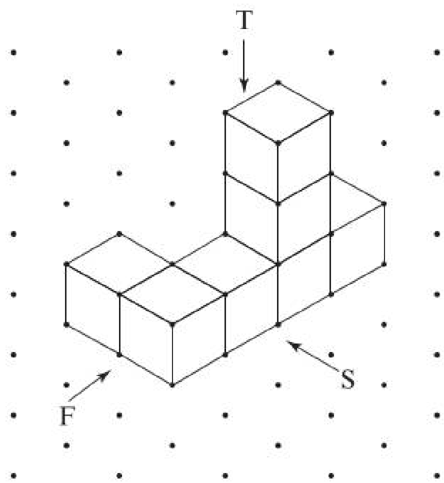
ii



iii



iv



Review this exercise.
How confident do you feel in your understanding of this section?
What can you do to increase your level of confidence?

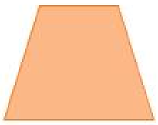
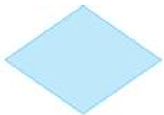
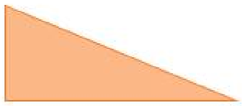
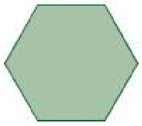
Summary checklist

- [ ] I can identify and describe 3D shapes.
- [ ] I can draw top, front and side views of 3D shapes.

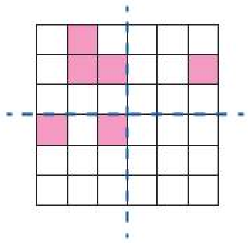
Check your progress

1 **a** Write down the number of lines of symmetry for each of these shapes.

b Write down the order of rotational symmetry for each of these shapes.

i  ii  iii  iv 

2 Copy and complete the pattern so that the dashed lines are lines of symmetry.

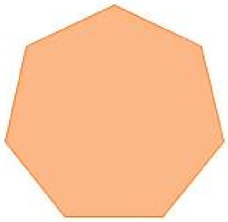


3 The diagram shows a regular heptagon.

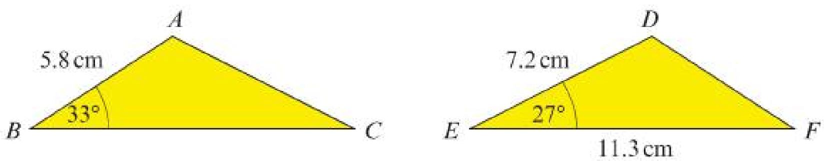
Copy and complete the properties that characterise a regular heptagon.

A regular heptagon has:

- ☐ sides the same length
- ☐ angles the same size
- ☐ lines of symmetry
- rotational symmetry of order ☐.



4 Triangle ABC is congruent to triangle DEF.



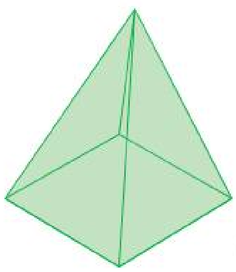
a Write down the length of side AC.

b Write down the size $\angle DFE$.

c Work out the size of $\angle BAC$.

5 **a** Write down the properties that characterise a square-based pyramid. You must include information about the faces, edges and vertices.

b Draw the top view, front view and side view of a square-based pyramid.



9 Sequences and functions

Getting started

1 Work out:

a $8+6$ **b** $17-5$ **c** $-2+6$ **d** $9-12$

e 5×7 **f** $24\div6$ **g** -3×6 **h** $35\div-7$

2 Copy and complete these calculations.

All the missing numbers are given in the rectangle on the right.

9	−5
14	7
5	10
4	6
8	3

a $4+\square=11$ **b** $23-\square=18$ **c** $\square+3=12$

d $-3+\square=3$ **e** $\square-8=-5$ **f** $10-\square=-4$

g $3\times\square=12$ **h** $\square\times12=96$ **i** $\square\div-2=\square$

3 Work out the following. Remember to use the correct order of operations.

a $5+4\times3$ **b** $(5+4)\times3$ **c** $6+4\div2$ **d** $(6+4)\div2$

e $\frac{12}{3}+9$ **f** $\frac{12+9}{3}$ **g** $-2\times3+10$ **h** $-5-\frac{10}{5}$

4 Work out the value of each expression.

a $n+4$ when $n=6$ **b** $6n$ when $n=3$

c $n-9$ when $n=20$ **d** $n-10$ when $n=8$

Leonardo Pisano, a famous mathematician, was born around 1170 in Pisa, Italy. Later, he became known as Fibonacci.

Fibonacci wrote several books. In one of his books, he included a number pattern that he discovered in 1202.

The number pattern is known as the Fibonacci **sequence**.

1, 1, 2, 3, 5, 8, 13, 21, 34, …

Can you see the pattern?

To find the next number in the pattern, you add the previous two numbers.

So, $1 + 1 = 2$

$1 + 2 = 3$

$2 + 3 = 5$

$3 + 5 = 8$

$5 + 8 = 13$ and so on.

Fibonacci (1170–1250)

The numbers in the Fibonacci sequence are called the Fibonacci numbers.

The Fibonacci numbers often appear in pairs in nature. For example, the numbers of petals on flowers often follow the Fibonacci numbers. The numbers of spirals in seed heads or pine cones often follow the Fibonacci numbers.

A sunflower can have 34 spirals turning clockwise and 21 spirals turning anticlockwise.
1, 1, 2, 3, 5, 8, 13, 21, 34, 55 ...

A pine cone can have eight spirals turning clockwise and 13 spirals turning anticlockwise.
1, 1, 2, 3, 5, 8, 13, 21, 34, 55 ...

9.1 Generating sequences (1)

In this section you will ...

- find and use term-to-term rules for number sequences.

Key words

consecutive terms
finite sequence
infinite sequence
sequence
term
term-to-term rule

3, 6, 9, 12, 15, … is a sequence of numbers.

Each number in the sequence is called a **term**. The first term is 3, the second term is 6, and so on.

Terms that follow each other are called **consecutive terms**. 3 and 6 are consecutive terms, 6 and 9 are consecutive terms, and so on. Each term is 3 more than the term before, so the **term-to-term rule** is 'Add 3'.

Three dots (called ellipses) written at the end of a sequence show that the sequence continues forever. A sequence that continues forever is called an **infinite sequence**.

When a sequence doesn't have the three dots at the end, then it doesn't continue forever. This type of sequence is called a **finite sequence**.

Worked example 9.1

a Write down the term-to-term rule and the next two terms of this sequence.

2, 6, 10, 14

b The first term of a sequence is 5.

The term-to-term rule of the sequence is: Multiply by 2 and then add 1.

Write down the first three terms of the sequence.

Answer

a Term-to-term rule is 'Add 4.'

You can see that the terms are going up by 4 every time, as $2+4=6$, $6+4=10$ and $10+4=14$.

Next two terms are 18 and 22.

Keep adding 4 to find the next two terms: $14+4=18$ and $18+4=22$.

b First three terms are 5, 11, 23.

Write down the first term, which is 5. Then use the term-to-term rule to work out the second and third terms.

Second term $=5\times2+1=11$,
third term $=11\times2+1=23$.

Exercise 9.1

1 Copy and complete the workings to find the term-to-term rule and the next two terms of each sequence.

a 7, 11, 15, 19, …

7 + 4 = 11, 11 + ☐ = 15, 15 + ☐ = 19
The term-to-term rule is: Add ☐.
The next two terms are: 19 + ☐ = ☐
☐ + ☐ = ☐

b 40, 35, 30, 25, …

40 − 5 = 35, 35 − ☐ = 30, 30 − ☐ = 25
The term-to-term rule is: Subtract ☐.
The next two terms are: 25 − ☐ = ☐
☐ − ☐ = ☐

2 For each of these infinite sequences, write down:

i the term-to-term rule **ii** the next two terms

a 2, 4, 6, 8, ☐, ☐
b 1, 4, 7, 10, ☐, ☐
c 5, 9, 13, 17, ☐, ☐
d 3, 8, 13, 18, ☐, ☐
e 30, 28, 26, 24, ☐, ☐
f 17, 14, 11, 8, ☐, ☐

3 Write down the first three terms of each of these sequences. Show your working.

	First term	Term-to-term rule
a	1	add 5
b	45	subtract 7
c	6	multiply by 2
d	60	divide by 2

Tip

For part **a**, work out:
1 + 5 = 6
then 6 + 5 = ☐
then ☐ + 5 = ☐

4 Copy these finite sequences and fill in the missing terms.

a 2, 5, ☐, 11, ☐, 17, 20
b 5, 11, 17, ☐, ☐, 35, ☐
c 26, 23, ☐, ☐, 14, ☐, 8
d 90, 82, ☐, 66, ☐, 50, ☐
e 8, ☐, ☐, 32, 40, ☐, ☐
f ☐, ☐, 28, 23, ☐, ☐, 8

5 Write down whether each of these sequences is finite or infinite.

a 4, 6, 8, 10, … **b** 3, 5, 7, 9, 11, 13, 15

c 85, 75, 65, 55, 45, 35 **d** 100, 97, 94, 91, 88, …

6 Write down the first three terms of each of these sequences.
The first one has been started for you.

a First term is 4. Term-to-term rule is: Multiply by 2 then add 1.

> *First term* $= 4$
> *Second term* $= 4 \times 2 + 1 = 8 + 1 = 9$
> *Third term* $= 9 \times 2 + 1 = \square + 1 = \square$

b First term is 10. Term-to-term rule is: Subtract 8 then multiply by 6.

c First term is 24. Term-to-term rule is: Divide by 2 then add 4.

7 Match each first term to its sequence and its term-to-term rule.
The first one has been done for you.

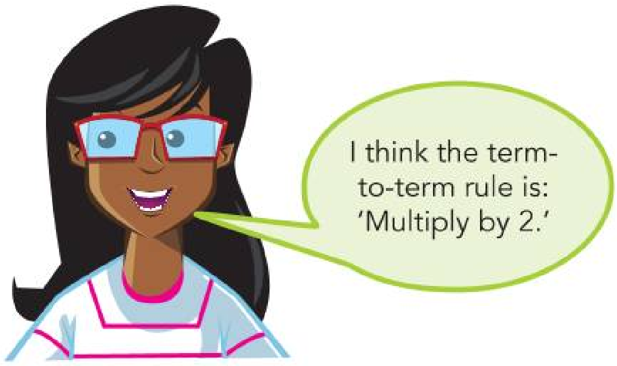
First term	Sequence	Term-to-term rule
A 3	a 11, 14, 17, 20, …	i subtract 2
B 80	b 17, 15, 13, 11, …	ii divide by 2
C 64	c 3, 6, 12, 24, …	iii multiply by 5 then add 1
D 11	d 80, 40, 20, 10, …	iv multiply by 2
E 17	e 1, 6, 31, 156, …	v divide by 2 then add 4
F 1	f 64, 36, 22, 15, …	vi add 3

Tip

Remember, an infinite sequence continues forever. A finite sequence does not continue forever.

8 Sofia and Zara are looking at this number sequence: 4, 8, 20, 56, 164, …

Sofia says: Zara says:



They are both incorrect. Explain why.
What must Sofia and Zara do to answer the question correctly?

Think like a mathematician

9 **a** In general, how many terms of a sequence must you know to be able to work out the term-to-term rule?

b What are the term-to-term rules for the sequences 2, 6, … and 3, 10, …?

10 Write down three possible term-to-term rules for each of these sequences and work out the next two terms for each rule. The first one has been started for you.

a 3, 7, …

Rule 1: Add 4.
Next two terms are: $7 + 4 = 11$
$11 + 4 = 15$

Rule 2: Multiply by 2 then add 1.
Next two terms are: $7 \times 2 + 1 = 15$
$15 \times 2 + 1 = 31$

Rule 3: Multiply by 3 then subtract 2.
Next two terms are: $7 \times 3 - 2 = \square$
$\square \times 3 - 2 = \square$

b 4, 8, … **c** 2, 9, … **d** 7, 17, …

11 Ryker is trying to solve this problem. Work out the answer to the problem.

> *The second term of a sequence is 13.*
> *The term-to-term rule is: Multiply by 2 then subtract 3.*
> *What is the first term of the sequence?*

Look back at your answer to Question **11**.

Write a short explanation of the method you used to solve this problem.

Discuss your method with a partner. Did they use the same method?

Can you think of a better method?

Activity 9.1

On a piece of paper, write down three questions that are similar to Question **2**, and three questions that are similar to Question **6**.

Work out the answers on a separate piece of paper.

Make sure the questions can be answered without using a calculator.

Swap your questions with those of a partner. Work out the answers to your partner's questions.

Swap your papers back and mark each other's work.

If you think your partner has made a mistake, discuss their mistake with them.

Summary checklist

- [ ] I can find term-to-term rules for number sequences.
- [ ] I can use term-to-term rules for number sequences.

9.2 Generating sequences (2)

In this section you will …

- find and use term-to-term rules for number sequences drawn as patterns.

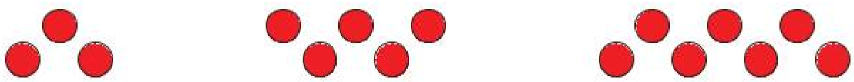
Key words

sequence of patterns

term-to-term rule

Here is a **sequence of patterns** made from dots.

pattern 1 pattern 2 pattern 3



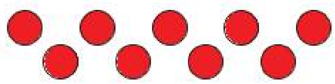
3 dots 5 dots 7 dots

The numbers of dots used to make each pattern form the sequence 3, 5, 7, …

You can see that, as you go from one pattern to the next, one extra dot is added to each end of the pattern. So, each pattern has two more dots than the previous pattern. The **term-to-term rule** is 'Add 2'.

The next pattern in the sequence has nine dots because $7+2=9$.

pattern 4



9 dots

Worked example 9.2

Here is a sequence of patterns of triangles made from matchsticks.

pattern 1 pattern 2 pattern 3

3 matchsticks 6 matchsticks 9 matchsticks

a Draw the next pattern in the sequence.

b Write down the sequence of numbers of matchsticks.

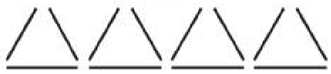
c Write down the term-to-term rule.

d Explain how the sequence is formed.

Continued

Answer

a pattern 4



12 matchsticks

The next pattern will have another triangle added to the end.
So pattern 4 has 12 matchsticks.

b 3, 6, 9, 12

Write down the number of matchsticks for each pattern.

c Add 3

Each term is 3 more than the previous term.

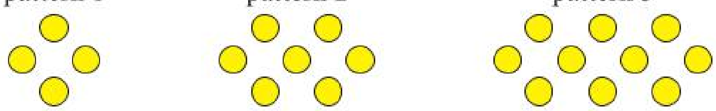
d An extra triangle is added, so three more matchsticks are added.

Describe in words how the pattern increases from one term to the next.

Exercise 9.2

1 This sequence of patterns is made from dots.

pattern 1 | pattern 2 | pattern 3



4 dots | 7 dots | 10 dots

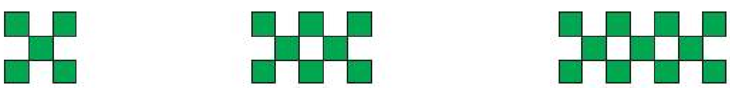
a Draw the next two patterns in the sequence.
b Write down the sequence of numbers of dots.
c Write down the term-to-term rule.
d Explain how the sequence is formed.

Tip

In part **d**, say how many dots are added to each pattern to get the next pattern.

2 This sequence of patterns is made from squares.

pattern 1 | pattern 2 | pattern 3



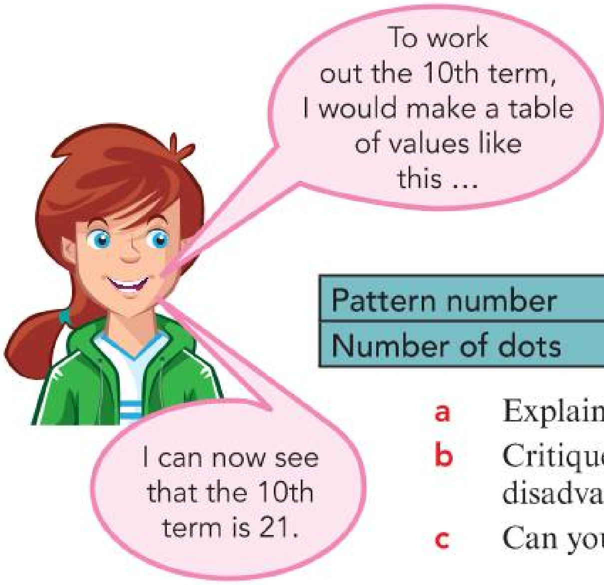
5 squares | 8 squares | 11 squares

a Draw the next two patterns in the sequence.
b Copy and complete the table to show the number of squares in each pattern.

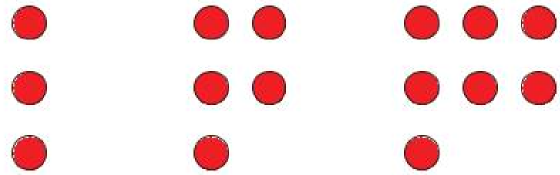
Pattern number	1	2	3	4	5
Number of squares	5	8	11		

c Write down the term-to-term rule.
d How many squares will there be in:
i pattern 6? **ii** pattern 8?

3 This sequence of patterns is made from dots.



pattern 1 pattern 2 pattern 3



3 dots 5 dots 7 dots

Pattern number	1	2	3	4	5	6	7	8	9	10
Number of dots	3	5	7	9	11	13	15	17	19	21

a Explain the method that Sofia has used.

b Critique this method by explaining the advantages and disadvantages of this method.

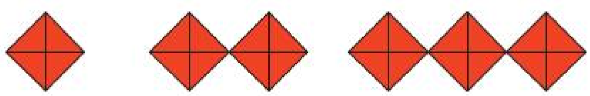
c Can you think of a better method that Sofia could use?

Think like a mathematician

4 In pairs, groups or as a class, compare your answers to Question **3**. Discuss the advantages and disadvantages of Sofia's method, as well as any other methods that you could use.

5 This sequence of patterns is made from triangles.

pattern 1 pattern 2 pattern 3



4 triangles 8 triangles 12 triangles

a Copy and complete the table to show the number of triangles in each pattern.

Pattern number	1	2	3	4	5
Number of triangles	4				

b Yusef thinks that one of the patterns will have 42 triangles in it.
Is he correct? Explain your answer.

c Frances thinks that one of the patterns will have 93 triangles in it.
Without doing any calculations, explain how you can tell that she is incorrect.

d What can you say about the number of triangles used in this sequence of patterns?
Discuss your answer with a partner.

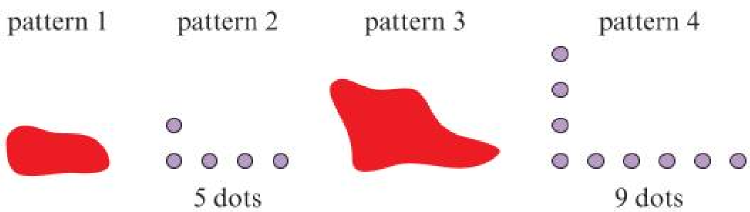
Activity 9.2

Design your own sequence of patterns made from a shape of your choice.
Draw the first five patterns in your sequence.
Draw a table to show the number of shapes in each of your patterns.
Work out the number of shapes in your:

a 10th pattern **b** 15th pattern

Ask a partner to check that your work is correct.

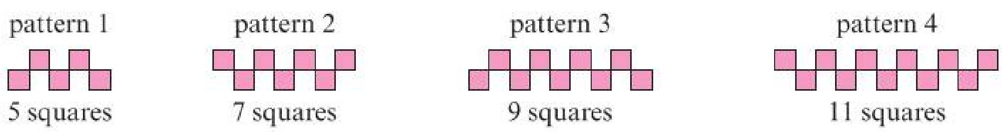
6 Jacob is using dots to draw a sequence of patterns.
There is a mark over the first and third patterns in his sequence.



a Draw the first and the third patterns of Jacob's sequence.

b How many dots will there be in pattern 7?

7 Sofia and Marcus are looking at this sequence of patterns made from squares.



Who is correct? Explain your answer.

Summary checklist

☐ I can find the term-to-term rules for number sequences drawn as patterns.
☐ I can use the term-to-term rules for number sequences drawn as patterns.

9.3 Using the *n*th term

In this section you will ...

- use algebra to describe the *n*th term of a sequence.

Key words

fewest
*n*th term
position number

Look at this sequence of numbers: 3, 6, 9, 12, 15, …
You can write this sequence in a table of values, as shown.

Position number	1	2	3	4	5
Term	3	6	9	12	15

You can see the terms in this sequence are the 3 times table.
So, if you multiply the position number by 3, you will get the terms of the sequence like this:

Position number	1	2	3	4	5
× 3	× 3	× 3	× 3	× 3	× 3
Term	3	6	9	12	15

Tip

The **position number** is where each term is in the sequence.

So the first term is 3.

The second term is 6.

The third term is 9, and so on.

The first term $= 1 \times 3 = 3$

The second term $= 2 \times 3 = 6$

The third term $= 3 \times 3 = 9$, etc.

You can say that the rule is: Term = 3 × position number.

You can use this rule to work out any term in the sequence.

For example, the eighth term $= 3 \times 8 = 24$.

If you let n represent the unknown term, you can use algebra to write the rule as: ***n*th term** $= 3 \times n$ or simply: *n*th term $= 3n$

Worked example 9.3

a Work out the *n*th term rule for each of these sequences:

i 12, 24, 36, 48, 60, … **ii** 3, 4, 5, 6, 7, …

b Work out the 20th term for each sequence.

Answer

a i

Position number (n)	1	2	3	4	5
Term	12	24	36	48	60

Write out the sequence in a table.

You can see that the sequence is the 12 times table.

Position number (n)	1	2	3	4	5
× 12	× 12	× 12	× 12	× 12	× 12
Term	12	24	36	48	60

So the rule is:

Term = position number × 12.

Continued

nth term $= 12n$ — So, nth term $= 12 \times n$ or simply $12n$.

ii

Write out the sequence in a table.

Position number (n)	1	2	3	4	5
Term	3	4	5	6	7

This time you can see that each term is 2 more than the position number.

Position number (n)	1	2	3	4	5
+ 2	+ 2	+ 2	+ 2	+ 2	+ 2
Term	3	4	5	6	7

nth term $= n + 2$ — So, nth term $= n + 2$ or $2 + n$.

b i 20th term $= 12 \times 20 = 240$ — Substitute $n = 20$ into each rule.

ii 20th term $= 20 + 2 = 22$

Exercise 9.3

1 **a** Copy and complete the workings shown to work out the nth term rule for this sequence.

6, 12, 18, 24, 30, …

Position number	1	2	3	4	5
× ☐	× ☐	× ☐	× ☐	× ☐	× ☐
Term	6	12	18	24	30

Term = ☐ × position number, so nth term rule is: nth term = ☐.

b Use your rule to work out the:

i 10th term   ii 15th term

2 For each of these sequences, work out its nth term rule.

a 5, 10, 15, 20, 25, …   **b** 8, 16, 24, 32, 40, …   **c** 15, 30, 45, 60, 75, …

3 For each of the sequences in Question **2**, use its nth term rule to work out the:

i eighth term   ii 20th term

4 **a** Copy and complete the workings to find out the *n*th term rule for this sequence.

6, 7, 8, 9, 10, …

Position number	1	2	3	4	5
+ □	+ □	+ □	+ □	+ □	+ □
Term	6	7	8	9	10

Term = position number + □, so nth term rule is: nth term = □.

b Use your rule to work out the:

i 20th term **ii** 35th term

5 For each of these sequences, work out its *n*th term rule.

a 10, 11, 12, 13, 14, … **b** 5, 6, 7, 8, 9, …

c 22, 23, 24, 25, 26, … **d** 43, 44, 45, 46, 47, …

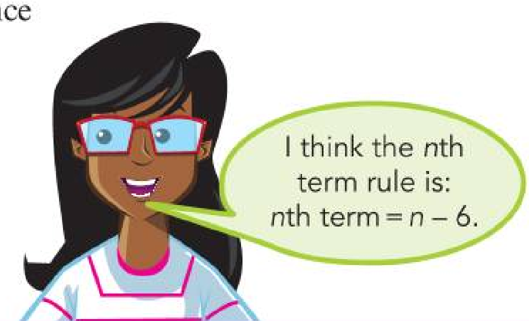
6 For each of the sequences in Question **5**, use its *n*th term rule to work out the:

i eighth term **ii** 20th term

7 Zara is looking at the sequence −5, −4, −3, −2, …

Is Zara correct?

Explain your answer and show all your working.



Tip

You could start by writing the sequence in a table.

Think like a mathematician

8 Look again at Question **7**. Discuss in groups, or as a class, the methods you used to show that Zara is correct. Did you all use the same method or did some learners use a different method?

9 Mei is using the nth term rule, nth term $= n - 2$. She uses it to work out the first four terms of the sequence like this:

nth term $= n - 2$ when $n = 1$, $1 - 2 = -1$
when $n = 2$, $2 - 2 = 0$
when $n = 3$, $3 - 2 = 1$
when $n = 4$, $4 - 2 = 2$
Sequence is $-1, 0, 1, 2, \ldots$

Work out the first five terms of each of these sequences.

a nth term $= n - 4$ **b** nth term $= n + 8$ **c** nth term $= 10n$

10 Match each sequence (**A** to **F**) with its correct nth term rule (**i** to **vi**).
The first one is done for you.

A $0, 1, 2, 3, 4, \ldots$	**i** $n + 3$
B $2, 4, 6, 8, 10, \ldots$	**ii** $n - 3$
C $4, 5, 6, 7, 8, \ldots$	**iii** $n - 1$
D $7, 14, 21, 28, 35, \ldots$	**iv** $n + 7$
E $-2, -1, 0, 1, 2, \ldots$	**v** $2n$
F $8, 9, 10, 11, 12, \ldots$	**vi** $7n$

11 Copy and complete this table.

nth term rule	5th term in sequence	10th term in sequence	20th term in sequence
nth term $= n + 12$	17		
nth term $= n - 5$		5	
nth term $= 4n$			80
nth term $= n + 35$		45	
nth term $= n - 15$	−10		
nth term $= 16n$			

12 Here are some number cards.

20	22	36	40	63	100

Here are some *n*th rule cards for four sequences.

Sequence A nth term $= 9n$	Sequence B nth term $= n + 6$	Sequence C nth term $= n + 20$	Sequence D nth term $= 20n$

a All six of the numbers can be used in only one of the sequences. Is the sequence A, B, C or D?

Show that all of the numbers are used in the sequence by working out their positions in the sequence.

b Which numbers can be used in three of the sequences?

Show your working.

c Which sequence uses the **fewest** of these numbers?

Explain how you worked out your answer.

Tip

Work out the value of *n* that gives each number on the cards.

How well do you think you understand the *n*th term rules?
Give yourself a score from 1 (I still need lots of practice) to 5 (I am feeling very confident).

	1	2	3	4	5
I can work out the *n*th term rule of a sequence of numbers.					
I can use the *n*th term rule to work out the sequence of numbers.					

Summary checklist

- ☐ I can find the *n*th term rule for a simple number sequence.
- ☐ I can use the *n*th term rule for a simple number sequence.

9.4 Representing simple functions

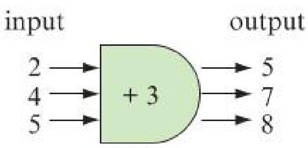
In this section you will …

- work out input and output numbers from function machines.

Key words

function
function machine
input
inverse operation
mapping diagram
maps
one-step function machine
output

A **function** is a relationship between two sets of numbers. You can show a function as a **function machine** like this.



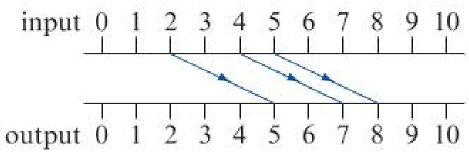
Tip

This function machine adds 3 to any number that enters the machine. It is called a **one-step function machine**.

The numbers that you enter into the function machine are called the **input**.

The numbers that you get out of the function machine are called the **output**.

You can also show a function as a **mapping diagram** like this.

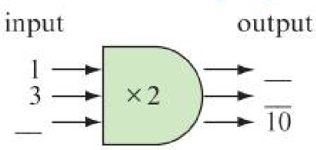


Tip

You say that 2 **maps** to 5, 4 maps to 7, and 5 maps to 8.

Worked example 9.4

a Find the missing inputs and outputs in this function machine.

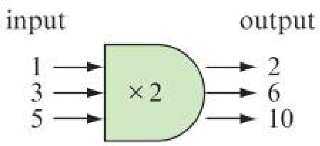


b Draw a mapping diagram to show the function in part **a**.

Continued

Answer

a



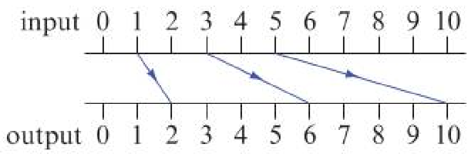
To work out the outputs, multiply the inputs by 2.

$1 \times 2 = 2$, $3 \times 2 = 6$

To work out the input, work backwards using **inverse operations**. Instead of multiplying the input by 2 to get the output, divide the output by 2 to get the input.

$10 \div 2 = 5$

b

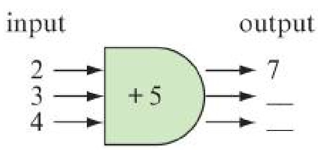


1 maps to 2, 3 maps to 6, and 5 maps to 10.

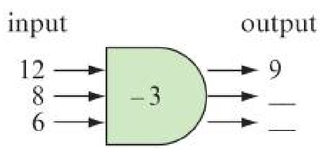
Exercise 9.4

1 Copy these function machines and work out the missing outputs.

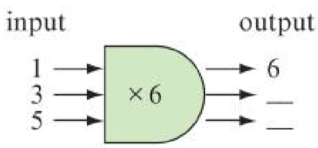
a



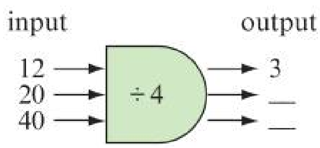
b



c

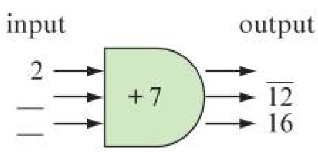


d

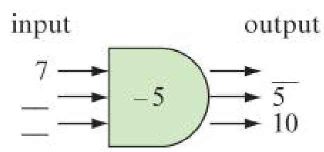


2 Copy these function machines and work out the missing outputs. Use inverse operations to work out the missing inputs.

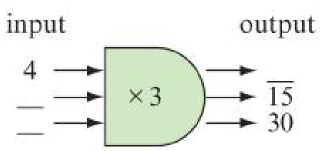
a



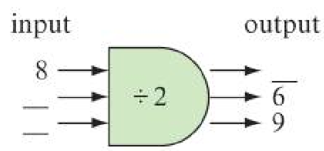
b



c



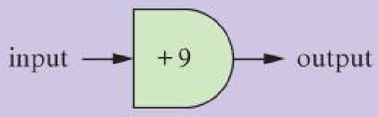
d



> **Tip**
>
> An inverse operation is when you work backwards doing the opposite of the instructions given; e.g. the opposite of add 2 is subtract 2.

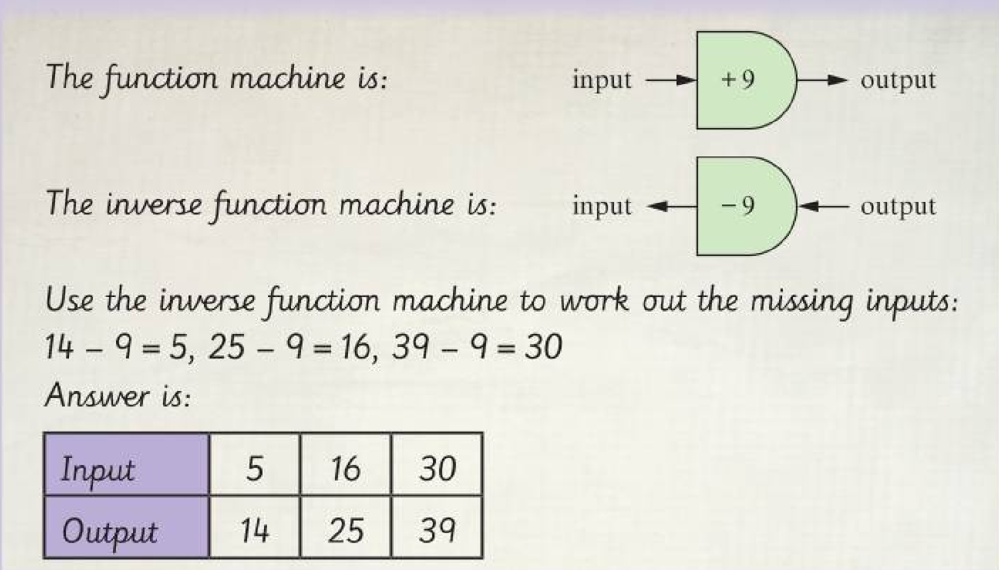
Think like a mathematician

3 Chang works out the answer to this question.
Work out the missing values in the table for this function machine.



Input			
Output	14	25	39

This is what he writes.



The function machine is: input → +9 → output

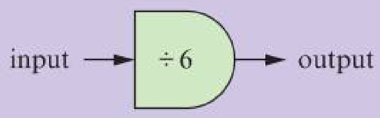
The inverse function machine is: input ← −9 ← output

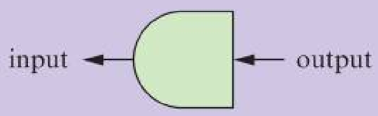
Use the inverse function machine to work out the missing inputs:
14 − 9 = 5, 25 − 9 = 16, 39 − 9 = 30
Answer is:

Input	*5*	*16*	*30*
Output	*14*	*25*	*39*

a What do you think of Chang's method?
What are the advantages and disadvantages of his method?
Discuss your answers.

b Use Chang's method to answer this question.
Copy and complete this inverse function machines and table of values.



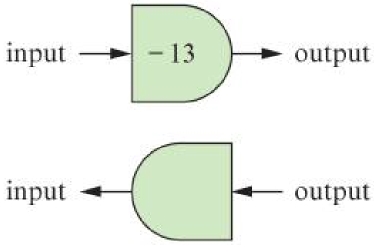


Input			
Output	4	9	20

Compare your inverse function and table of values with other learners in the class.

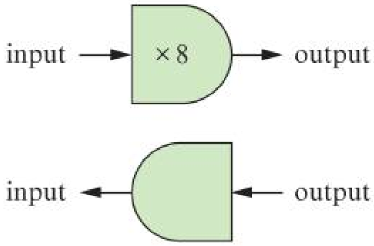
4 Copy and complete these inverse function machines and tables of values.

a



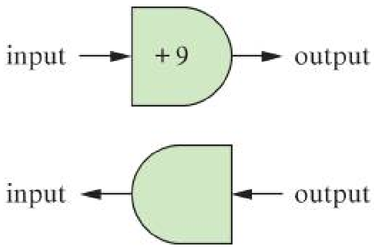
Input			
Output	7	12	38

b



Input			
Output	24	56	120

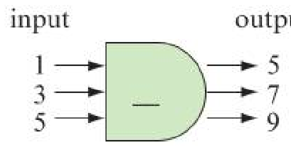
c



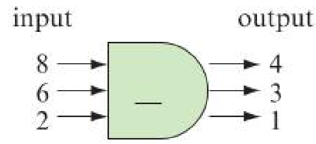
Input			
Output	17	24	45

5 **a** In the function machines shown, the functions are missing.

i



ii

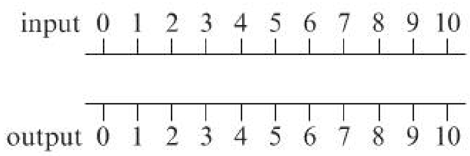


What do you think are the missing functions?

Test your functions to see if they are correct.

If they aren't correct, try a different function and test again.

b Make two copies of the diagram below.

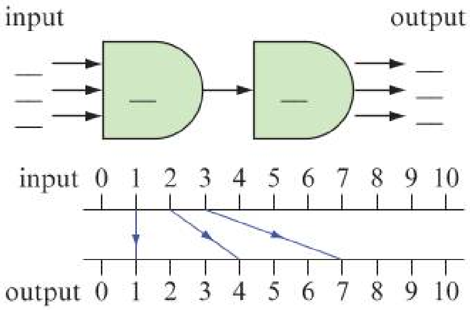


Draw a mapping diagram for each of the functions in part **a**.

Tip

To test your function, enter the input numbers and see if you get the correct output numbers.

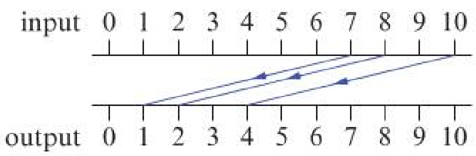
6 Chin-Mae draws this mapping diagram and function machine for the same function.



Copy the function machine. Fill in the missing numbers and the rules in the function machine. Show your working.

> **Tip**
>
> Fill in the input and output numbers in the function machine first. Then try different functions until you get the correct combination.

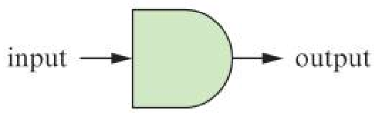
7 Chin-Mae draws this mapping diagram for a function.



Copy and complete this function machine and table of values for the same function.

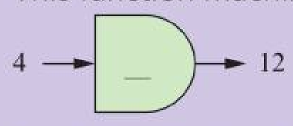
Explain how you worked out your answer.

Input			
Output			



Think like a mathematician

8 This function machine shows only one input and one output.



a Show that the function could be + 8.

b Write down another possible function for this function machine.

c What is the smallest number of input and output numbers that you must have in order to be able to work out the correct function. Explain your answer.

d Discuss your answers to parts a, b and c with other learners in your class.

Summary checklist

- [ ] I can work out output values of a function machine.
- [ ] I can work out input values of a function machine.

Check your progress

1 For each of these infinite sequences, work out:

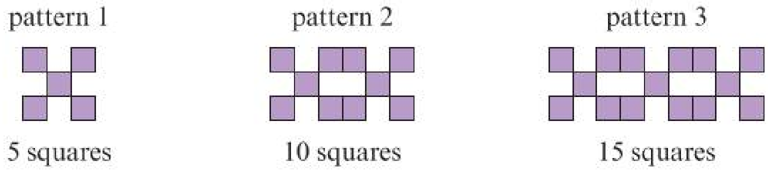
i the term-to-term rule **ii** the next two terms **iii** the 10th term

a 6, 8, 10, 12, … **b** 9, 15, 21, 27, … **c** 28, 25, 22, 19, …

2 Write down the first four terms of the sequence that has a first term of 5 and a term-to-term rule of: Multiply by 3 then subtract 5.

3 This sequence of patterns is made from squares.

a Draw the next pattern in the sequence.



b Copy and complete the table to show the number of squares in each pattern.

Pattern number	1	2	3	4	5
Number of squares	5	10			

c Write down the term-to-term rule.

d How many squares will there be in pattern 10?

4 Work out the *n*th term rules for both of these sequences.

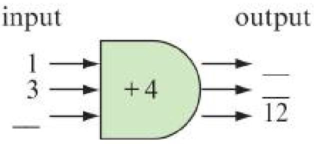
a 3, 6, 9, 12, 15, … **b** 8, 9, 10, 11, 12, …

5 For both sequences in Question **4**, use your *n*th term rules to work out the 10th term.

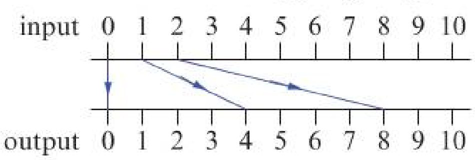
6 Work out the first four terms of both of these sequences.

a *n*th term $= 5n$ **b** *n*th term $= n - 7$

7 Copy this function machine and work out the missing input and outputs.

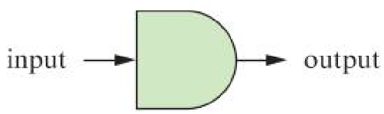


8 Tarun draws this mapping diagram for a function.



Copy and complete this function machine and table of values for the same function.

Input			
Output			



Project 4

Mole and goose

Shown is a section of a horizontal number line.

A goose walks along the number line and stops sometimes to lay an egg.

She likes to space her eggs out equally. For example, she might lay her eggs at 2, 7, 12, 17, 22, …

A mole digs a tunnel below the number line. It likes to dig up to the surface every so often, to see how far it has dug. It always pokes out its head at equally spaced intervals. For example, it might poke out its head at 1, 4, 7, 10, 13, …

When the mole visits 7, it finds an egg there! Where else will the mole find an egg?

Choose a starting number for each animal, and decide how far each animal will travel at each step of the sequence.

Does the mole find an egg? Does it find more than one egg?

Can you find some pairs of sequences in which the mole finds more than one egg?

How can you predict how far apart the mole finds the eggs?

Can you find some pairs of sequences in which the mole <u>never</u> finds an egg?

What is special about the size of the steps in these sequences?

10 Percentages

Getting started

1 Write these fractions in their simplest form.

a $\frac{24}{40}$ b $\frac{25}{30}$ c $\frac{36}{27}$ d $\frac{60}{180}$

2 Write these fractions as decimals.

a $\frac{5}{8}$ b $\frac{8}{5}$

3 Work out:

a $\frac{1}{5} \times 45$ b $\frac{3}{8} \times 24$

4 Find:

a 25% of 120 people b 60% of \$60

Percentages are everywhere.

You often see percentages in news reports. Percentages can be used to describe price changes, test results, the chance of bad weather, the results of surveys or opinion polls, and many other things.

Where have you seen percentages used in real life recently?

You can write percentages as fractions or as decimals; for example, 40% is equivalent to 0.4 or to $\frac{2}{5}$. You need to be able to recognise when percentages, fractions and decimals are equivalent.

You need to be able to work out a percentage of a quantity. You can do this by first changing the percentage to a fraction or a decimal.

You first learned about percentages as parts out of 100. Percentages can be very small, less than 1%. Percentages can also be larger than 100% in some cases. You will see examples of both of those in this unit.

10.1 Fractions, decimals and percentages

In this section you will …

- learn to recognise and use the fact that fractions, decimals and percentages have the same value.

Key words

decimal
denominator
equivalent
fraction
numerator
percentage

You can write a percentage as a fraction or as a decimal.

$32\% = \frac{32}{100}$ 0.32 $60\% = \frac{60}{100}$ 0.6 $83\% = \frac{83}{100}$ 0.83

Sometimes you can write the fraction in a simpler form.

For $\frac{32}{100}$ you can divide the numerator and the denominator by 4 to get $\frac{32}{100} = \frac{8}{25}$.

For $\frac{60}{100}$ you can divide the numerator and the denominator by 20 to get $\frac{60}{100} = \frac{3}{5}$.

You cannot simplify $\frac{83}{100}$ because the only common factor of 83 and 100 is 1.

Tip

$\frac{32}{100}$ is **equivalent** to $\frac{8}{25}$.

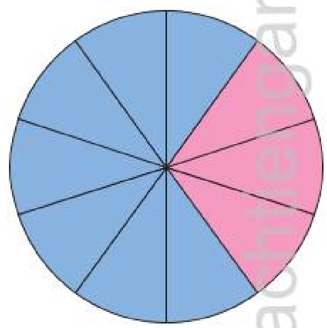
Worked example 10.1

a In the circle diagram shown, how much of the circle is:

i pink? **ii** blue?

Give your answers as decimals, fractions and percentages.

b The area of the circle is 60 cm². What is the area of the pink region?



Answer

a **i** 3 parts out of 10 are pink.

The fraction is $\frac{3}{10}$ and the decimal is 0.3.

$\frac{3}{10} = \frac{30}{100}$, so that is 30%.

ii The rest of the circle is blue, so that is $\frac{7}{10}$ or 0.7 or 70%.

Continued

b The area that is pink is 30% of 60 cm².

You can use a fraction or a decimal to find the area.

Using the fraction: $\frac{3}{10} \times 60 = 60 \div 10 \times 3 = 6 \times 3 = 18$ cm²

Using the decimal: $0.3 \times 60 = 18$ cm²

In part **b** of Worked example 10.1, you could use algebra to write an expression linking the area of the pink region to the area of the circle; for example: $r = 0.3c$ or $r = \frac{3}{10}c$, where r is the area of the pink region and c is the area of the circle.

Tip

See Unit 2 for a reminder on using algebra.

Exercise 10.1

1 You could work with a partner on this question.

70% is equivalent to $\frac{70}{100}$.

a Show that $\frac{70}{100}$ in its simplest form is $\frac{7}{10}$.

The denominator of $\frac{7}{10}$ is 10.

b 70% as a fraction in its simplest form has 10 as its denominator.

25% = $\frac{1}{4}$, so as a fraction in its simplest form it has 4 as a denominator.

Starting with a whole-number percentage, what other denominators can you find when you write the percentage as a fraction in its simplest form?

2 Write these percentages as fractions. Write your answers in the simplest form.

a 60% **b** 61% **c** 62% **d** 64% **e** 65% **f** 70%

3 Joshua writes $0.3 = 3\%$.

a Explain why this is incorrect.

b Write 3% correctly as a decimal and as a fraction.

4 Write these percentages as decimals and as fractions.

a 40% **b** 4% **c** 9% **d** 90% **e** 5%

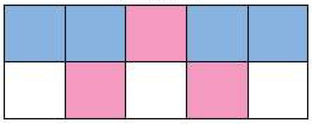
5 Write these fractions as percentages.

a $\frac{1}{4}$ **b** $\frac{2}{5}$ **c** $\frac{4}{5}$

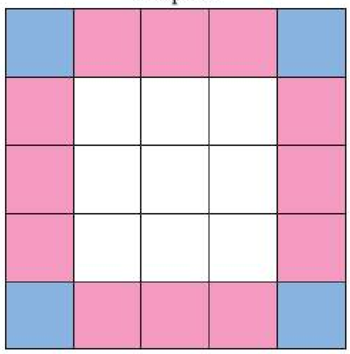
d $\frac{7}{50}$ **e** $\frac{7}{20}$ **f** $\frac{7}{25}$

6 Here are two shapes:

shape 1



shape 2



a Write down as a fraction, a percentage and a decimal the part of shape 1 that is coloured pink.

b Write down as a fraction, a percentage and a decimal the part of shape 1 that is coloured blue.

c Write down as a fraction, a percentage and a decimal the part of shape 2 that is coloured pink.

d Write down as a fraction, a percentage and a decimal the part of shape 2 that is coloured blue.

e The area of shape 1 is 45 cm^2. Work out the area that is shaded pink.

f The area of shape 2 is 75 cm^2. Work out the area that is shaded blue.

7

4%	6%	60%	40%	30%
$\frac{1}{25}$	$\frac{2}{5}$	$\frac{3}{10}$	$\frac{3}{50}$	$\frac{3}{5}$
0.4	0.6	0.3	0.04	0.06

Sort these percentages, fractions and decimals into five groups.

Think like a mathematician

8 The numerator of $\frac{3}{4}$ is 3 and $\frac{3}{4} = 75\%$.

a Find more fractions with a numerator of 3 for which the equivalent percentage is a whole number.

b Share your results with a partner. Can you change or improve your list?

c Think about how you did this question. If you did a similar question with a different numerator, would you use the same method?

9 Work out:

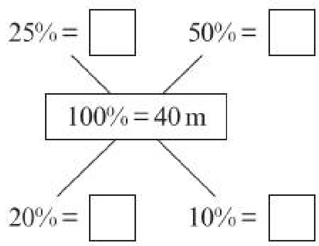
a $\frac{1}{2}$ of 60 grams b $\frac{3}{4}$ of 60 grams c $\frac{4}{5}$ of 60 grams

d $\frac{7}{10}$ of 60 grams e $\frac{11}{20}$ of 60 grams

10 Work out:

a 50% of \$300 b 20% of \$300 c 30% of \$300 d 15% of \$300

11 a Copy and complete this diagram.



b Now add four more lines to your diagram. You choose the percentages.

12 30% of \$70 = \$21

a How can you use this result to find 60% of \$70?

b What other percentages of \$70 can you find using this result? Show your method each time.

c Compare your results with a partner's. Have you got different results?

13 a Write $\frac{1}{4}$ as a percentage.

b Use your result from part **a** to write $\frac{1}{8}$ as a percentage.

c Write equivalent percentages for other fractions with 8 as a denominator.

Summary checklist

☐ I can recognise when fractions, decimals and percentages are equivalent and use this to calculate percentages.

10.2 Percentages large and small

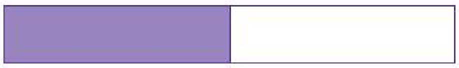
In this section you will …

- recognise and use percentages, including percentages less than 1 or greater than 100.

Key words

common factor
mixed number

$50\% = 0.5$ $\qquad$ $50\% = \frac{50}{100} = \frac{1}{2}$



$5\% = 0.05$ $\qquad$ $5\% = \frac{5}{100} = \frac{1}{20}$

$0.5\% = 0.005$ $\qquad$ $0.5\% = \frac{0.5}{100} = \frac{5}{1000} = \frac{1}{200}$

0.5% is too small to show on a diagram.

To write $\frac{0.5}{100}$ as a fraction in its simplest form:

- First, multiply the numerator and denominator by 10 to get $\frac{5}{1000}$.
- Then divide the numerator and denominator by 5 to get $\frac{1}{200}$.

Worked example 10.2

Write 17.5% as a:

a decimal $\qquad$ **b** fraction

Answer

a Divide 17.5 by 100 to get $17.5\% = 0.175$.

b $17.5\% = \frac{17.5}{100}$

$\frac{17.5}{100} = \frac{175}{1000}$ — To get whole numbers, multiply the numerator and denominator by 10.

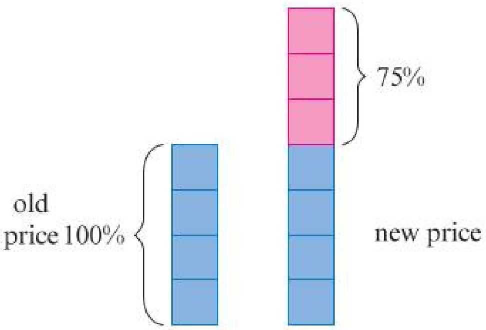
When you divide by 5, you get $\frac{35}{200}$. — Now divide by **common factors**.

When you divide by 5 again, you get $\frac{7}{40}$. — This is in its simplest form.

Percentages can be more than 100%.

Suppose that over 20 years the price of a house increases by 75%.

You can draw a diagram to show this.



$100\% + 75\% = 175\%$

The new price is 175% of the old price.

Worked example 10.3

The height of a young child is 140% of its height 2 years ago.

The child's height 2 years ago was 70 centimetres. Work out the height of the child now.

Answer

You can use decimals or fractions to solve this problem.

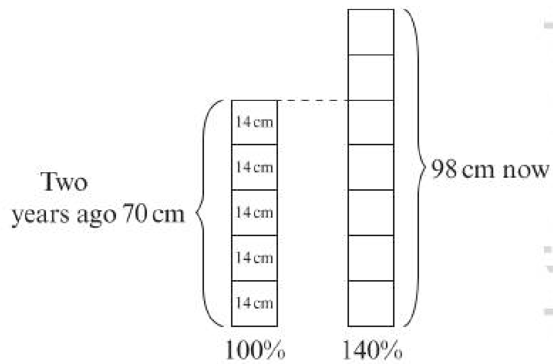
Using decimals: $140\% = 1.4$

The height is $1.4 \times 70\text{ cm} = 98\text{ cm}$

Using fractions: $140\% = \frac{140}{100}\quad \frac{7}{5} =$

The height is $\frac{7}{5} \times 70 = 70 \div 5 \times 7 = 14 \times 7 = 98\text{ cm}$.

You can show the different heights in a diagram like this:



In Worked example 10.3, you could use algebra to write an expression linking the child's height now and the child's height 2 years ago; for example: $n = 1.4t$ or $n = \frac{7}{5}t$, where n is the child's height now and t is the child's height 2 years ago.

Worked example 10.4

Arun has \$20.

He gives \$5 to Marcus and \$13 to Sofia.

a What percentage does each person get?

b What percentage is left?

Answer

a The fraction of the money that Marcus gets $= \frac{7}{5} \quad \frac{1}{4} = 25\%$

The fraction that Sofia gets is $\frac{13}{20} = \frac{65}{100} \quad 65\%$

The amount left is \$20 – \$5 – \$13 = \$2

The percentage is $\frac{2}{20} = \frac{1}{10} \quad 10\%$

The sum of the 3 percentages is 100%

Exercise 10.2

1 Worked example 10.2 shows how to write 17.5% as a fraction.
Choose some more percentages that end in .5.
Write these percentages as fractions in their simplest form.
What different denominators do you get? What is the largest denominator? What is the smallest denominator?

2 Write these percentages as decimals and as fractions. Write each fraction in its simplest form.

a 7.5% **b** 62.5% **c** 1.5% **d** 47.5% **e** 32.5%

3 Copy and complete this table.

100%	\$80	\$300		
50%			\$45	
5%				\$3.20
0.5%				

4 Write these percentages as fractions in their simplest form.

a 2% **b** 0.2% **c** 8% **d** 0.8% **e** 7% **f** 0.7%

5 **a** Work out:

- **i** 1% of 600 kilograms
- **ii** 1.5% of 600 kilograms
- **iii** 3.5% of 600 kilograms
- **iv** 2.2% of 600 kilograms

b In part **a**, why was it useful to work out 1% first?

6 **a** Work out 1% of 7000 metres.

b Use your answer to part **a** to work out:

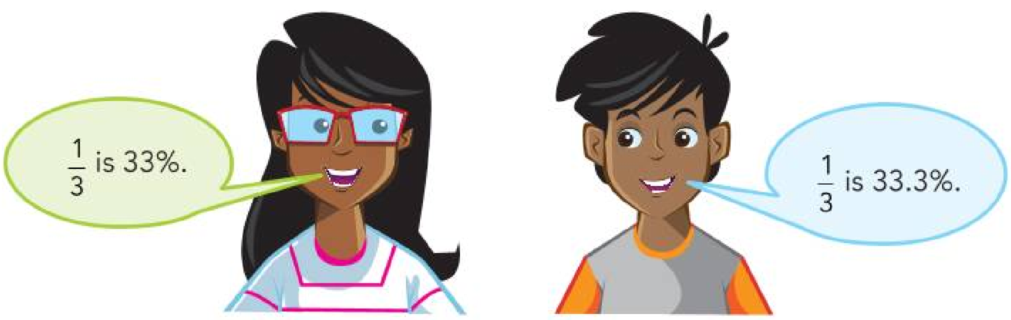
- **i** 2% of 7000 metres
- **ii** 0.5% of 7000 metres
- **iii** 0.1% of 7000 metres
- **iv** 0.3% of 7000 metres

7 How would you work out 1.3% of \$7500?
Can you think of any different methods?

8 **a** Work out $\frac{1}{3}$ of 100.

b Write $\frac{1}{3}$ as a percentage.

Zara says: Arun says:



c How could Zara be correct?

d How could Arun be correct?

e Write $\frac{2}{3}$ as a percentage.

9 Write each of these **mixed numbers** as a percentage. The first one has been done for you.

a $1\frac{1}{2} = 150\%$ **b** $1\frac{1}{4}$ **c** $1\frac{3}{4}$

d $1\frac{3}{10}$ **e** $1\frac{7}{10}$ **f** $2\frac{1}{4}$

10 **a** Write $\frac{1}{5}$ as a percentage.

b Write these mixed fractions as percentages.

i $1\frac{1}{5}$ **ii** $1\frac{3}{5}$ **iii** $1\frac{4}{5}$ **iv** $2\frac{3}{5}$

Think like a mathematician

11 **a** Work out 30% of $60.

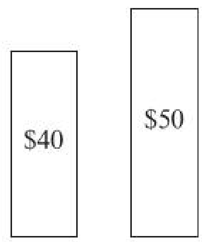
b Use your answer to part **a** to work out some percentages of $60 that are greater than 100%.

c Give your percentages to a partner to check.

d What percentage of $60 is $102?

12 **a** Look at the diagram. What percentage of $50 is $40?

b What percentage of $40 is $50?



13 Marcus has 500 g of flour. He uses 200 g to bake bread and 120 g to make biscuits.

Work out the percentage of the flour:

a he uses for bread

b he uses for biscuits

c he does not use

14 Sofia works for 40 hours in a week. She spends 25 hours in her office, 8 hours in the factory and the rest of the time in a workshop.

Work out the percentage of the time that she spends:

a in her office

b in the factory

c in a workshop

15 Zara earns $250.

She pays $95 in tax and pays a bill of $65.

Work out the percentage of her money:

a she pays in tax **b** she has left after both payments

16 An airline ticket costs $40.

The price is increased by 200%.

a Work out:

i the increase **ii** the price after the increase

b Find the number to complete this sentence:

The new price is … times the original price

17 The mass of a puppy is 5 kg. As it grows, the mass increases by 300%

a Work out:

i the increase in mass ii the new mass

b Find the number to complete this sentence:

The new mass is … times the original mass.

18 a The cost of a car increases by 5%. What percentage of the original price is the new price?

b The cost of petrol increases by 80%. What percentage of the original price is the new price?

19 Ruqiyah is trying to answer this question:

What percentage of $125 is $200?

She thinks the answer is a whole number.

She cannot find the answer. Can you help her?

20 Vikram wants to work out 135% of $40 but he does not know how.

Describe some different methods to do this calculation.

Which method do you prefer? Why?

Summary checklist

- [ ] I can use percentages that are less than 1.
- [ ] I can understand and calculate percentages greater than 100.

Check your progress

1 Write:

a 70% as a fraction in its simplest form

b 9% as a decimal

c $\frac{5}{8}$ as a percentage

d $1\frac{1}{20}$ as a percentage

2 Work out:

a 115% of \$30

b 190% of \$6

3 Work out:

a 103% of \$45

b 0.3% of \$450

4 Copy and complete this table.

100%	30	2500		
120%				
12.5%				6
0.5%			4	

Describe how you can use fractions to work out 80% of 65 kilograms.
Describe two different methods to work out 0.2% of 2000.

5 Arun has 80 stamps. He uses 12 of them and gives 60 to other people. Work out the percentage:

a he gives to other people

b he has left

6 The number of people working for a company increases by 400%. Find the number to complete this sentence:

The number of people working for the company has multiplied by …

11 Graphs

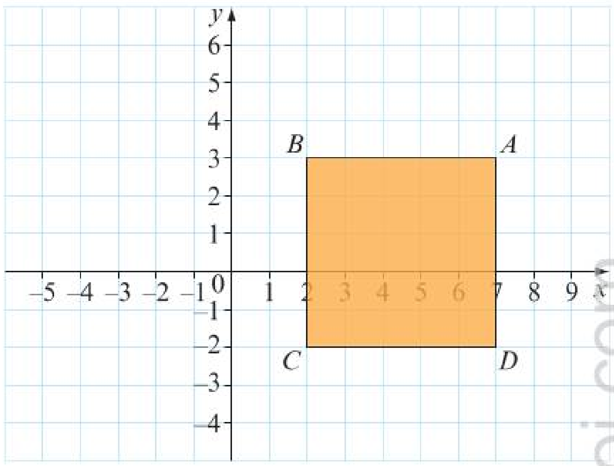
Getting started

1 A coordinate grid is shown.

a Write down the **coordinates** of the vertices, *ABCD*, of the square.

b Where does *BC* cross the **x-axis**?

c *BCFE* is a rectangle. *E* has the coordinates (−4, 3). Work out the coordinates of *F*.



2 For the expression $x + 5$, work out the value of the expression when:

a $x = 4$ **b** $x = 1.5$

c $x = -2$ **d** $x = -9$

3 For the expression $4x$, work out the value of $4x$ when:

a $x = 2$ **b** $x = 7$ **c** $x = -3$ **d** $x = -5$

You can use a **graph** to show the connection between two variables.

Graphs show information as images that are easy to understand.

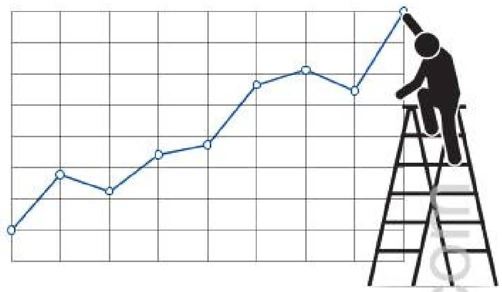
Here are some examples of when you could use a graph to show a connection:

- How does the mass of a baby increase with age?
- How does the cost of a taxi journey vary with the distance travelled?
- How does the temperature of water increase as you heat it?
- How does the cost of a phone call depend on the length of the call?
- How does the amount of fuel used by a car vary with the distance travelled?
- How has the population of a country changed over the past 10 years?

- How do prices of vegetables vary from one month to the next?
- How does the amount of tax you pay vary with your income?
- How much energy does an athlete use while running a marathon?
- How does the temperature change over the day?
- How far has a walker travelled as time goes by?
- How much will you get when you change one currency to another?
- How does your time to run 100 metres vary with age?
- What is a healthy mass for an adult of a particular height?
- How does the output of a solar panel vary with temperature?
- How are the profits of a company varying?

Can you imagine what a graph representing any of the situations above might look like?

In this unit you will look at the simplest type of graph on a coordinate grid: a straight-line graph.



11.1 Functions

In this section you will ...

- represent situations in words or as linear functions.

Key words

function
negative gradient
positive gradient
variable
zero gradient

A **function** shows the relationship between two **variables**.

You often use x and y to represent two variables, but you can use other letters.

Example:

Maryam is 6 years older than her brother Jamal.

When Jamal is 2 years old, then Maryam is 8 years old.

When Jamal is 11 years old, then Maryam is 17 years old.

You do not know their ages now.

Using algebra, you can represent their ages with the letters m and j.

You say that Maryam is m years old and Jamal is j years old.

You know that m is 6 more than j. You can write this as $m = j + 6$, where m and j are variables. They can have different values.

The function $m = j + 6$ shows the relationship between the two variables m and j.

Tip

See Unit 2 for a reminder on using algebra.

Worked example 11.1

a You can exchange 1 Hong Kong dollar (HK\$1) for 9 Indian rupees (₹9).

i How many Indian rupees can you get in exchange for HK\$20?

ii Write a function to show the relationship between d Hong Kong dollars and r Indian rupees.

b You can exchange x US dollars for y Indian rupees. A function to show this is $y = 70x$.

Describe what this function means, in words.

Answer

a **i** $20 \times 9 = 180$ Indian rupees

ii $r = 9d$

b You can exchange 1 US dollar (US\$1) for 70 Indian rupees (₹70).

Exercise 11.1

1 **a** Angelo is twice as old as Sabrina. How old is Angelo when Sabrina is:

i 4 years old? **ii** 6 years old?

iii 10 years old? **iv** 12 years old?

b When Sabrina is s years old, how old is Angelo?

c Sabrina is s years old and Angelo is a years old.

Copy and complete this function: $a =$ ....

Tip

'twice' means 'two times'.

2 Lilah's mass is $\frac{3}{4}$ of the mass of Abdul.

a If Abdul's mass is 40 kg, what is the mass of Lilah?

b Write down two more examples of the possible masses of Abdul and Lilah.

c Abdul's mass is x kg and Lilah's mass is y kg.

Copy and complete this function: $y =$ ....

3 A book costs \$10 more than a diary.

a Work out how much the book costs if the diary costs:

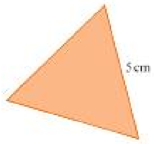
i \$6 **ii** \$10 **iii** \$13

iv \$8.50 **v** \$14.95

b The diary costs \$$x$. Write down the cost of the book.

c The cost of the book is \$$y$. Write down a function to show the relationship between y and x.

4 a This is an equilateral triangle. Each side is 5 cm. What is the perimeter of the triangle?



b The side of an equilateral triangle is 20 cm. What is the perimeter of this triangle?

c What is the connection between the length of one side of an equilateral triangle and the perimeter of the triangle?

d Each side of an equilateral triangle is s cm. The perimeter is p cm. Write down a function to show the relationship between p and s.

Think like a mathematician

5 a The length of a rectangle is 3 cm more than its width.

i Draw some rectangles that follow this rule.

ii The width of the rectangle is w cm and the length is l cm. Write down a function generalising the relationship between l and w.

b The length of a rectangle is three times its width.

i Draw some rectangles that follow this rule.

ii Write down a function generalising the relationship between the length and the width.

iii Find a function generalising the relationship between the perimeter and the width.

c Choose a rule of your own that connects the length and width of a rectangle. Draw some examples and find a function to generalise the relationship.

d Swap your answer to part **c** with a partner's answer. Mark each other's work.

6 x and y are two variables and are connected by the function $y = x - 5$.

a Find three possible pairs of values for x and y.

b Give three examples of what x and y could represent.

c Compare your answers to part **b** with a partner's answers. Do you agree that your examples are sensible?

7 A litre of petrol costs US$150.

a Work out the cost of 10 litres of petrol.

b Write down a function to show the cost (y dollars) of x litres of petrol.

8 You can exchange 10 euro (€10) for 210 peso.

a How many pesos will you get in exchange for 1 euro (€1)?

b Write down a function to show the relationship between y pesos and x euros.

9 The relationship between US dollars (x) and Libyan dinars (y) is $y = 1.5x$.

a Describe this relationship in words.

b Explain how you would calculate the value of US$570 in dinars.

c Explain how you would calculate the value of 570 dinars in US dollars.

Summary checklist

☐ I can understand how a situation can be represented either in words or with a simple function involving two variables x and y.

11.2 Graphs of functions

In this section you will ...

- plot graphs of linear functions.

Key words

axes

coordinates

graph

Here is the function $y = x + 3$.

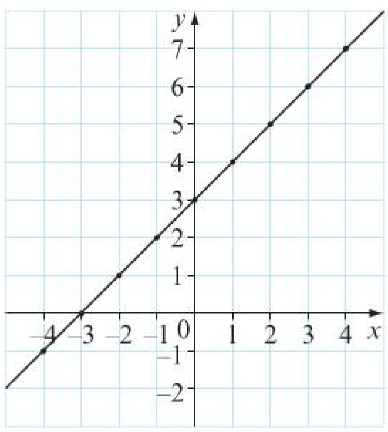
When $x = 2$, then $y = 2 + 3 = 5$.

When $x = -4$, then $y = -4 + 3 = -1$.

You can display values of x and the corresponding values of y in a table.

x	−4	−3	−2	−1	0	1	2	3	4
y	−1	0	1	2	3	4	5	6	7

You can use these values as coordinates: to plot the points (−4, −1), (−3, 0), (−2, 1), and so on.



The points are in a straight line. This is a graph of $y = x + 3$.

Worked example 11.2

For the function $y = 0.5x$:

a Copy and complete this table of values.

x	−6	−3	0	3	6
y	−3				

b Draw a graph of the function.

c Show that the point (−24, −12) is on the line, but that the point (15, 30) is not on the line.

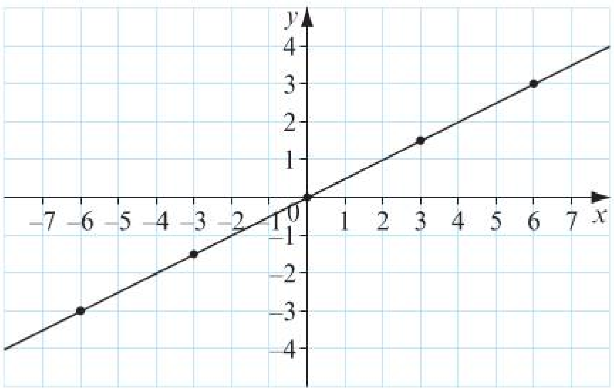
Answer

a

x	−6	−3	0	3	6
y	−3	−1.5	0	1.5	3

The value of y is half the value of x.

b



The diagram shows only a part of the line. The line continues forever.

c When $x = -24$, then $y = 0.5 \times -24 = -12$, so the point (−24, −12) is on the line.

When $x = 15$, then $y = 0.5 \times 15 = 7.5$, so the point (15, 7.5) is on the line, but the point (15, 30) is not on the line.

The point (−24, −12) is on the line. The point (15, 30) is above the line.

Worked example 11.3

You can exchange 5 Turkish lira for 90 Japanese yen.

a How many Japanese yen can you get in exchange for 1 Turkish lira?

b You can exchange x Turkish lira for y Japanese yen. Write down a function to show this.

c Draw a graph of this function.

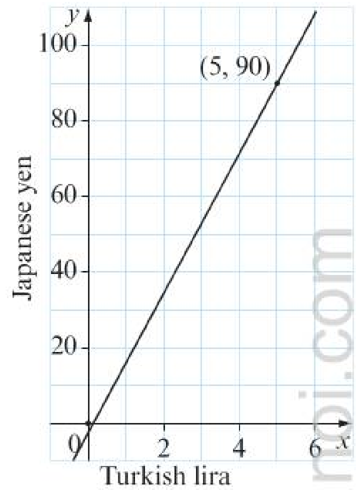
Answer

a For 1 Turkish lira you can get $90 \div 5 = 18$ Japanese yen

b $y = 18x$

c The graph is a straight line through (0, 0) and (5, 90), as shown.

In this graph, there is a different scale on each axis.



Exercise 11.2

1 For the function $y = x + 1$:

a Copy and complete this table of values.

x	−3	−2	−1	0	1	2	3
y		−1					4

b Use the table to draw a graph of $y = x + 1$.

c Show that (−20, −19) is on the line $y = x + 1$.

d Is (20, 19) on the line $y = x + 1$? Give a reason for your answer.

2 For the function $y = x - 2$:

a Copy and complete this table of values.

x	−4	−2	0	2	4	6
y						

b Draw a graph of $y = x - 2$.

c Is the point (25, 23) on the line? Give a reason for your answer.

3 For the function $y = x + 8$:

a Copy and complete this table of values.

x	−3	−2	−1	0	1	2	3	4
y	5					10		

b Draw a graph of the line.

c The points (20, □) and (−20, □) are on the line. Work out the missing numbers.

4 **a** Copy and complete this table of values for the function $y = 2x$.

x	−3	−2	−1	0	1	2	3
y							

b Use the table to draw the line $y = 2x$.

c Here are some points on the line. Copy and complete the coordinates.

i (4.5, □) **ii** (−5, □)

iii (8.5, □) **iv** (−8, □)

d (1, 3) is above the line and (−2, −6) is below the line. Is each of the following points above or below the line?

i (2.5, 8.5) **ii** (−9, −15)

iii (25, 65) **iv** (−20, −70)

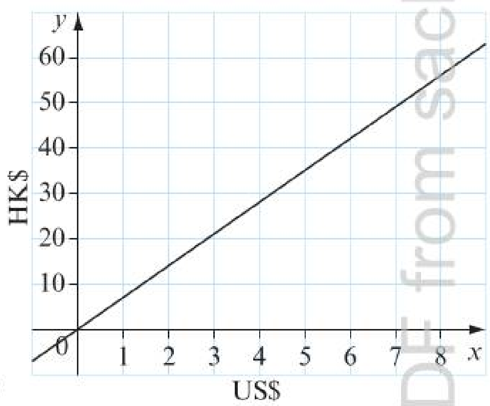
5 The equation of a line is $y = 5x$.

a Copy and complete this table of values for the function $y = 5x$.

x	−2	−1	0	1	2	3	4
y							

b Draw a graph of $y = 5x$.

6 This graph converts between US dollars (US$) and Hong Kong dollars (HK$).



a How many Hong Kong dollars can you get in exchange for US$5?

b Copy and complete this table.

US$	1	2	3	4	5	6	7
HK$							

c Write down a function to show the connection between HK$ (y) and US$ (x).

d How many HK$ can you get in exchange for US$50?

You can use the internet to find the conversion rate between any two currencies. Describe how you could use the conversion rate to draw a graph that shows the conversion between two currencies.

7 This question is about lines with an equation of the form $y = x + c$, where c is a number.

- **a** Draw a graph of the line $y = x$.
- **b** On the same **axes**, draw a graph of the line $y = x - 2$.
- **c** Choose a value for c and draw a graph of the line $y = x + c$ on the same axes.
- **d** Choose two more values for c and draw the line $y = x + c$ on the same axes each time.
- **e** Describe the similarities and the differences between lines of the form $y = x + c$.

8 This question is about lines with an equation of the form $y = mx$, where m is a positive number.

- **a** Draw a graph of the line $y = x$.
- **b** On the same axes, draw a graph of the line $y = 0.5x$.
- **c** Draw more graphs on the same axes of lines of the form $y = mx$, where m is a positive number.
- **d** When m is a positive number, describe what the line with the equation $y = mx$ looks like.

9 You can exchange 10 Swiss francs for 40 Polish zloty.

- **a** Draw a graph to show the conversion between these two currencies.
- **b** You can exchange s Swiss francs for z Polish zloty. Write a function to illustrate this.

10 Sangeeta is 30 years younger than her mother.

- **a** Show the relationship between their ages on a graph. Put Sangeeta's mother's age on the horizontal axis.
- **b** Sangeeta is y years old and her mother is x years old. Write down a function connecting their ages.

Think like a mathematician

11 a On the same axes, draw the lines $y = 2x$ and $y = x + 1$.

b For the graph drawn in part **a**, where do the lines cross?

c Investigate where the lines $y = 2x$ and $y = x + k$ cross when k is an integer.

Tip

Choose a value of k to use.

Summary checklist

☐ I can plot the graphs of functions in the form $y = x + c$ or $y = mx$ by constructing tables of values and plotting coordinate pairs.

11.3 Lines parallel to the axes

In this section you will ...

- learn how to recognise lines parallel to the x-axis or y-axis.

Key words

vertices

vertex

x-axis

y-axis

This is a rectangle:

The coordinates of the **vertices** are: A is (4, 2), B is (4, −3), C is (−2, −3) and D is (−2, 2).

A and B are on a line parallel to the **y-axis**. This line passes through 4 on the x-axis. The name of this line is $x = 4$.

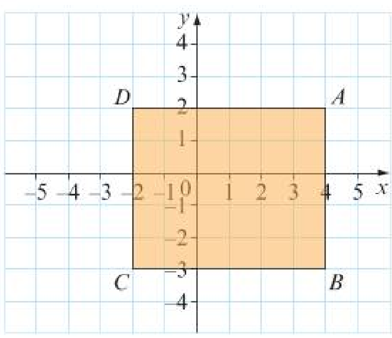
C and D are on a line parallel to the y-axis. This line passes through −2 on the x-axis. The name of this line is $x = -2$.

A and D are on a line parallel to the x-axis. This line passes through 2 on the y-axis. The name of this line is $y = 2$.

B and C are on a line parallel to the x-axis. This line passes through −3 on the y-axis. The name of this line is $y = -3$.

Tip

The 'name of a line' is sometimes called the 'equation of a line'.



Worked example 11.4

a Plot the points $P(2, -5)$ and $Q(-6, -5)$ on a coordinate grid.

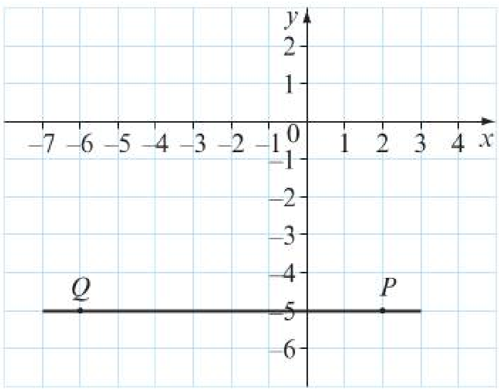
b Draw a straight line that passes through P and Q.

c Write down the coordinates of two more points on the line you drew in part **b**.

d Write down the name of the line in part **b**.

Answer

a, b



c There are lots of possible answers; for example, $(4, -5)$ or $(0, -5)$ or $(-2.5, -5)$. A point is on the line when the y-coordinate is -5.

d The name of the line is $y = -5$.

The x-axis has the name $y = 0$. The y-axis has the name $x = 0$.

Exercise 11.3

1 a i Draw a square on a coordinate grid with the following properties.
- The centre of the square is at (1, 2).
- The sides of the square are parallel to the coordinate axes.

ii Each side of the square is on a horizontal or vertical line. Write down the name of each line.

b i Draw a different square but with the same properties as the square in part **a**.

ii Each side of your new square is on a horizontal or vertical line. Write down the name of each line.

c Compare your work with a partner's. Do you both have correct answers?

2 a Draw the rectangle with vertices at $A(-1, 3)$, $B(2, 3)$, $C(2, -3)$ and $D(-1, -3)$.

b Write down the name of the straight line through A and B.

c Write down the coordinates of two more points on the line through A and B.

d Write down the name of the straight line through A and D.

e Write down the equation of the straight line through C and D.

3 a Draw these lines on a coordinate grid:

$x=5$ $\quad x=-4$ $\quad y=1$ $\quad y=-3$

b The lines in part **a** form a rectangle. Write down the coordinates of the vertices of the rectangle.

4 One vertex of a square is (1, 2). Each side of the square is 4 units long. The sides of the square are parallel to the coordinate axes.

What could be the coordinates of the other vertices? (There is more than one possible answer.)

5 a Draw the line $x=1$ on a grid. Label this line L.

b Draw a line parallel to L and 3.5 units away from L.

c Write down the equation of the parallel line.

d Draw a line perpendicular to L that passes through (1, 4.5).

e Write down the equation of the perpendicular line.

f Draw a line perpendicular to L that passes through (3.5, 1.5). Write down the equation of this line.

6 a On a grid, draw these four lines.

i $x=3$ $\quad$ ii $y=5$ $\quad$ iii $x=-3$ $\quad$ iv $y=-1$

b You can see a square inside the four lines. Work out the area of the square.

c Draw four lines that enclose a rectangle with an area of 18 units.

d Write down the equations of your four lines.

e Compare your answers to parts **c** and **d** with a partner's answers. Check that you both have correct answers.

f Suppose you know the equations of your four lines in part **c**. Can you use the equations to show that the area of the rectangle is 18 without drawing it? Explain how to do this.

7 Here are the coordinates of some points: $A(10, 10)$, $B(-10, 15)$, $C(10, -15)$, $D(-15, 15)$, $E(-10, -10)$ and $F(15, -15)$.

Which points are on each of these lines?

a $y=10$ $\quad$ b $x=-10$

c $x=15$ $\quad$ d $y=-15$

Think like a mathematician

8 Two lines parallel to the x-axis and two lines parallel to the y-axis make a square. The centre of the square is at the origin (0, 0).
Each side of the square is 8 units.

a Find the equation of each of the four lines.

b A second square is the same size but its centre is in a different place. How would you find the equations of the four lines of this square?

c Suppose you know the centre of a square and the length of each side. How would you find the equations of the four lines?

Summary checklist

☐ I can recognise straight-line graphs that are parallel to the x-axis or the y-axis.

11.4 Interpreting graphs

In this section you will …

- interpret graphs related to rates of change.

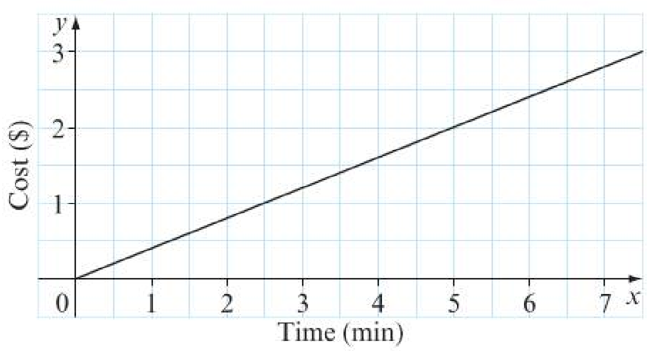
Key word

negative gradient
positive gradient
rate
zero gradien

Graphs can show how the value of a variable changes with time.

Worked example 11.5

This graph shows the cost of making a phone call.



Continued

a Use the graph to work out the cost of a 4 minute call.

b Copy and complete this table of values.

Length of call (min)	1	2	3	4	5	6	7
Cost ($)						2.4	

c What is the charge for each minute of a phone call?

d Write down a function to show the cost (c), in dollars, for a call that lasts t minutes.

e Work out the cost of a 10 minute call.

Answer

a The cost of a 4 minute call is $1.60.

b

Length of call (min)	1	2	3	4	5	6	7
Cost ($)	0.4	0.8	1.2	1.6	2	2.4	2.8

c $0.4

d $c = 0.4t$

e If $t = 10$, then the cost $c = 0.4 \times 10 = \$4$.

Use your algebra skills from Unit 2.

The graph in Worked example 11.5 is a straight line. This shows that the cost increases by the same amount each minute.

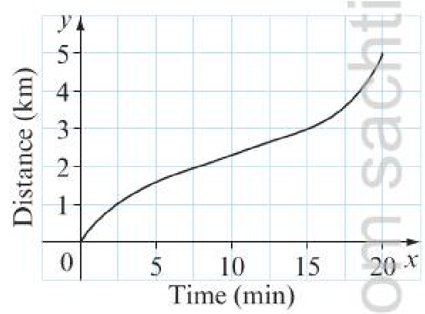
The **rate** for the call is $0.4 per minute.

Tip

This is the cost for each minute.

Worked example 11.6

This graph shows the distance travelled by a runner in a 5 kilometre race.



a How long does the runner take to run the race?

b How far has the runner gone after 5 minutes?

c When is the runner travelling fastest? Explain how you know.

Continued

Answer

a The runner takes 20 minutes to travel 5 kilometres.

b 1.6 kilometres

c The runner is going fastest when the graph is steepest. This is near the beginning and near the end of the race. The runner is going more slowly in the middle of the race, where the graph is less steep.

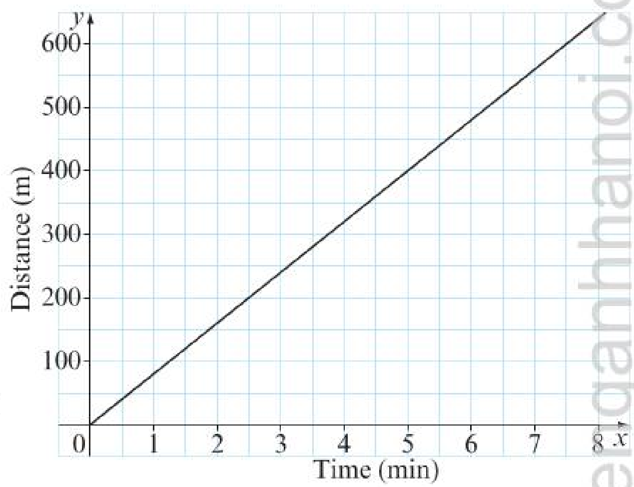
In the graph in Worked example 11.6 the distance increases as the time increases. The graph slopes upwards from left to right. We say that the graph has a **positive gradient**.

When a graph slopes downwards from left to right it has a **negative gradient**.

When a graph is horizontal it has **zero gradient**. An example is the line $y = 5$.

Exercise 11.4

1 A boy is walking. The graph shows how far he walks.



a How far does he walk in 5 minutes?

b Copy and complete this table.

Time (min)	1	2	3	4	5	6	7
Distance (m)							

c What is the rate of walking, in metres per minute?

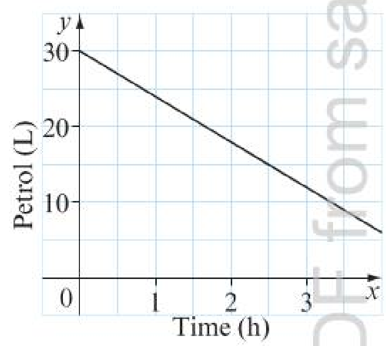
d How far does the boy walk in 10 minutes?

e Write down a function for the distance walked (d metres) in t minutes.

f Show that the boy will take more than 12 minutes to walk 1 kilometre.

g Is the gradient positive, negative or zero?

2 The graph shows the litres (L) of petrol in a car throughout a journey.



a How much petrol is in the car at the start of the journey?

b How much petrol is in the car after 1.5 hours?

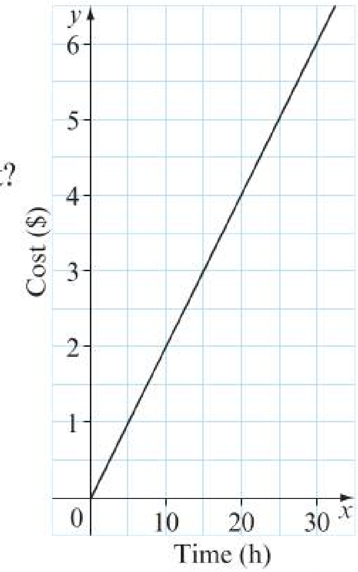
c Work out the rate at which the car uses petrol, in litres per hour.

d If the car uses petrol at the same rate, how long will it take until the car has no petrol?

e Is the gradient positive, negative or zero?

3 This graph shows the cost of the electricity needed to operate a machine.

- **a** Find the cost of the electricity needed to operate the machine for 24 hours.
- **b** The graph is a straight line. What does this tell you about the cost?
- **c** Work out the cost per hour of the electricity.
- **d** Work out the cost of the electricity needed to use the machine for a week.
- **e** Is the gradient positive, negative or zero?

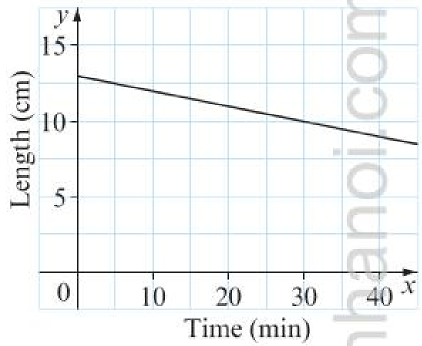


4 This graph shows the length of a burning candle.

- **a** Work out the length of the candle at the start.
- **b** Copy and complete this table.

Time (min)	0	10	20	30	40
Length (cm)					

- **c** How much does the length of the candle decrease every minute?
- **d** If the candle burns at the same rate, how long will it take to burn completely?
- **e** Is the gradient positive, negative or zero?

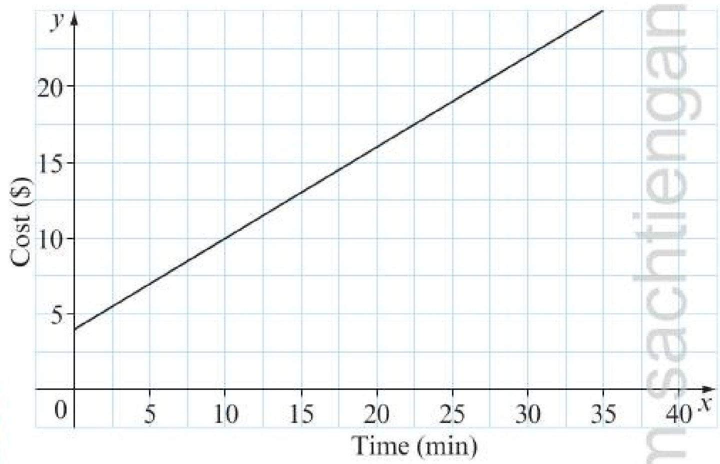


5 A taxi has an initial charge and then a charge for each minute of the journey. This graph shows the total cost.

- **a** How does the graph show that the initial charge is $4?
- **b** Copy and complete this table.

Time (min)	10	20	30	40
Cost ($)				

- **c** Work out the extra cost of each extra minute of the taxi journey.
- **d** Sofia says:

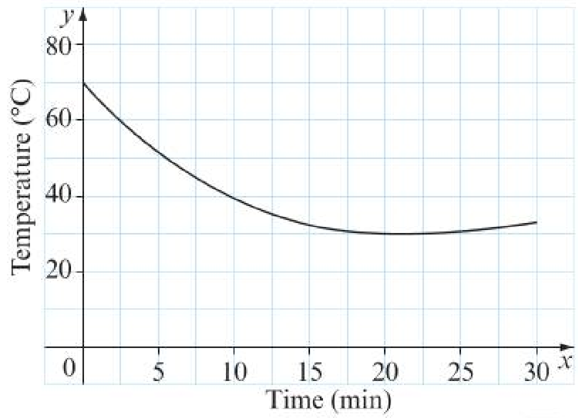


Show that Sofia is not correct.

e Show your answer to part **d** to a partner. Ask your partner if your explanation is clear. Can it be improved?

f Is the gradient positive, negative or zero?

6 The graph shows how the temperature of some water is changing.



a What is the temperature at the start?

b How does the graph show that the water is cooling down?

c Copy and complete this table.

Time (min)	0	5	10	15	20	25
Temperature (°C)				33		31

d The water is not cooling at the same rate all the time. Describe how the rate of cooling changes. Give a reason for your answer.

e The water stops cooling when it is at room temperature. Use the graph to estimate room temperature.

f The gradient of the curve varies.

Describe when the gradient is:

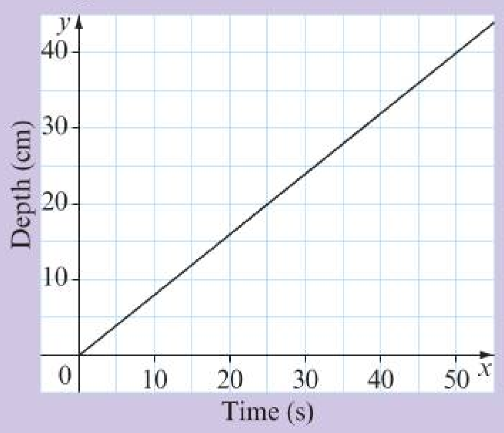
i positive **ii** negative **iii** zero

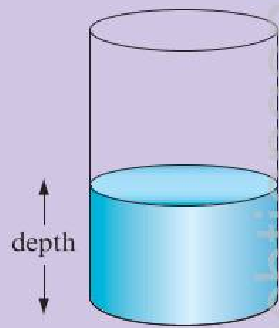
Think like a mathematician

7 A vase has the shape of a cylinder.

The vase is being filled at a constant rate with water from a tap.

This graph shows how the depth of the water in the vase changes with time.





a Work out the depth of water after 20 seconds.

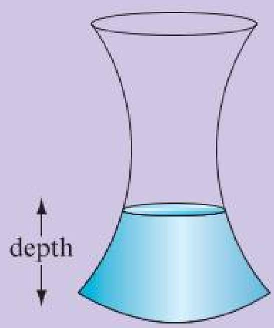
Continued

b When is the depth 30 centimetres?

c Work out the rate at which the depth is increasing, in centimetres per second.

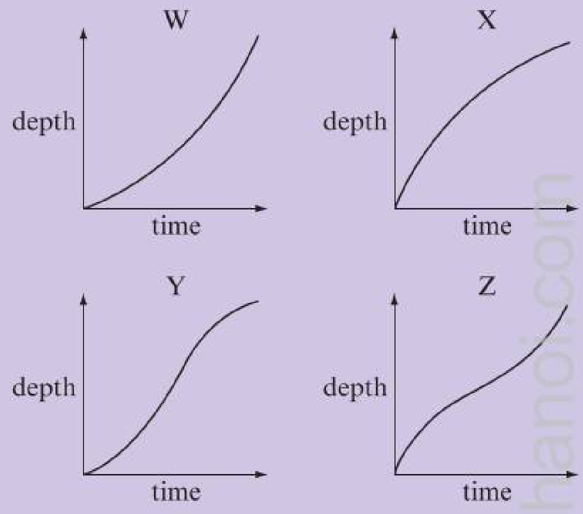
Here is another vase.

The vase is being filled at a constant rate with water from a tap.



d Explain why the graph of depth against time for this vase is not a straight line.

Here are four graphs.



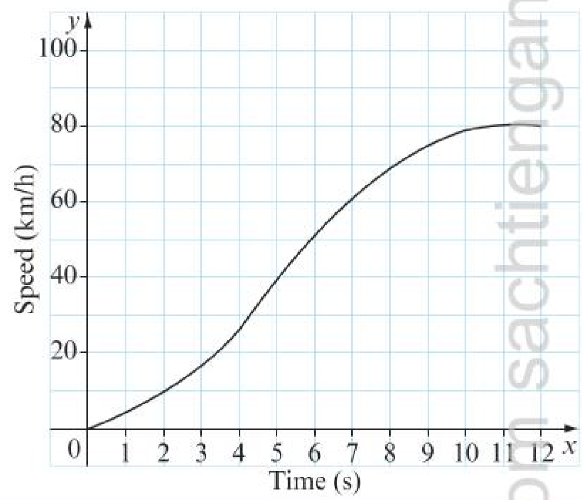
e Which graph shows how the depth of the water changes for the second vase?

f Draw a vase to match the shape of each of the other three graphs.

g Draw other vases that will produce graphs that are different from the ones shown above.

h Share your work with a partner. Do you both agree on your answers to parts **e**, **f** and **g**?

8 A car starts from rest. Its speed then increases for 12 seconds.



a What is the car's speed after 12 seconds?

b Find the car's speed after 6 seconds.

c How long does it take for the speed of the car to be 74 km/h?

d How much does the car's speed change in the first 4 seconds?

e When is the car's speed changing most quickly? Give a reason for your answer.

Summary checklist

- [ ] I can interpret graphs that show a variable changing over time.
- [ ] I can explain why graphs showing rates of change have a specific shape.

Check your progress

1 Jeff and Larissa both have some money. Larissa has \$4 more than Jeff.

a Copy and complete this table to show how much money they could each have. Add one column of your own to the table.

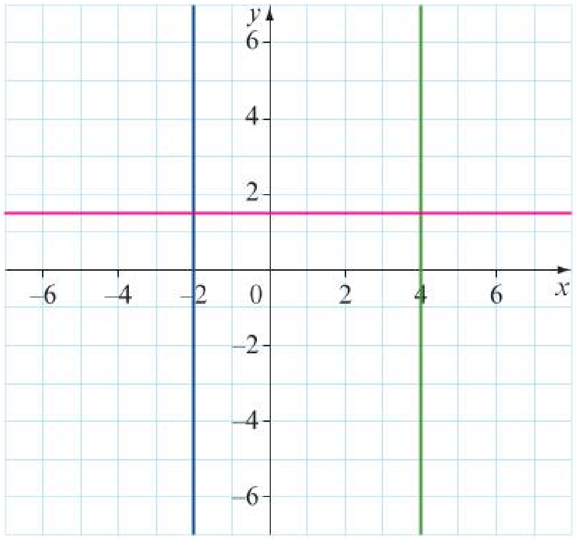
Jeff (\$)	2	3	4	5	6
Larissa (\$)		7			

b Use the table in part **a** to draw a graph.

c Jeff has \$$x$ and Larissa has \$$y$. Write a formula for y in terms of x.

2 a Write down the equations of the three lines on this grid.

b A fourth line is added so that the four lines form a square. Find the equation of the fourth line.



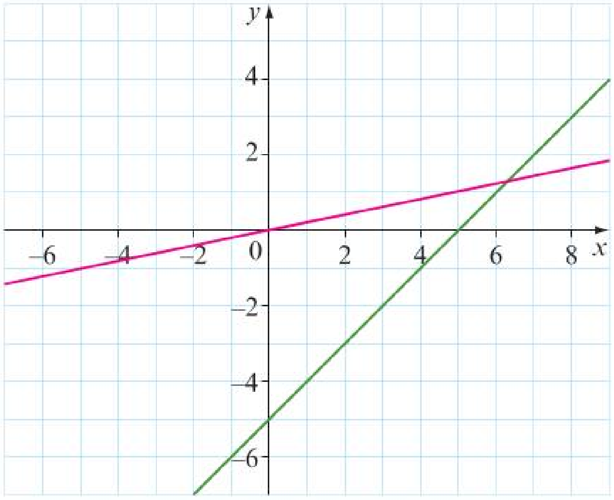
3 Two lines parallel to the axes cross at (−9.5, 7.5). Write down the equations of the lines.

4 a Draw a graph of the line $y = x - 4$.

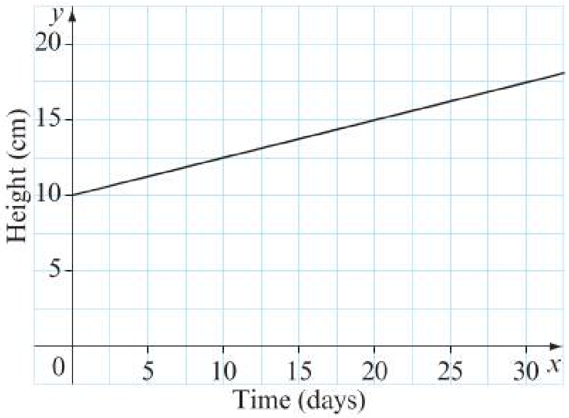
b On the same axes, draw the line $y = \frac{1}{3}x$.

c Where do the lines cross?

5 Work out the equations of these two lines.



6 The height of a plant is measured every day. This graph shows the results.



a Find the initial height of the plant.

b Find the height after:

i 10 days **ii** 20 days **iii** 30 days

c How do you know that the plant grows the same amount each day?

d How much does the plant grow each day?

› Project 5

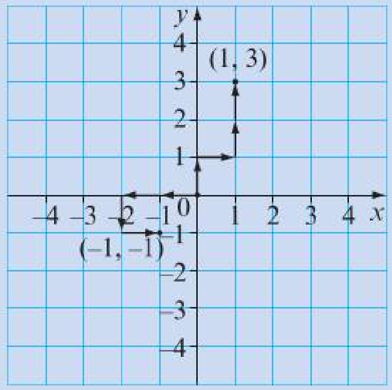
Four steps

I wonder where I will finish when I start at the origin (0, 0) and take four steps along the gridlines going up, down, left or right …

Two possible paths are shown.

When I go up, right, up, up, I finish at the point (1, 3).

When I go left, left, down, right, I finish at the point (–1, –1).



Explore some other four-step paths, starting at the origin.

At how many different points can I finish?

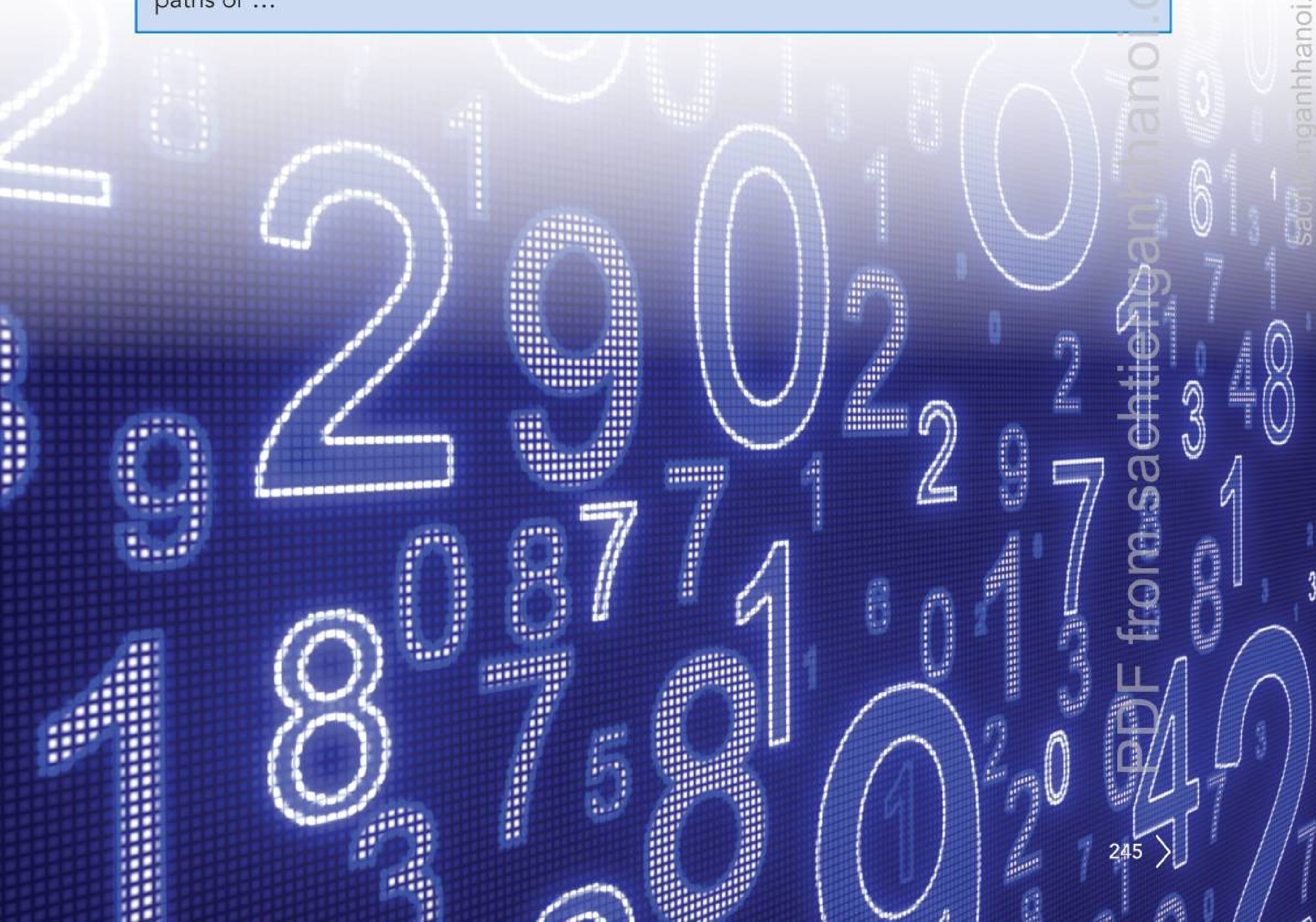
What shape is made by the points that I can reach?

What is special about the coordinates of the points where I finish?

Are there any points that I can reach in only one way?

Explore the number of ways you can reach the points on the boundary.

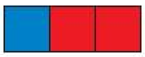
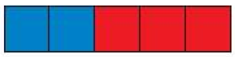
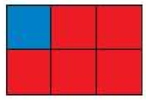
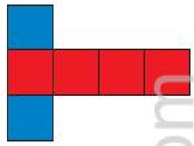
Once you've explored four-step paths, you might like to explore five-step paths or six-step paths or …



12 Ratio and proportion

Getting started

1 For each of these shapes, write down the ratio of blue squares to red squares.

a  **b**  **c**  **d** 

2 Which of these ratios are equivalent to the ratio 1 : 2?

2 : 4   3 : 5   7 : 14   6 : 3   10 : 20   8 : 9

3 Put these cards into pairs of equivalent ratios.

1 : 3   4 : 1   2 : 5   2 : 6   3 : 2   6 : 15   6 : 4   12 : 3

4 One pen costs 80 cents. How much does it cost to buy:

a two pens? **b** five pens? **c** 20 pens?

5 A 5 litre tin of paint covers 20 square metres of wall. How many square metres of wall will 15 litres of the same paint cover?

A ratio is a way of comparing two or more numbers or quantities.

You use the symbol : to show ratio.

Ratios were used a long time before this symbol was invented.

Pythagoras was a famous Greek mathematician who lived around 500 BCE. Historians and mathematicians believe that he discovered the golden ratio.

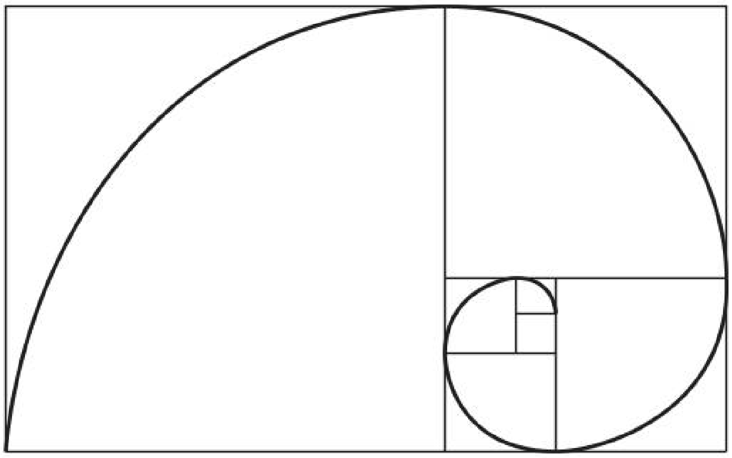
If the lengths of the sides of a rectangle are in the golden ratio, the rectangle is called the golden rectangle. Some sizes of paper, such as A4, are based on the golden rectangle.

Pythagoras (about 570 BCE to 495 BCE)

$$\frac{a+b}{a} = \frac{a}{b} = \varphi \approx 1{,}61803$$

A rectangle is golden if, when you cut a square from it, the piece left over is mathematically similar to the original. This means that the sides are in the same **proportion**.

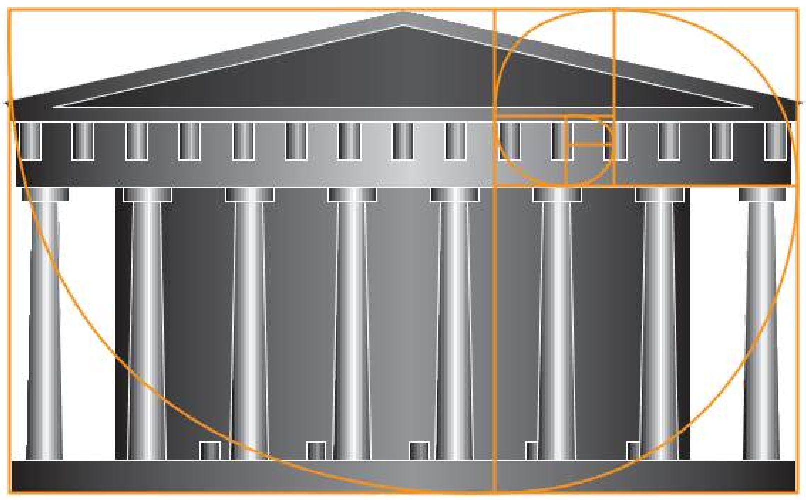
When you divide a rectangle in this way several times, you will see that a spiral shape forms.



The golden rectangle

You can see this spiral shape in nature; for example, in the shell of a chambered nautilus.

You can also see this spiral shape in architecture, such as the Parthenon in Athens.



Today you use ratios in everyday life, often without knowing it! Builders use ratios when mixing sand and cement for concrete. Chefs use ratios when mixing ingredients for cakes.

12.1 Simplifying ratios

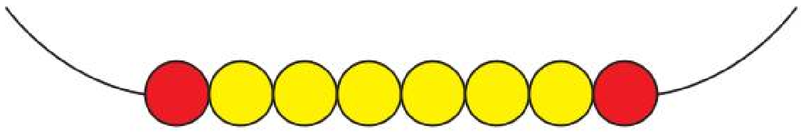
In this section you will …

- simplify and compare ratios.

Key words

highest common factor
proportion
ratio
simplify
simplest form

In this necklace there are two red beads and six yellow beads.



You can write the **ratio** of red beads to yellow beads as 2:6. You say 'The ratio of red beads to yellow beads is two to six.'

For every one red bead there are three yellow beads, so you can **simplify** the ratio 2:6 to 1:3.

You write a ratio in its **simplest form** by dividing the numbers in the ratio by the **highest common factor**. Here, the highest common factor of 2 and 6 is 2, so you divide both the numbers by 2.

red : yellow
2 : 6
÷2 ÷2
1 : 3

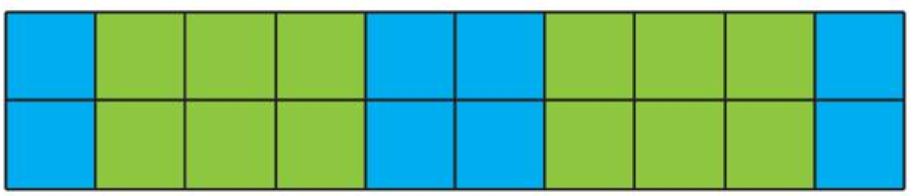
A ratio written in its simplest form must not contain any decimals or fractions.

Worked example 12.1

a The rectangle shown is made up of blue squares and green squares.

i Write down the ratio of blue squares to green squares.

ii Write the ratio in its simplest form.



b A model of a car is 6 cm high. The real car is 160 cm high.

Write the ratio of the height of the model car to the real car, in its simplest form.

Continued

Answer

a **i** blue : green = 8 : 12	There are eight blue squares and 12 green squares in the rectangle.
ii 8 : 12 (÷4) → 2 : 3 (÷4)	The highest common factor of 8 and 12 is 4, so divide both numbers by 4.
b 6 cm : 160 cm	Start by writing the two heights in the correct order, model car : real car.
6 : 160	As the units are the same, you can ignore them.
3 : 80	The highest common factor of 6 and 160 is 2, so divide both numbers by 2.

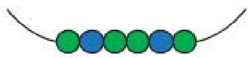
Exercise 12.1

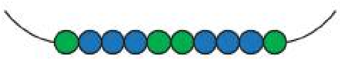
1 For each of these necklaces, write down the ratio of green beads to blue beads, in its simplest form. The first one has been done for you.

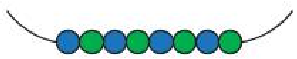
a 

green : blue

2 : 4 (÷2) → 1 : 2 (÷2)

b 

c 

d 

2 Write down the correct answer (**A**, **B** or **C**) for each of these. The first one has been done for you.

a 6 : 12 written in its simplest form is:

A 3 : 6 **B** 1 : 6 **C** 1 : 2

Answer: **C** 1 : 2

b 3 : 18 written in its simplest form is:

A 1 : 6 **B** 1 : 5 **C** 1 : 4

c 20 : 4 written in its simplest form is:

A 10 : 2 **B** 1 : 5 **C** 5 : 1

d 24 : 6 written in its simplest form is:

A 12 : 3 **B** 4 : 1 **C** 8 : 2

3 Write each of these ratios in its simplest form.

a	2 : 8	**b**	2 : 12	**c**	3 : 6
d	3 hours : 15 hours	**e**	4 kg : 8 kg	**f**	4 cm : 12 cm
g	25 : 5	**h**	60 : 5	**i**	36 : 6
j	14 days : 7 days	**k**	24 g : 8 g	**l**	54 mL : 9 mL

4 Look at this number sequence: 32, 28, ☐, 20, 16, ☐, 8, 4.

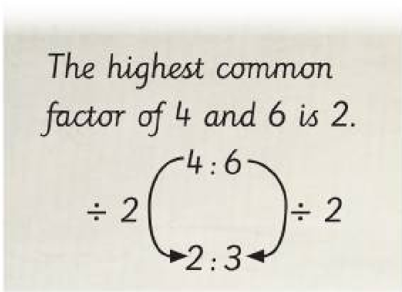
Two of the numbers are missing.

Write the ratio of the first missing number to the second missing number, in its simplest form.

5 Pierre makes a model of the Eiffel Tower in Paris.
The Eiffel Tower is 324 metres tall.
Pierre's model is 3 metres tall.

Write the ratio of the height of the real Eiffel Tower to the height of the model, in its simplest form.

6 This is how Zara writes the ratio 4 : 6 in its simplest form.



Write down the correct answer, **A**, **B** or **C** for each of these.

a 8 : 12 written in its simplest form is:

A 4 : 6 **B** 2 : 3 **C** 1 : 1.5

b 4 : 18 written in its simplest form is:

A 1 : 4.5 **B** 9 : 2 **C** 2 : 9

c 32 : 12 written in its simplest form is:

A 8 : 3 **B** 16 : 6 **C** 3 : 8

7 Write each of these ratios in its simplest form.

a	6 weeks : 21 weeks	**b**	8 mL : 10 mL
c	8 tonnes : 14 tonnes	**d**	24 °C : 10 °C
e	28 km : 12 km	**f**	21 litres : 9 litres

8 Helena sees this recipe for pastry.
She says, 'The ratio of margarine to flour is 4 : 1.'

a Explain the mistake that Helena has made.

b Complete this statement correctly for Helena:
The ratio of _________ to _________ is __________.

Pastry	
200 g flour	Pinch salt
50 g margarine	Water to mix
50 g lard	

Compare your answer to Question **8b** with a partner's answer.

a Do you have the same answer? If you do, explain how you can give a different answer that is also correct.

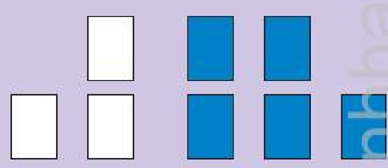
b Do you have a different answer? If you do, are you both correct? Explain why.

c Compare your answers with those of other learners in the class.

9 In a recipe, Vipul uses 45 millilitres of vinegar and 120 millilitres of oil. Write the ratio of vinegar to oil, in its simplest form.

Think like a mathematician

10 Bryn mixes tins of white paint with blue paint in the ratio 1 : 2.
Alun mixes tins of white paint with blue paint in the ratio 2 : 3.



Who has the darker blue paint, Bryn or Alun?
With a partner or in a group, discuss the different methods you could use to answer this question.
Decide which is your favourite method. Would this method work for all questions that are like this?

11 Melania makes a drink by mixing orange juice with water in the ratio 2 : 5.
Boris makes a drink by mixing orange juice with water in the ratio 1 : 3.
Who has the drink with the higher proportion of orange juice?
Explain how you worked out your answer.

Tip

The drink with the higher proportion of orange juice is the drink that has more orange juice.

12 **a** One year, at the Borrowdale golf club, there are 15 women and 60 men members. Write the ratio of women : men, in its simplest form.

b In the same year, at the Avondale golf club, there are 12 women and 56 men members.

Write the ratio of women : men, in its simplest form.

c Which golf club has the greatest proportion of men members? Explain how you worked out your answer.

13 Use the information in Question **12** to answer this question.

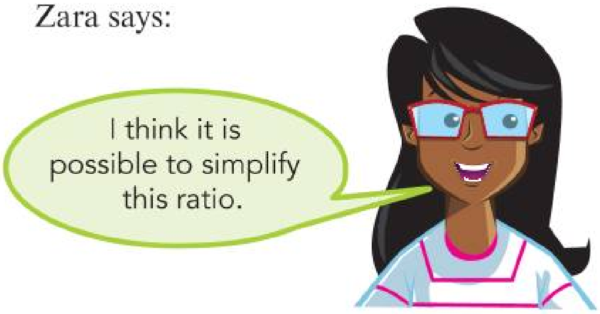
In the following year, seven women and six men join the Borrowdale golf club. Also 12 women and 10 men join the Avondale golf club.

Which golf club now has the greatest proportion of men members? Explain how you worked out your answer.

14 Both Zara and Sofia are trying to answer this algebra question:

Is the ratio $20x : 15x$ written in its simplest form?

Zara says:



Sofia says:

a Explain whether you think that Zara is correct or incorrect.

b Discuss your answer with a partner. Do you agree with each other?

If you agree, check your answer with other learners in your class. If you disagree, explain your reasons to each other.

Summary checklist

- [ ] I can simplify ratios.
- [ ] I can compare ratios.

12.2 Sharing in a ratio

In this section you will …

- divide an amount into two parts in a given ratio.

Key words

conjecture
divide
share
twice as much

You can use ratios to divide things up or to share them.

Example: Sally and Bob buy a car for \$15 000.

Sally pays \$10 000 and Bob pays \$5000.

You can write the amounts they paid as a ratio.

Sally : Bob

$$\div 5000 \left(\begin{array}{c} 10\,000 : 5000 \\ 2 : 1 \end{array} \right) \div 5000$$

So, Sally paid **twice as much** as Bob.

Five years later they sell the car for \$9000. They need to share the money fairly between them.

Sally paid twice as much as Bob, so she should get twice as much as him.

How do you work out how much each of them gets?

To share in a given ratio:

- Add the numbers in the ratio to find the total number of parts.
- Divide the amount to be shared by the total number of parts to find the value of one part.
- Use multiplication to work out the value of each share.

Tip

The highest common factor of 10 000 and 5000 is 5000, so you divide by 5000 to simplify the ratio.

Worked example 12.2

Share \$9000 between Sally and Bob in the ratio 2 : 1.

Answer

Total number of parts:	$2 + 1 = 3$	Add the numbers in the ratio to find the total number of parts.
Value of one part:	\$9000 ÷ 3 = \$3000	Divide the amount to be shared by the total number of parts to find the value of one part.
Sally gets:	\$3000 × 2 = \$6000	Multiply by 2 to get Sally's share.
Bob gets:	\$3000 × 1 = \$3000	Multiply by 1 to get Bob's share.
		Check: \$6000 + \$3000 = \$9000 ✓

Exercise 12.2

1 **a** Copy and complete the workings to share \$45 between Ethan and Julie in the ratio 1 : 4.

Total number of parts: $1 + 4 = \square$

Value of one part: $\$45 \div \square = \square$

Ethan gets: $1 \times \square = \square$

Julie gets: $4 \times \square = \square$

b Copy and complete this table to show the information.

	Number of parts	Amount
Ethan	1	\$□
Julie	4	\$□
Total	5	\$45

2 Share these amounts between Dong and Chul in the ratios given. Show all your workings. Use the same method as that used in Question **1**.

a \$24 in the ratio 1 : 2 **b** \$65 in the ratio 1 : 4

c \$48 in the ratio 3 : 1 **d** \$30 in the ratio 5 : 1

e \$21 in the ratio 1 : 6 **f** \$64 in the ratio 7 : 1

Tip

If it helps you, put the information in a table, as in Question **1b**.

Look again at your answers to questions **1** and **2**.

a Think about a method that you can use to check that you have shared each amount correctly.

b Compare your method with a partner's method. Do you both have the same method?

If you don't have the same method, which method do you think is best?

3 Share these amounts between Lin and Kuan-yin in the ratios given.

a \$35 in the ratio 2 : 3 **b** \$49 in the ratio 3 : 4

c \$32 in the ratio 5 : 3 **d** \$90 in the ratio 7 : 3

4 This is part of Lily's homework.

Question

Raine and Abella share an electricity bill in the ratio 5 : 4.
The electricity bill is \$72.
How much does each of them pay?

Solution

Total number of parts:	$5 + 4 = 9$
Value of one part:	$\$72 \div 9 = \$8$
Raine pays:	$5 \times \$8 = \square$
Abella pays:	$4 \times \$8 = \square$
Check:	

Copy and complete Lily's solution. Make sure you include a check.

Think like a mathematician

5 Work with a partner or in a small group to answer this question.

a A shop sells bags of Brazil nuts and walnuts in the ratio 4 : 5.
What fraction of the bags of nuts that the shop sells are:

i Brazil nuts? **ii** walnuts?

b Explain how you can use any ratio to work out each part of the ratio as a fraction of the total.

6 A school choir is made up of girls and boys in the ratio 4 : 3.
There are 35 students in the choir altogether.

a How many of the students are boys?

b What fraction of the students are boys?

Activity 12.1

On a piece of paper, write down four questions that are similar to Question **6** in this exercise.

Work out the answers on a separate piece of paper.

Make sure the questions can be answered without using a calculator.

Swap your questions with those of a partner. Work out the answers to your partner's questions.

Swap your papers back and mark each other's work.

If you think your partner has made a mistake, discuss their mistake with them.

Think like a mathematician

7 In pairs or groups, look at the following question and solution.
This is part of Rafina's homework.

Question

Brad and Lola buy a painting for \$120.
Brad pays \$80 and Lola pays \$40.
They later sell the painting for \$630.
How much should each of them get?

Solution

Ratio for Brad : Lola is 80 : 40.

Total number of parts:	$80 + 40 = 120$
Value of one part:	$\$630 \div 120 = 5.25$
Brad gets:	$\$80 \times 5.25 = \$420$
Lola gets:	$\$40 \times 5.25 = \$210$
Check:	$\$420 + \$210 = \$630$ ✓

Rafina has got the answer correct.

However, some of her calculations were quite difficult and she had to use a calculator.

a How can she make the calculations easier?

b Rewrite the solution for her. Do not use a calculator.

c Compare your answers to parts **a** and **b** with those of other learners. Did you have the same idea or different ideas?

Tip

What extra step could Rafina add to simplify her solution?

8 Arun and Marcus are going to share \$240, <u>either</u> in the ratio of their ages <u>or</u> in the ratio of their masses.

arun is 14 years old and has a mass of 58 kg. Marcus is 16 years old and has a mass of 62 kg.

a **Conjecture** which ratio, age or mass, you think will be better for Arun. Explain your decision.

b Work out whether your decision is correct.

9 Every New Year, Auntie Bea gives \$320 to be shared between her nieces in the ratio of their ages. This year the nieces are aged 3 and 7.

Show that in 5 years' time the younger niece will get \$32 <u>more</u> than she got this year.

10 Carys and Damien share \$$E$ in the ratio $c:d$. Use algebra to write an expression for the amount that:

a Carys will get

b Damien will get

Tip

In question **10**, start by writing an expression for:

i the total number of parts

ii the value of one part.

Summary checklist

☐ I can share an amount into a given ratio with two parts.

12.3 Using direct proportion

In this section you will …

- use the unitary method to solve problems involving ratio and direct proportion.

Key words

direct proportion
half as much
twice as much
unitary method

Two quantities are in **direct proportion** when their ratio stays the same as the quantities increase or decrease.

One packet of rice costs \$3.25, so two packets of rice cost twice as much.

Two packets of rice cost $2 \times \$3.25 = \$6.50$.

Six tickets to a concert cost \$120, so three tickets cost **half as much**.

Three tickets cost $\$120 \div 2 = \$60$.

Worked example 12.3

a Three books cost \$12. Work out the cost of 10 books.

b A recipe uses two eggs to make 12 cupcakes. How many eggs are needed to make 36 cupcakes?

Answer

a	$\$12 \div 3 = \$4$	First, work out the cost of one book by dividing the total cost of the books by 3.
	$10 \times \$4 = \$40$	Now work out the cost of 10 books by multiplying the cost of one book by 10.
b	$36 \div 12 = 3$	You can see that to make 36 cupcakes you need to multiply the recipe by 3.
	$2 \times 3 = 6$ eggs	So, multiply the number of eggs by 3.

The method used in part **a** of Worked example 12.3 is called the **unitary method** because you find the cost of one book then use this to find the cost of 10 books.

Exercise 12.3

Tip

You can assume that all of the bananas have the same mass.

Tip

Currency information:

USD are United States dollars.

ZAR are South African rand.

1. The mass of three bananas is 375 grams.
   Copy and complete the working to find the mass of eight bananas.
   One banana weighs: $375 \div 3 = \square$
   Eight bananas weigh: $8 \times \square = \square$
2. Tom buys two bags of chips for \$2.40.
   Work out the cost of:
   a one bag of chips **b** five bags of chips
3. Joy exchanges USD 100 for ZAR 1400.
   a How many ZAR can she exchange for 1 USD?
   b How many ZAR can she exchange for 6 USD?
4. Ivan buys four shirts for \$37.80. Work out the cost of nine shirts.
5. Dakri buys seven tickets to a show for \$112. How much do three tickets cost?
6. A recipe uses 360 grams of flour for a serving of four people. How much flour is needed for three people?

7 Look at this question.

Question

A recipe uses 450 grams of rice for six people.
How much rice is needed for:
a 24 people? b nine people?

Jamila and Xun answer this question in different ways.
Jamila uses the unitary method.

Solution

Six people need 450 grams.
One person needs $450 \div 6 = 75$ grams.
a 24 people need $24 \times 75 = 1800$ grams or 1.8 kilograms.
b Nine people need $9 \times 75 = 675$ grams.

Xun uses a build-up method.

Solution

a Six people need 450 grams.
24 people need four times as much, as $6 \times 4 = 24$.
24 people need $4 \times 450 = 1800$ grams or 1.8 kilograms.
b Six people need 450 grams.
Three people need $450 \div 2 = 225$ grams.
Nine people need $450 + 225 = 675$ grams.

a Look through both solutions and make sure that you understand the methods.
b Critique the methods, writing down the advantages and disadvantages of each method.
c If you were using a calculator, which method would you choose? Explain why.
d If you didn't have a calculator to use, which method would you choose? Explain why.
e Are your answers to parts **c** and **d** the same or different? Explain why.

Think like a mathematician

8 In pairs, groups, or as a class, compare your answers to question **7**.
When is it best to use the unitary method / the build-up method?
Discuss your reasons why, and the advantages and disadvantages of each method.

9 A recipe for four people uses 800 grams of potato.
Copy and complete the workings, using the build-up method, to find the mass of potato needed for:

a 20 people **b** six people

a Mass of potato for four people is 800 grams.
The connection between 4 and 20 is: $20 \div 4 = \square$
Mass of potato for 20 people: 800 grams $\times \square = \square$ grams

b Mass of potato for four people is 800 grams.
Mass of potato for two people: 800 grams $\div 2 = \square$ grams
Mass of potato for six people: 800 grams + $\square$ grams = $\square$ grams

10 This is part of Irene's homework.

Question
A recipe for six people uses 300 grams of rice.
How much rice is needed for 15 people?

Solution
The recipe is for six people, and 6 + 9 = 15.
Six people need 300 grams of rice.
Three people need 300 grams ÷ 2 = 150 grams of rice
6 + 3 = 9, so 300 + 150 = 450 grams
Altogether, 450 grams of rice is needed.

Explain Irene's mistake and write out the correct solution.

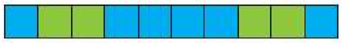
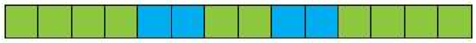
11 A teacher buys homework books for her class of 30 students. She paid a total of \$105 for the books. Two more students join her class, so she then buys two extra books. The teacher works out that the total cost of the books is now \$121. Is she correct? Explain your answer. If the teacher is incorrect, what mistake do you think she has made?

Summary checklist

☐ I can use the unitary method to solve problems involving direct proportion.

Check your progress

1 For each of these shapes, write down the ratio of green squares to blue squares. Write each ratio in its simplest form.

a 
b 

2 Write each of these ratios in its simplest form.

a 2:6 b 18:3 c 6:9 d 24:16

3 Guy makes a drink by mixing apple juice with water in the ratio 1:4.

Kim makes a drink by mixing apple juice with water in the ratio 2:7.

Who has the drink with the higher proportion of apple juice? Explain how you worked out your answer.

4 Share these amounts between Tao and Chris in the ratios given.

a \$15 in the ratio 1:2 b \$25 in the ratio 4:1

c 45 kilograms in the ratio 3:2 d 24 litres in the ratio 5:3

5 Li-Ming buys two packets of chips for \$1.40. Work out the cost of:

a one packet of chips b five packets of chips

6 The mass of four tins of beans is 1.6 kilograms. Work out the mass of:

a 12 tins of beans b 10 tins of beans

13 Probability

Getting started

1 Three possible **outcomes** when you throw a dice are: throw a 6, throw an even number, and throw less than 5.

- **a** Which of these outcomes is most **likely**?
- **b** How would you describe the **likelihood** of throwing an even number?
- **c** Which two of these outcomes are **mutually exclusive**?

2 Idzi makes a dice out of a piece of wood. He rolls the dice 50 times and gets these scores.

1 4 2 3 1	5 2 4 2 1	5 6 4 2 1	5 2 6 2 4	2 1 2 4 3
2 5 5 1 2	3 3 6 5 2	3 2 1 1 1	4 5 3 1 2	3 2 5 6 2

He thinks that the dice is not fair.

- **a** What does Idzi mean when he says that the dice is 'not fair'?
- **b** What proportion of the throws are:
  - **i** 1? **ii** 2? **iii** 6?
- **c** On the basis of these results, which number is most likely?

3 **a** Write $\frac{4}{5}$ as a decimal and as a percentage.

- **b** Write 30% as a decimal and as a fraction.

Uncertainty and chance exist in many situations in real life.

For example, if you are planning an outdoor activity, it is important to know what type of weather to expect. Will it be sunny? Will it be windy? Will it rain?

You cannot be certain what the weather will be. However, a weather forecast often gives you an idea of what type of weather to expect.

Here are some questions for which the answers involve uncertainty.

- Will my team win their next match?
- What is the chance of an earthquake?
- What is the likelihood of having an accident on a particular road?
- If I throw two dice, will I score a double 6?
- Is it safe to go skiing?
- Will my train be late?

- What are my chances of getting a top exam grade?
- Am I likely to live to be 100 years old?

The branch of maths that deals with uncertainty is called probability. In this unit you will learn how to calculate and use probabilities in simple situations.

› 13.1 The probability scale

In this section you will ...

- use the language associated with probability
- understand how probabilities range from 0 to 1.

Key words

event
even chance
likely
likelihood
outcome
probability
unlikely

Rolling an odd number on a dice or winning a football match are examples of **events**.

Events can have a number of different outcomes.

The outcomes that give an odd number when you roll a dice are 1, 3 and 5.

The outcomes when you win a football match are the possible scores.

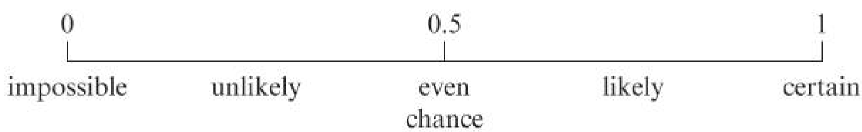
You can use words such as certain, likely, **unlikely** and impossible to describe the likelihood that an event will happen.

You can also use a number between 0 and 1 to represent the likelihood that an event will happen. This number is called a **probability**.

An event that is certain to happen has a probability of 1.

An outcome that is impossible has a probability of 0.

You can show a probability on a probability scale.



You can write a probability as a fraction, a decimal or a percentage.

For example, an **even chance** means a probability of 0.5 or $\frac{1}{2}$ or 50%.

Worked example 13.1

Tigers and Lions are two football teams.

The probability that Tigers will win their next match is 45%.

The probability that Lions will win their next match is $\frac{4}{5}$.

a Which team is more likely to win their next match? Give a reason for your answer.

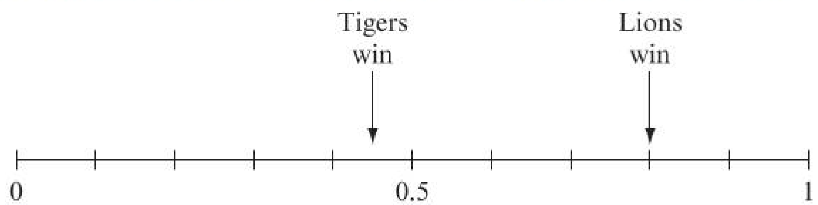
b Show each team's probability of winning on a probability scale.

Answer

a $\frac{4}{5} = 0.8 = 80\%$

Lions are more likely to win because 80% > 45%.

b If you mark the line in tenths, it is easy to put in the events.



Exercise 13.1

1 Here are some words that describe likelihood.

likely | unlikely | very likely | very unlikely | certain | impossible | even chance

a Find some outcomes that can be described by each of these words or phrases.

b Can you think of any other words or phrases to describe likelihood? If you can, give some examples.

2 Choose the best word or phrase to describe these events.

a When you flip a coin, it will land showing heads.

b The day after Monday will be Tuesday.

c You have the same birthday as your teacher.

d It will rain one day next week.

e You will do well in your next maths test.

3 You throw a dice. Put these events (**A** to **E**) in order of likelihood. Put the least likely first.

A You throw the number 3.
B You throw the number 3 or more.
C You throw a number less than 3.
D You throw an odd number.
E You throw a number less than 1.

4 The probability of rain tomorrow is 25%.
The probability of sunny weather tomorrow is 60%.

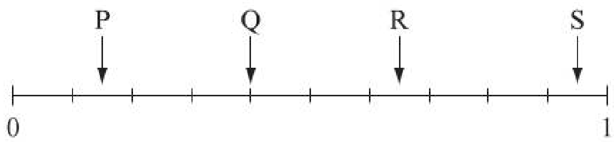
a Write both probabilities as fractions.
b Show each weather event's probability on a probability scale.

5 Here are the probabilities that three teams will win their next match.

City $\frac{2}{3}$, Rovers 60%, United 0.7

a Which team is most likely to win? Give a reason for your answer.
b Which team is least likely to win?

6 Here is a probability scale.



Estimate the probability of each event.

7 a Draw a probability scale. Mark these events on the diagram.

A Zhing will be late for school: 25%
B Rain will fall in the town: 0.6
C The football match will be a draw: $\frac{1}{5}$
D A plant will flower: 80%
E Roshni will study maths at university: $\frac{9}{10}$
F The train will be late: 0.05

b Compare your probability scale with a partner's.

8 a Suggest some events that could have the following probabilities.

i $\frac{1}{10}$ ii 50% iii 0.85 iv 100%

Compare your examples with a partner's examples. Which examples do you prefer?

Think like a mathematician

9 **a** A weather forecast says:

The probability of rain tomorrow is 80%.

How do you think this probability was worked out?

b Try to find some examples of probabilities being used in the news. How do you think the probabilities were worked out?

10 Sogand flips a coin. The probability that it lands with heads facing up is 50%.

Sogand flips the coin five times. She gets heads every time.

Look at these three statements.

A The probability that the next flip lands with heads facing up is 50%.

B The probability that the next flip shows heads is more than 50%.

C The probability that the next flip shows heads is less than 50%.

Which statement do you think is correct? Give a reason for your answer.

Summary checklist

- [ ] I can use language associated with probability to describe the likelihood of outcomes.
- [ ] I can use the probability scale from 0 to 1 with fractions, decimals or percentages.

13.2 Mutually exclusive outcomes

In this section you will ...

- learn how to identify mutually exclusive outcomes
- use equally likely outcomes to find theoretical probabilities.

Key words

equally likely
mutually exclusive
theoretical probability

When two outcomes cannot happen at the same time they are mutually exclusive. For example, if you get a 1 when you throw a dice, you cannot also get a 3.

When you roll a dice, each number is equally likely. 1, 2, 3, 4, 5 and 6 are **equally likely** outcomes.

For the event of winning a football match, do you think that winning 1 – 0 and winning 6 – 0 are equally likely outcomes?

When the outcomes are equally likely, you do not need to do an experiment to find the probabilities. You can use the fact that outcomes are equally likely to calculate **theoretical probabilities**.

Worked example 13.2

Here are 10 number cards.

2	5	8	5	9	8	5	3	8	5

The cards are placed face down. One card is taken without looking.

What is the probability that the number chosen is:

a 3? **b** 5? **c** 8?

d 1? **e** an odd number? **f** not 2?

Answer

There are 10 cards. A card is taken without looking, so each number is equally likely to be chosen.

a One card has the number 3. The probability of choosing a 3 is $\frac{1}{10}$ or 10%.

b Four cards have the number 5. The probability of choosing a 5 is $\frac{4}{10}$ or $\frac{2}{5}$ or 40%.

c Three cards have the number 8. The probability of choosing an 8 is $\frac{3}{10}$ or 30%.

d No cards have the number 1. Choosing a 1 is impossible. The probability of choosing a 1 is 0.

Continued

e Six cards have an odd number. The probability of choosing an odd number is $\frac{6}{10}$ or $\frac{3}{5}$ or 60%.

f Only one card has the number 2. The other nine cards do not have the number 2.

The probability of not choosing 2 is $\frac{9}{10}$ or 90%.

In Worked example 13.2 there are five mutually exclusive outcomes. They are the numbers 2, 3, 5, 8 and 9. Here are the probabilities for each outcome.

Outcome	2	3	5	8	9
Probability	10%	10%	40%	30%	10%

The sum of these probabilities is
$10\% + 10\% + 40\% + 30\% + 10\% = 100\% = 1$.

The probabilities add up to 1 because exactly one of the numbers is certain to be chosen.

Exercise 13.2

1 Chen throws a fair coin.

a What are the possible outcomes?

b Are the outcomes mutually exclusive?

c Are the outcomes equally likely?

2 A weather forecast says that the probability of rain tomorrow is 20% and the probability of strong winds tomorrow is 30%.

a Explain why rain and strong winds are not mutually exclusive outcomes.

b Explain why rain and strong winds are not equally likely outcomes.

3 Balsem throws an unbiased dice.

Work out the probability that she throws:

a the number 4

b the number 3

c an odd number

d an even number

e the number 3 or more

f a number more than 6

4 Each letter of the word MATHEMATICS is written on a separate card.

| A | M | T | H | E | M | A | T | S | C | I |

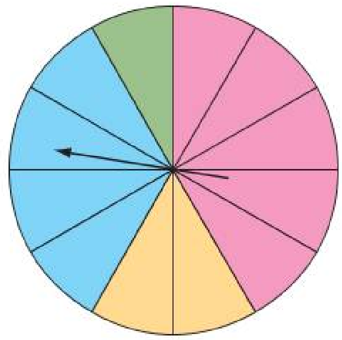
The cards are placed face down. One card is chosen without looking.

Work out the probability that the card is:

a H   **b** A

c S   **d** M

e B   **f** a letter in the word SCIENCE



5 A coloured spinner has 12 equal sections. When spun, the pointer has an equal likelihood of stopping in any of the **sectors**.

a Work out the probability outcome of each of the four colours.

b Show that the sum of the probabilities in part **a** is 1.

6 The letters of the word BANANA are printed on cards.

| B | A | N | A | N | A |

The cards are placed face down. A card is chosen without looking.

a Find the probability that the card is the letter:

i B   **ii** A   **iii** N

Show that your three answers in part **a** add up to 1.
Why does this happen?

7 The names of six boys and four girls are put in a bag.

One of the boys' names is Blake. One of the girls' names is Crystal.

One name is chosen without looking.

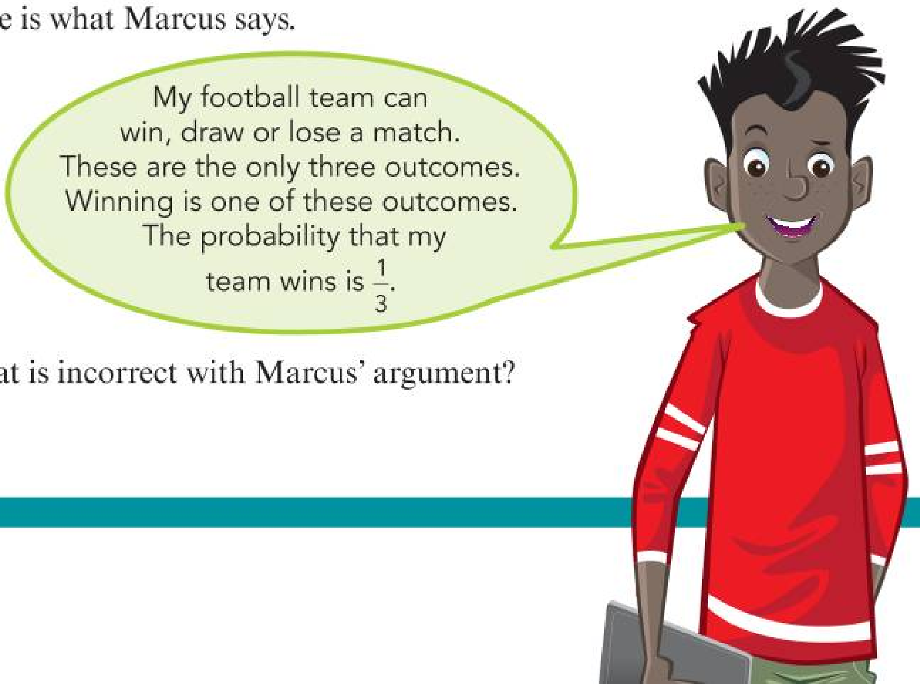
a Work out the probability that the name chosen is:

i Blake   **ii** a boy's name   **iii** a girl's name   **iv** not Crystal

v a boy's name or a girl's name

b Explain why the chosen name is 50% more likely to be a boy's than a girl's name.

8 Here is what Marcus says.



What is incorrect with Marcus' argument?

9 An unbiased dice with 20 faces has the numbers from 1 to 20.
The dice is thrown.

a Work out the probability of throwing:
- **i** the number 3 **ii** a multiple of 3
- **iii** an odd number **iv** a factor of 30

b Work out the probability of throwing:
- **i** the number 6 **ii** a number less than 6
- **iii** a number more than 6

c Explain why the three probabilities in part **b** must add up to 1.

d **i** Describe an outcome that has a probability of 75%.
ii Describe an outcome that has a probability of 15%.

10 Here are eight number cards.

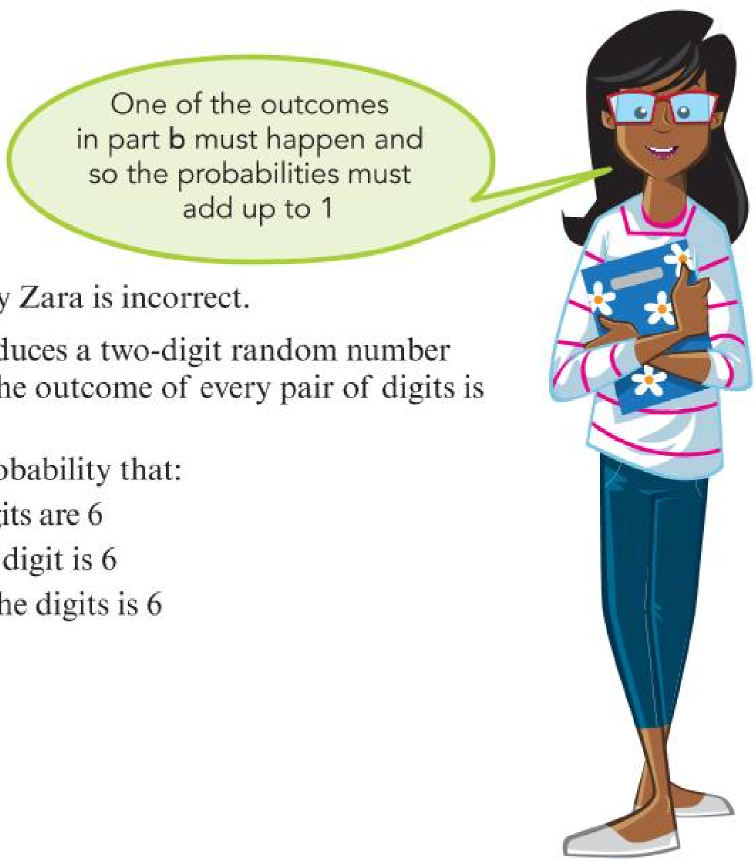
2	3	5	7	11	13	17	19

The cards are placed face down. A card is chosen without looking.

a Work out the probability that the card chosen is:
- **i** an odd number **ii** a prime number **iii** a factor of 100

b Work out the probability that the card chosen is:
- **i** 5 **ii** 5 or less **iii** 5 or more

c Zara says:



Explain why Zara is incorrect.

11 A computer produces a two-digit random number from 00 to 99. The outcome of every pair of digits is equally likely.
Work out the probability that:

a both the digits are 6
b exactly one digit is 6
c neither of the digits is 6

Think like a mathematician

12 Caleb has a set of cards. Each card has a number on it. The cards are placed face down and he takes a card without looking.

The probability that the chosen card is 3 is $\frac{1}{3}$.

The probability that the chosen card is 4 is $\frac{1}{4}$.

- **a** Find a possible list of Caleb's cards.
- **b** What can you say about the number of cards in the set?
- **c** An-Mei has a different set of cards, where each card has a number on it. She places them face down and takes a card without looking.

  In her set, the probability that the chosen card is 4 is $\frac{1}{4}$ and the probability that the chosen card is 5 is $\frac{1}{5}$.

  What can you say about An-Mei's set?
- **d** Could An-Mei have the same set as Caleb?

Summary checklist

- [ ] I can identify mutually exclusive outcomes.
- [ ] I can use equally likely outcomes to find theoretical probabilities.

13.3 Experimental probabilities

In this section you will …

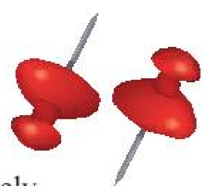
- use experiments to calculate experimental probabilities.

Key words

experiment
experimental probability
relative frequency
theoretical probability
trial

When outcomes are not equally likely, you can use an experiment to estimate probabilities.

If you drop a drawing pin, it can land point up or point down.



You cannot assume that these two outcomes are equally likely.

What is the probability of the drawing pin landing point up?

You can do an experiment to find the **experimental probability** of the drawing pin landing point up.

To do this, drop a large number of drawing pins.

Work out the relative frequency of point up $= \dfrac{\textit{frequency of point up}}{\textit{total number of drawing pins}}$

This tells you an experimental probability for point up.

Tip

Each drop of a drawing pin is a trial.

Worked example 13.3

124 drawing pins are dropped. 48 land point up. 76 land point down.

Find the experimental probability of a drawing pin landing:

a point up **b** point down

Answer

a The relative frequency of point up is $\frac{48}{124} = 0.39$, to 2 decimal places.

The experimental probability of a drawing pin landing point up is 39%.

Use a calculator to work out $48 \div 124$.

b The relative frequency of point down is $\frac{76}{124} = 0.61$, to 2 decimal places.

The experimental probability of a drawing pin landing point down is 61%.

You can also get the answer to part **b** of Worked example 13.3 by finding $100\% - 39\% = 61\%$.

When you do an experiment to find a probability, the answer might not be exactly the same each time. A large number of trials gives a more reliable answer than a small number of trials.

Exercise 13.3

1 A survey of 400 cars travelling along a road is carried out. The results show that 140 cars were travelling faster than 60 km/h.

Find the experimental probability that the next car will be:

a travelling faster than 60 km/h **b** travelling at 60 km/h or less

2 There are 320 students in a school. 16 students travel to school by car. 96 students walk to school.

Find the experimental probability that a particular student:

a travels by car to school

b walks to school

c does not walk to school

d does not walk or travel by car to school

3 Mrs Patel drives to work each day. Sometimes she must stop at a set of traffic lights. In the past 50 working days she has stopped at this set of traffic lights 32 times.

a Find the experimental probability that Mrs Patel will have to stop at this set of lights tomorrow.

b Find the experimental probability that she will not have to stop at the lights next Wednesday.

4 Jasmine goes to school five days a week. In the past eight weeks she has been late for school on three days.

Estimate the probability that tomorrow Jasmine will be:

a late for school

b on time

5 Carlos looks at the weather records for November for his town.

Over the past five years (150 days) there has been rain on 36 days in November. Use this information to estimate the following probabilities. Write your answers as percentages.

a Estimate the probability that it will rain on 1 November next year.

b Estimate the probability that it will not rain on 30 November next year.

6 Arun says:

a How did Arun work out this probability?

b This is not a good way to estimate the probability. Why?

c Compare your answer to part **b** with a partner's answer. Have you both given the same explanation? Can you improve your explanation?

7 Here are the results of a survey of 240 students attending a college.

Item	Has a mobile phone	Has a computer in their bedroom	Wants to be in a band	Is a member of a sports team
Number of students	232	167	92	68

a Find the experimental probability that a student chosen at random from the college:

i has a mobile phone ii has a computer in their bedroom

iii is not a member of a sports team

Give each answer as a percentage, rounded to the nearest whole number.

b Why is the following argument incorrect?

A good estimate of the probability that a student at the college wants to be in a band or is a member of a sports team is $\frac{92 + 68}{240} = \frac{160}{240} = \frac{2}{3}$ or 67%.

8 Varun flips a coin. The two possible outcomes are heads and tails.

a If the outcomes are equally likely, what are the **theoretical probabilities** of each outcome?

b Varun's results are shown in the table.

Outcome	heads	tails	total
Frequency	24	16	40

Use the results to find the relative frequency of each outcome.

c Varun's friend Toby says that Varun is not flipping the coin fairly because the probabilities from the experiment are incorrect.

What does Toby mean? Do you think Toby is correct?

9 A bag contains one white ball, one black ball and some red balls. A ball is chosen without looking.

a If there are three red balls, work out the theoretical probability of choosing each colour.

Daniella takes out one ball, records the colour and replaces it in the bag. She does this 50 times.

She records her results in a table.

Outcome	white	black	red	total
Frequency	6	8	36	50

b Use the results to find the experimental probability of choosing each of the three colours.

c Daniella knows that there are an odd number of red balls. What is the most likely number? Give a reason for your answer.

10 Work with a partner to answer this question.

This table shows 200 random digits generated by a spreadsheet.

0	2	7	5	0	8	6	9	0	7	8	0	3	2	9	2	9	7	4	7
5	7	3	0	2	9	8	4	5	3	9	6	4	9	7	5	3	1	1	5
2	8	3	0	4	3	7	6	5	5	0	5	7	9	4	1	8	8	1	4
8	9	2	6	5	4	1	0	9	6	3	4	1	0	1	5	4	9	2	3
2	7	9	7	1	0	1	5	7	1	6	6	6	2	2	6	2	6	3	4
8	1	8	0	1	4	2	0	3	7	6	0	1	9	0	9	6	7	8	2
0	5	9	0	6	1	8	7	9	4	8	0	2	7	9	3	8	3	9	5
2	4	6	1	7	3	8	0	1	1	4	1	0	7	3	4	9	1	5	5
0	8	9	9	0	6	7	8	2	6	2	6	2	1	2	2	4	9	3	8
1	2	4	6	6	3	0	1	1	7	0	8	5	9	0	8	1	5	0	1

a If every digit from 0 to 9 is equally likely, what is the theoretical probability that a digit is 0?

b Use the first row of the table as a sample of 20 digits. Use it to work out the experimental probability that a digit is 0. Write the answer as a decimal.

c Now use the first two rows as a sample of 40 digits. Use it to work out the experimental probability that a digit is 0.

d Continue in this way to get samples of 60, 80 and so on, up to 200. In each case, work out the experimental probability that the digit is 0.

e Look at the ten experimental probabilities you have calculated so far. What do you notice about them?

f Choose a different digit and do the same calculations. Comment on your results.

Think like a mathematician

11 Work with a partner or in a group.

The number π is very important in mathematics. When π is written as a decimal it does not terminate. You can find the first few digits on your calculator. You can find many more digits on the internet. Here are the first 250 digits.

3.1415 92653 58979 32384 62643 38327 95028 84197
16939 93751 05820 97494 45923 07816 40628 62089
98628 03482 53421 17067 98214 80865 13282 30664
70938 44609 55058 22317 25359 40812 84811 17450
28410 27019 38521 10555 96446 22948 95493 03819
64428 81097 56659 33446 12847 56482 33786 78316
52712 01909

Tip

You will learn about π (pi, you say it so that it rhymes with 'sky') later in your studies.

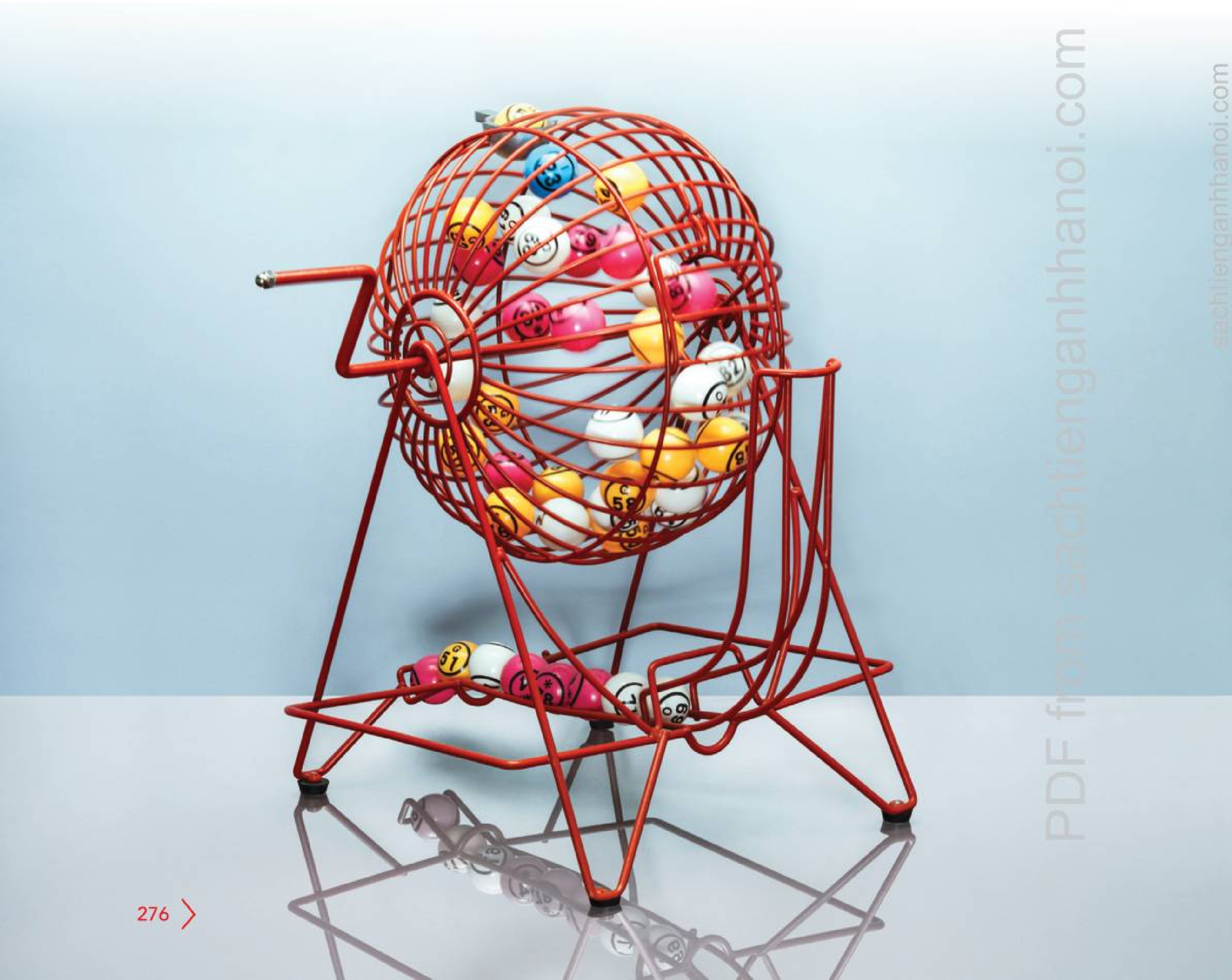
Continued

Investigate this question: Is every digit from 0 to 9 equally likely, or are some digits more likely than others?

Decide how you will investigate this question. Give evidence for any conclusion you reach.

Summary checklist

- [ ] I can use experiments or simulations to calculate experimental probabilities.



Check your progress

1 Describe a typical probability for an outcome that:

a is very likely **b** is impossible **c** has an even chance **d** is unlikely

2 Here is a set of cards with letters.

A	R	I	T	H	M	E	T	I	C

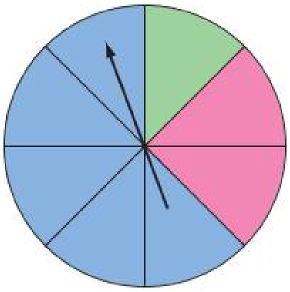
The cards are placed face down. One card is chosen without looking.

a Find the probability that the letter chosen is:

i A **ii** I **iii** N **iv** in the word SUBTRACT

b Write down two equally likely outcomes for this set of cards.

3 Here is a spinner.



a Find the theoretical probability of the spinner landing on each colour.

A computer simulation gives these results for 200 spins.

Colour	green	pink	blue
Frequency	18	54	128

b Find the experimental probability for each colour.

c Compare the theoretical and the experimental probabilities.

Here are the results of a computer simulation of another 300 spins.

Colour	green	pink	blue
Frequency	36	66	198

d Find experimental probabilities based on all 500 trials and comment on the results.

4 Seeds of a plant are sown in two different types of soil.

120 seeds are sown in soil A and 83 seeds grow successfully.

75 seeds are sown in soil B and 62 seeds grow successfully.

Which type of soil has a better success rate? Give a reason for your answer.

14 Position and transformation

Getting started

1 Copy and complete these distance conversions.

a 400 cm = ☐ m   b 8 km = ☐ m   c 3250 m = ☐ km

2 On a coordinate grid:

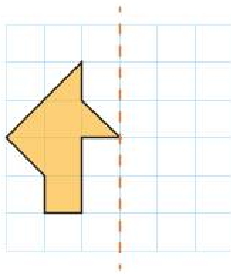
a plot and label the points $A(2, 1)$, $B(1, -2)$ and $C(-2, 0)$

b join the points to make triangle ABC

c **translate** triangle ABC 2 squares right and 3 squares up.

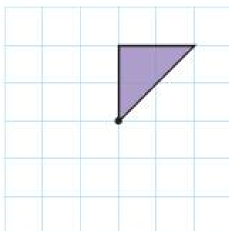
3 Copy this diagram.

Reflect the shape in the orange **mirror line**.

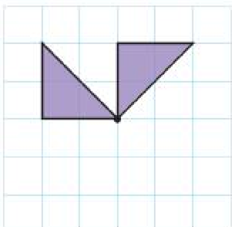


4 The diagram shows a triangle on a grid. One of the vertices of the triangle is marked with a black dot.

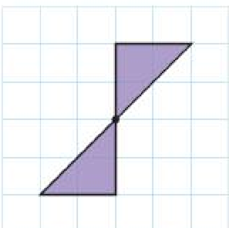
The triangle is rotated 90° **clockwise** around the black dot. Which of these diagrams, **A**, **B** or **C**, shows the correct rotation?



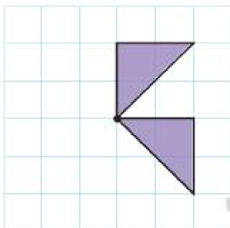
A



B



C



A **transformation** is when you move a shape by **reflecting**, **translating** or **rotating** it.

The shape you start with is called the **object**. The shape you finish with, after a transformation, is called the **image**.

You see transformations in everyday life.

Look at a lake, a river or the sea on a calm day and you will see a **reflection**. In the picture below you can see the mountains, clouds, jetty, trees and stone wall all reflected in the lake.

In the other picture you can see someone firing an arrow in an archery competition. To reach the target, the arrow goes through a **translation** from where it started to where it finished.

The Singapore Flyer is one of the largest Ferris wheels in the world. It is 165 metres tall, has 28 capsules and can carry a maximum of 784 people at a time.

The wheel turns or rotates about its centre. Each complete **rotation** takes about 32 minutes.

During a transformation, a shape changes only its position. It doesn't change its shape and size. An object and its image are always identical or **congruent**.

14.1 Maps and plans

In this section you will …

- use scales on maps and plans.

Key words

scale drawing

scale

A **scale drawing** is a drawing that represents an object in real life.

The **scale** gives the relationship between the lengths on the drawing and the real-life lengths.

You can write a scale in three ways:

- using the word 'represents'; for example, '1 cm represents 100 cm'
- using the word 'to'; for example, '1 to 100'
- using a ratio sign; for example, '1 : 100'.

When you write a scale using 'to' or the ratio sign, the numbers you use must be in the same units.

So you must write the scale '1 cm represents 10 m' as either '1 to 1000' or '1 : 1000'.

Tip

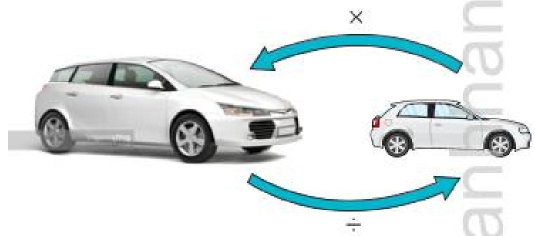
1 m = 100 cm

10 m = 1000 cm

The scale '1 to 10' or '1 : 10' means that every centimetre on the drawing represents 10 centimetres in real life.

To change a length on a drawing to a length in real life, multiply by the scale.

To change a length in real life to a length on a drawing, divide by the scale.



Worked example 14.1

a Razaan makes a scale drawing of the front of a building. She uses a scale of 1 cm represents 5 m.

i On her drawing, the building is 12 cm long. How long is the building in real life?

ii The building in real life is 120 m tall. How tall is the building on the scale drawing?

b A map has a scale of 1 : 50 000.

i On the map the distance between two villages is 8 cm. What is the distance between the two villages in real life?

ii The distance between two towns is 18 km in real life. What is the distance between the two towns on the map?

Continued

Answer

a **i** $12 \times 5 = 60\,m$	Multiply the length on the drawing by the scale of 5 to get the real-life length. Remember to include the units (m) with your answer.
ii $120 \div 5 = 24\,cm$	Divide the height in real life by the scale of 5 to get the height of the drawing. Remember to include the units (cm) with your answer.
b **i** $8\,cm \times 50\,000 = 400\,000\,cm$	Multiply the distance on the map (cm) by the scale of 50 000 to get the real-life distance (cm).
$400\,000\,cm \div 100 = 4000\,m$	Change centimetres to metres.
$4000\,m \div 1000 = 4\,km$	Change metres to kilometres.
ii $18\,km \times 1000 = 18\,000\,m$	Change kilometres to metres.
$18\,000\,m \times 100 = 1\,800\,000\,cm$	Change metres to centimetres.
$1\,800\,000 \div 50\,000 = 36\,cm$	Divide the real-life distance (cm) by the scale of 50 000 to get the distance on the map (cm).

Exercise 14.1

1 A scale drawing uses a scale of 1 cm represents 3 m.
Copy and complete the workings.

- **a** 2 cm on the drawing represents $2 \times 3 = \square$ m in real life.
- **b** 5 cm on the drawing represents $5 \times 3 = \square$ m in real life.
- **c** 8 cm on the drawing represents $\square \times 3 = \square$ m in real life.

2 A scale drawing uses a scale of 1 cm represents 4 m.
Copy and complete the workings.

- **a** 8 m in real life represents $8 \div 4 = \square$ cm on the drawing.
- **b** 12 m in real life represents $12 \div 4 = \square$ cm on the drawing.
- **c** 20 m in real life represents $\square \div 4 = \square$ cm on the drawing.

3 Marsile draws a scale drawing of a playing field. He uses a scale of 1 cm represents 10 m.

- **a** On his drawing the playing field is 18 cm long. How long is the playing field in real life?
- **b** The playing field in real life is 80 m wide. How wide is the playing field on the scale drawing?

4 The map shows part of Zimbabwe, in Africa. The scale of the map is 1 cm represents 40 km.

a Use a ruler to measure the distance, in cm, from Gweru to Masvingo. Write this down.

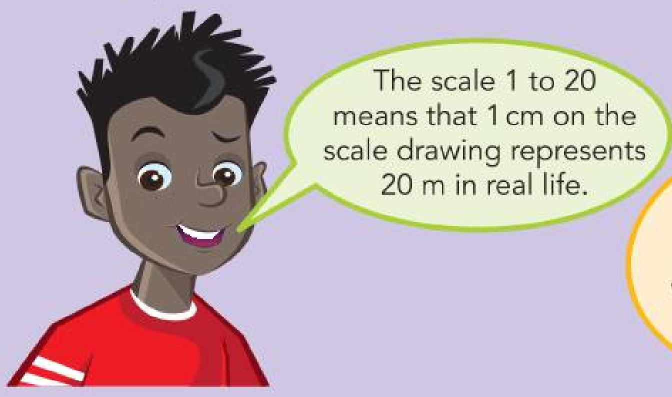
b Use your measurement from part **a** to work out the distance, in km, from Gweru to Masvingo in reallife.

Think like a mathematician

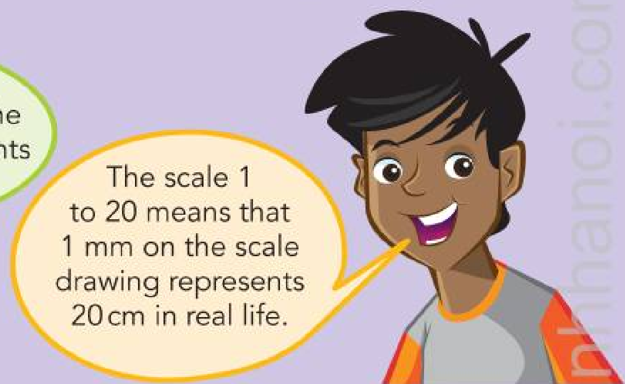
5 Look at this question and discuss with a partner or in a small group.

Marcus, Arun and Sofia are discussing the scale 1 to 20.

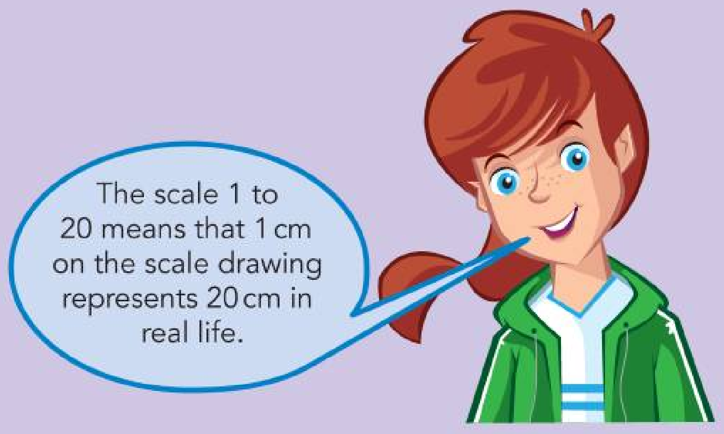
Marcus says:



Arun says:

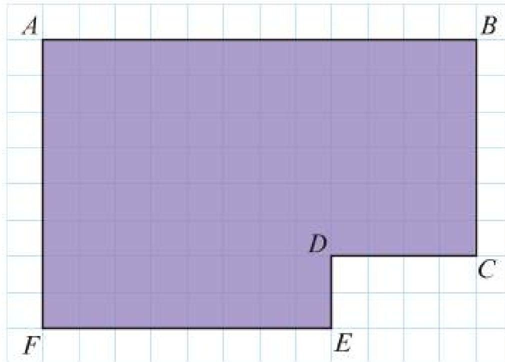


Sofia says:



Who is correct, Marcus, Arun and/or Sofia? Explain your answer.

6 This scale drawing of Visuri's bedroom is drawn on centimetre squared paper. The scale is 1 to 25.



a Work out the length in real life of the wall:

i *AB* **ii** *BC* **iii** *CD*

iv *DE* **v** *EF* **vi** *AF*

Give your answers in metres.

b The wardrobe in Visuri's room is 50 cm deep. What is this measurement on the scale drawing?

c The bed in Visuri's room is 1.75 m long. What is this measurement on the scale drawing? Give your answer in centimetres.

7 A map has a scale of 1 : 25 000.

a On the map the distance between two villages is 12 cm. What is the distance, in km, between the two villages in real life?

b The distance between two schools is 12 km in real life. What is the distance, in cm, between the two schools on the map?

8 Aika and Hinata use different methods to answer Question **7a**.

This is what they write:

Aika

Scale is 1 : 25 000,
so 1 cm represents 25 000 cm.
So 12 cm on the map would be:
$12 \times 25\,000 = 300\,000$ *cm*
$300\,000$ *cm* $\div 100 = 3000$ *m*
3000 *m* $\div 1000 = 3$ *km*

Hinata

Scale is 1 : 25 000,
so 1 cm represents 25 000 cm.
$25\,000$ *cm* $\div 100 = 250$ *m*
250 *m* $\div 1000 = 0.25$ *km*
so 1 cm represents 0.25 km.
$12 \times 0.25 = 3$ *km*

a Critique Aika's and Hinata's methods. What are the advantages and disadvantages of each method?

b Whose method do you prefer? Explain why.

9 This map has a scale of 1 : 80 000.

a Use a ruler to measure the distance, in cm, from Letterston to Wolf's Castle. Write this down.

b Use your measurement from part **a** to work out the distance, in km, from Letterston to Wolf's Castle in real life.

c The distance from Wolf's Castle to Fishguard is 12 km in real life. What is this measurement, in cm, on the map?

10 This is part of Faisal's homework.

Question	A map has a scale of 1 : 50 000. The distance between two train stations is 8.5 cm on the map. What is the distance between the two train stations in real life?
Solution	$8.5 \div 50\,000 = 0.00017$ cm $0.00017 \times 100 = 0.017$ m $0.017 \times 1000 = 17$ km The train stations are 17 km apart in real life.

a Explain the mistakes that Faisal has made.

b Write the correct solution.

Activity 14.1

Work with a partner for this activity.
Here are some red, white and blue cards.
The red cards are distances on a map. The white cards are map scales.
The blue cards are distances in real life.

3 cm	6 cm	7.5 cm	10 cm	12.8 cm
1 : 12 000	1 : 15 000	1 : 30 000	1 : 200 000	
4.5 km	6 km	7.5 km	9 km	15 km

a Take it in turns to choose one red card and one white card to give to your partner. Your partner must then change the distance on the red card to a distance, in km, using the map scale given on the white card. Check your partner's solutions and answers. Discuss any mistakes that they make. Do this two times each.

b Take it in turns to choose one blue card and one white card to give to your partner. Your partner must then change the distance on the blue card to a distance, in cm, using the map scale given on the white card. Check your partner's solutions and answers. Discuss any mistakes that they make. Do this two times each.

11 Babra takes part in an 18 km run. The distance of the route on a map is 24 cm. Work out if **A**, **B** or **C** is the correct map scale. Show your working.

A 1 : 65 000 **B** 1 : 70 000 **C** 1 : 75 000

In this section you have used scales written in three different ways:

- using the word 'represents'; for example, '1 cm represents 5 km'
- using the word 'to'; for example, '1 to 20'
- using a ratio sign; for example, '1 : 25 000'.

a Which of these ways do you prefer? Explain why.

b Do you find one of the ways more difficult to use than the others? How can you improve your skills in using the scale written in this way?

Summary checklist

☐ I can use a scale to convert distances on plans and maps to distances in real life.
☐ I can use a scale to convert distances in real life to distances on plans and maps.

14.2 The distance between two points

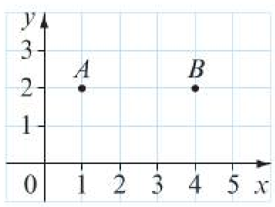
In this section you will …

- work out the distance between two points on a coordinate grid.

Key words

distance between two points
x-coordinate
y-coordinate

This coordinate grid shows the two points, $A(1, 2)$ and $B(4, 2)$.



The y-coordinate of both points is 2.

As the y-coordinate of both points is the same, A and B lie on the same horizontal line.

You can see that the distance between the two points is 3 units.

You can work out this distance by finding the difference between the **x-coordinates** of the two points.

x-coordinate of A is 1, x-coordinate of B is 4, so $4 - 1 = 3$ units.

Worked example 14.2

Without drawing a coordinate grid, work out the distance between the points:

a $C(9, 3)$ and $D(5, 3)$

b $E(2, 4)$ and $F(2, 7)$

Answer

a	$9 - 5 = 4$ units	Points C and D both have the same y-coordinate, so they lie on the same horizontal line. The distance between them is the difference between the x-coordinates of the two points.
b	$7 - 4 = 3$ units	Points E and F both have the same x-coordinate, so they lie on the same vertical line. The distance between them is the difference between the y-coordinates of the two points.

Exercise 14.2

1 Copy and complete the working to find the distance between these pairs of points. Each pair of points has the same y-coordinate.

a (9, 5) and (3, 5)
Difference between x-coordinates is: $9 - 3 = \square$ units.

b (12, 4) and (5, 4)
Difference between x-coordinates is: $12 - 5 = \square$ units.

2 Copy and complete the working to find the distance between these pairs of points. Each pair of points has the same x-coordinate.

a (1, 11) and (1, 5)
Difference between y-coordinates is: $11 - 5 = \square$ units.

b (8, 16) and (8, 7)
Difference between y-coordinates is: $16 - 7 = \square$ units.

3 Work out the distance between these pairs of points.
Choose the correct answer: **A**, **B** or **C**.

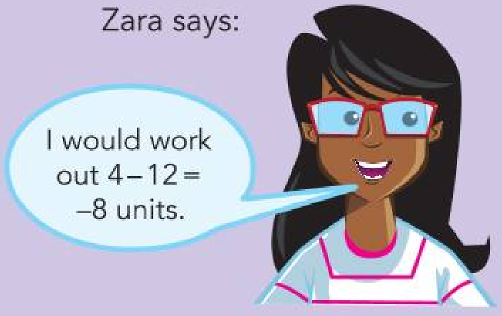
a	(7, 1) and (7, 6)	**A** −5 units	**B** 5 units	**C** 1 unit
b	(8, 2) and (4, 2)	**A** 6 units	**B** 2 units	**C** 4 units
c	(8, 15) and (15, 15)	**A** −7 units	**B** 0 units	**C** 7 units

Think like a mathematician

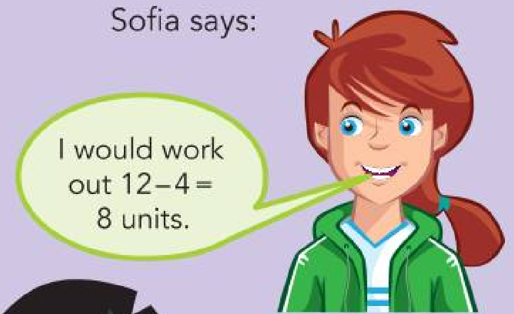
4 Discuss with a partner or in a small group the answers to these questions.

a Zara and Sofia are working out the distance between the points (7, 4) and (7, 12).

Zara says:

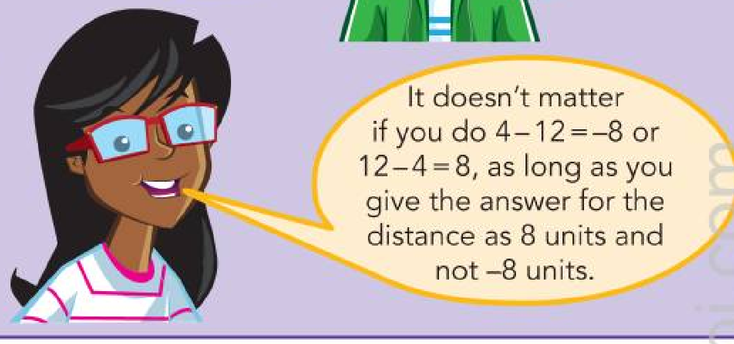


Sofia says:



Who is correct, Zara or Sofia? Explain your answer.

b This is what Zara says:



Do you agree or disagree with Zara? Explain why.

5 Here are a selection of cards showing the coordinates of the points A to J.

$A(2, 9)$	$B(6, 12)$	$C(11, 10)$	$D(2, 15)$	$E(0, 18)$
$F(6, 17)$	$G(6, 1)$	$H(3, 18)$	$I(6, 8)$	$J(19, 10)$

Write down the pairs of points that are the following distances apart.

a 7 units **b** 3 units **c** 8 units **d** 6 units

Activity 14.2

Work with a partner for this activity.

Make four coordinate cards similar to the cards in Question **5**. Do not show them to your partner yet.

Label the cards P, Q, R and S.

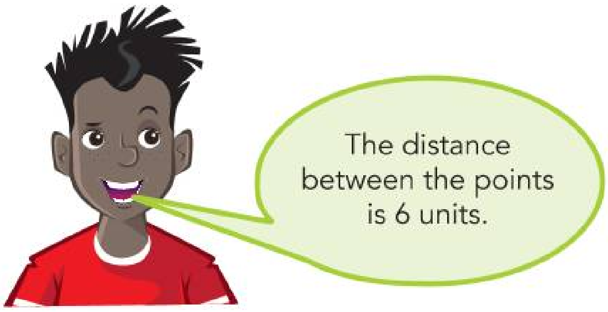
P and Q must have the same x-coordinates, but the y-coordinates must be different.

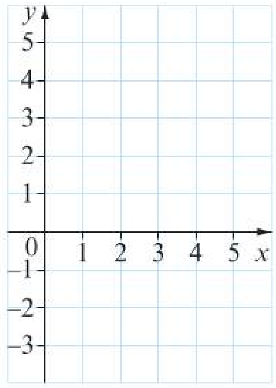
R and S must have the same y-coordinates, but the x-coordinates must be different.

a Swap cards P and Q with your partner. Work out the distance between each other's points. Then check each other's answers.

b Swap cards R and S with your partner. Work out the distance between each other's points. Then check each other's answers.

6 Marcus is working out the distance between the points (3, 4) and (3, −2).





a Make a copy of the coordinate grid shown on the right. Show, by plotting the points on the grid, that Marcus is correct.

b Show, by calculation, that Marcus is correct.

c Compare your calculation in part **b** with a partner's calculation. Did both of you do the same calculation? Discuss any differences in your methods.

> **Tip**
>
> In part **a**, work out:
>
> $3 + 5 = \square$
>
> Or $3 - -5 = 3 + 5 = \square$
>
> Or $-5 - 3 = -8$,
>
> so the distance $= \square$.

7 Work out the distance between these pairs of points.

a (4, 3) and (4, −5)

b (−1, 7) and (5, 7)

c (−3, 8) and (−3, −6)

d (2, 0) and (−8, 0)

8 This is part of Guillame's homework. He has worked out the correct answer.

Question	*Work out the distance between the points (−4, 2) and (−9, 2).*
Solution	*Points have the same y-coordinates, so find the distance between x-coordinates.* $-9 - -4 = -9 + 4 = -5$ *Distance = 5 units*

a What do you think of Guillame's method?

b Can you think of an easier method? If yes, write down this method.

9 The blue cards show pairs of points on a coordinate grid.
The yellow cards show the distances between two points.
Match each blue card to its correct yellow card.
The first one has been done for you.

A	$(-4, 3)$ and $(-7, 3)$	**i**	8
B	$(9, -1)$ and $(9, -9)$	**ii**	7
C	$(-10, 0)$ and $(-15, 0)$	**iii**	3
D	$(-2, -6)$ and $(-2, -15)$	**iv**	9
E	$(1, -7)$ and $(1, -14)$	**v**	5

10 Amelia draws a square, $ABCD$, on a coordinate grid.
The coordinates of A, B and C are $A(3, 5)$, $B(7, 5)$ and $C(7, 9)$.

a What is the side length of the square? Explain how you worked out your answer.

b What are the coordinates of D? Explain how you worked out your answer.

11 A netball coach draws a coordinate grid to show players where she wants them at different times during a match.
The centre of the court is the point $(0, 0)$ and 1 unit on the grid represents 2 m on the court.
So, a player at the point $(0, 3)$ is $3 \times 2 = 6$ m from the centre of the court.
The players in a netball team are:

GS (goal shooter) GA (goal attack)
WA (wing attack) WD (wing defence)
GK (goal keeper) GD (goal defence)
C (centre)

At one time in the match, the players are at these coordinates:

GS $(-5, 2)$ GA $(-5, -2)$ WA $(-3, 2)$ C $(-1, 0)$
GD $(4, -2)$ WD $(1, -2)$ GK $(6, 0)$

a What is the distance on the court between these pairs of players?

i GS and GA **ii** GS and WA

b Which two players are 14 m apart on the court?

c **i** Which three players are in a straight line?

ii What are the distances on the court between these three players?

d Draw a coordinate grid that goes from −6 to +6 on the x-axis and −3 to +3 on the y-axis. Plot the positions of the seven players using the coordinates given.

Use your grid to check that your answers to parts **a**, **b** and **c** are correct.

Remember that 1 unit on the grid represents 2 m on the court.

Summary checklist

☐ I can work out the distance between two points on a coordinate grid.

14.3 Translating 2D shapes

In this section you will …

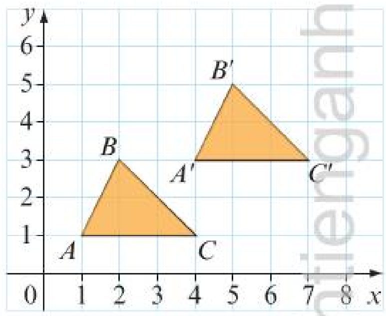
- work out the coordinates of shapes after a translation.

Key words

corresponds
image
object
translate

You already know that when you translate a 2D shape on a coordinate grid, you move it up or down and right or left.

Look at triangle ABC on this coordinate grid.



The vertices have coordinates $A(1, 1)$, $B(2, 3)$ and $C(4, 1)$.

When you translate triangle ABC 3 squares right and 2 squares up the vertices now have coordinates $A'(4, 3)$, $B'(5, 5)$ and $C'(7, 3)$.

You say A' as 'A dash'.

You say that 'the point A' **corresponds** to point A'.

You also say that 'triangle ABC is the object' and that 'triangle $A'B'C'$ is the image of triangle ABC'.

The following diagram shows how you can work out the coordinates of the vertices of triangle $A'B'C'$ without drawing a grid.

Translation is 3 squares right (+ 3) and 2 squares up (+ 2).

You need to add 3 to all the x-coordinates and add 2 to all the y-coordinates of the vertices.

A	(1 ,	1)	B	(2 ,	3)	C	(4 ,	1)
↓	↓ +3	↓ +2	↓	↓ +3	↓ +2	↓	↓ +3	↓ +2
A′	(4 ,	3)	B′	(5 ,	5)	C′	(7 ,	3)

Worked example 14.3

A square, $ABCD$, has vertices at the points $A(2, 1)$, $B(5, 2)$, $C(4, 5)$ and $D(1, 4)$.

$ABCD$ is translated 2 squares right and 1 square down.

The image of $ABCD$ is $A'B'C'D'$. Work out the coordinates of the vertices of $A'B'C'D'$.

Answer

$ABCD$ is translated 2 squares right and 1 square down, so you need to **add 2** to all the x-coordinates and **subtract 1** from all the y-coordinates of the vertices.

A	(2 ,	1)	B	(5 ,	2)	C	(4 ,	5)	D	(1 ,	4)
↓	↓+2	↓−1	↓	↓+2	↓−1	↓	↓+2	↓−1	↓	↓+2	↓−1
A'	(4 ,	0)	B'	(7 ,	1)	C'	(6 ,	4)	D'	(3 ,	3)

Exercise 14.3

1 A triangle, ABC, has vertices at the points $A(2, 1)$, $B(7, 1)$ and $C(2, 5)$.

ABC is translated 4 squares right and 5 squares up.

The image of ABC is $A'B'C'$.

Copy and complete the workings to find the coordinates of the vertices of $A'B'C'$.

A	(2 ,	1)	B	(7 ,	1)	C	(2 ,	5)
↓	↓+4	↓+5	↓	↓+4	↓+5	↓	↓+4	↓+5
A′	(6 ,	☐)	B′	(☐ ,	6)	C′	(☐ ,	☐)

2 The yellow cards have different translations written on them.
The white cards show what must be added or subtracted to the x- and y-coordinates of a shape to complete the translation.
Match each yellow card to its correct white card.
The first one has been done for you.

A 4 squares left and 1 square up

B 4 squares right and 1 square down

C 4 squares left and 1 square down

D 4 squares right and 1 square up

i x ↓−4, y ↓−1

ii x ↓+4, y ↓−1

iii x ↓−4, y ↓+1

iv x ↓+4, y ↓+1

3 A parallelogram, $PQRS$, has vertices at the points $P(3, 1)$, $Q(8, 1)$, $R(10, 4)$ and $S(5, 4)$.
$PQRS$ is translated 1 square left and 3 squares up. The image of $PQRS$ is $P'Q'R'S'$.
Work out the coordinates of the vertices of $P'Q'R'S'$.

Think like a mathematician

4 Look at this question in pairs or groups, then discuss the answers to parts **a**, **b** and **c** below.

This is part of Dan's homework.

<u>Question</u> A triangle, ABC, has vertices at the points A(1, 4), B(2, 7) and C(5, 6).
ABC is translated 6 squares right and 4 squares down.
The image of ABC is A′B′C′.
Work out the coordinates of the vertices of A′B′C′.

<u>Solution</u> A′ is at (7, 0). B′ is at (8, 11). C′ is at (1, 12).

a Dan has correctly worked out the coordinates of only one vertex. Which is the correct vertex: A', B' or C'?

b Explain the mistakes Dan made when he worked out the other vertices.

c What could Dan do to improve the solution that he has written?

d How could he check that his answers are correct?

5 A pentagon, $JKLMN$, has vertices at $J(1, 3)$, $K(3, 3)$, $L(3, 5)$, $M(2, 7)$ and $N(1, 5)$.
The pentagon is translated 3 squares right and 2 squares down to $J'K'L'M'N'$.

a Work out the coordinates of the vertices of $J'K'L'M'N'$.

b i On a square grid, draw some coordinate axes going from 0 to 8 on the x- and y-axes.

ii Draw the pentagon $JKLMN$ on the grid.

iii Translate the pentagon 3 squares right and 2 squares down to become $J'K'L'M'N'$.

iv Use your grid to check that your coordinates in part **a** for the vertices of $J'K'L'M'N'$ are correct. If they are incorrect, make sure you understand the mistakes that you have made.

6 Chaow translates triangle JKL to $J'K'L'$. JKL has vertices at $J(4, 2)$, $K(5, 5)$ and $L(3, 3)$.

Chaow works out that the vertices of $J'K'L'$ are at $J'(1, 7)$, $K'(10, 2)$ and $L'(0, 8)$.

Chaow has worked out two of the vertices correctly and one incorrectly.

- **a** Which vertex, J', K' or L', is incorrect? Explain how you worked out your answer.
- **b** What is the correct translation that Chaow used?
- **c** What is the incorrect translation that Chaow used?

7 This is part of Hathai's homework. She has spilt some juice on her work.

> Question: A square, ABCD, has vertices at the points A(−3, −3), B(2, −3), C([illegible]) and D([illegible])
> ABCD is translated [illegible]
> to A′B′C′D′.
> Work out the coordinates of the vertices of A′B′C′D′
>
> Answer: A′(−7, 3), B′([illegible]), C′(−2, 2), D′(−7, −2)

- **a** Work out the coordinates of vertices:
  - **i** B' **ii** C **iii** D
- **b** Explain how you worked out the answers to part **a**.

In this section you have carried out translations without drawing the shapes on a coordinate grid.

Did you prefer a different method? If yes, which method do you prefer?

Did you check your answers by drawing the shapes on a grid or were you confident that your answers were correct?

What could help you to improve your knowledge of this topic?

Summary checklist

- [ ] I can work out the coordinates of shapes after a translation.

14.4 Reflecting shapes

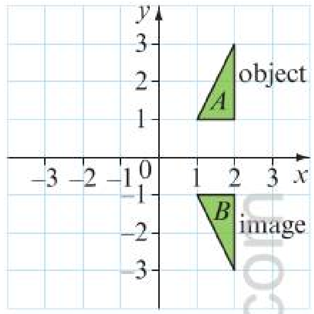
In this section you will …

- reflect shapes in the x-axis or y-axis on a coordinate grid.

Key words

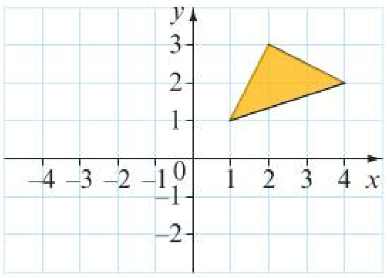
congruent
mirror line
reflected

The diagram shows a coordinate grid.
Triangle A is **reflected** in the x-axis.
The x-axis is the horizontal axis. This is the mirror line.
The image of triangle A is labelled triangle B.
Triangles A and B are **congruent**.

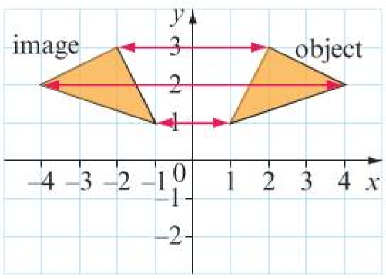


Worked example 14.4

Draw a reflection of this triangle in the y-axis.



Answer



Take each vertex of the object, one at a time, and plot its reflection in the mirror line.

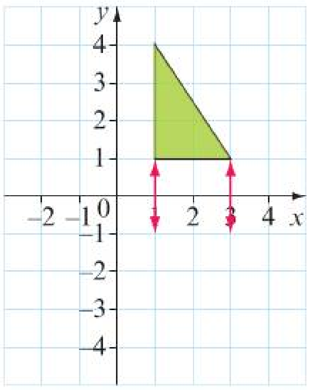
The y-axis is the vertical axis. This is the mirror line.

Use a ruler to join the reflected vertices with straight lines, to produce the image.

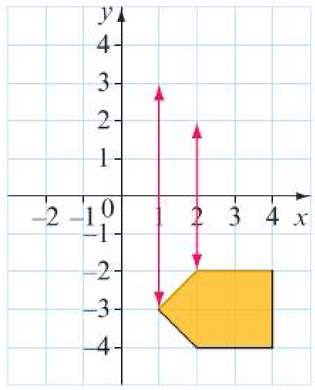
Exercise 14.4

1 Copy each diagram and reflect the shape in the *x*-axis.
Two have been started for you.

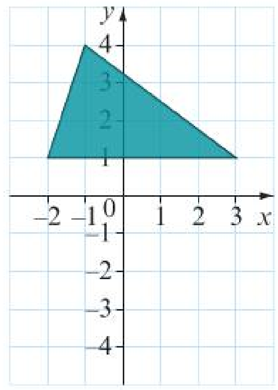
a



b

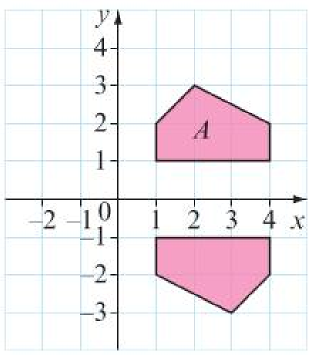


c

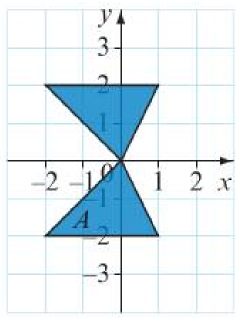


2 In which of these diagrams has shape *A* been correctly reflected in the *x*-axis?
If the reflection is incorrect, copy the diagram and draw the correct reflection.

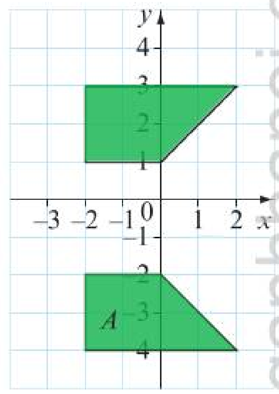
a



b

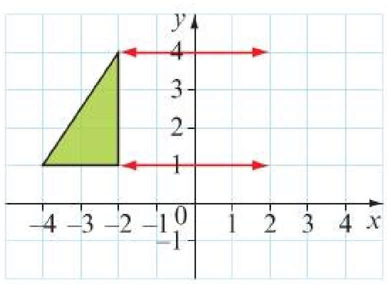


c

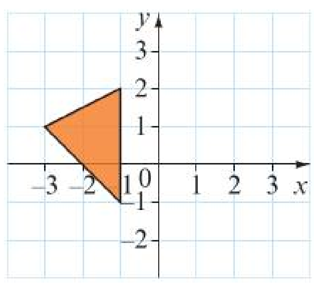


3 Copy each diagram and reflect the shape in the *y*-axis. One has been started for you.

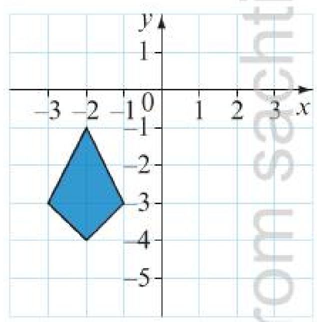
a



b



c

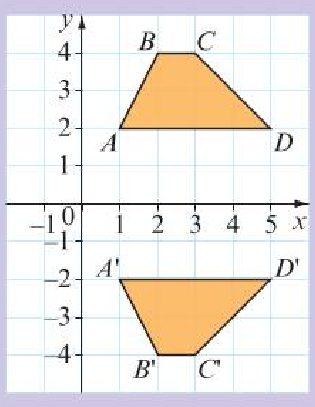


Think like a mathematician

4 Work with a partner, or in a small group, to answer this question.

Zara reflects trapezium *ABCD* in the x-axis.

The diagram shows the object, *ABCD*, and its image, $A'B'C'D'$.



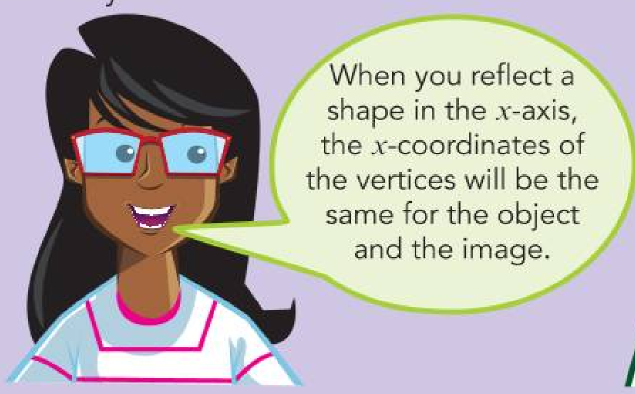
a The table shows the coordinates of the vertices of the object and its image.

Copy and complete the table.

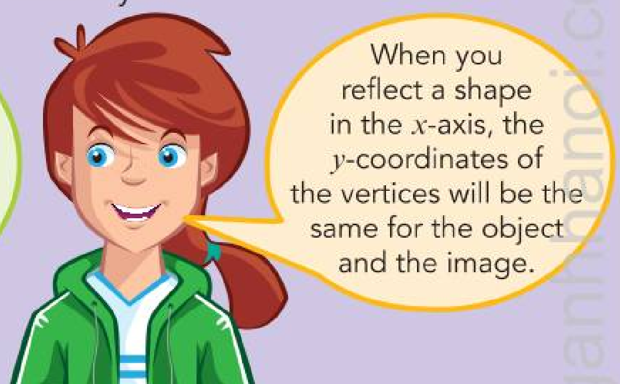
Object	*A*(1, 2)	*B*(2, 4)	*C*(□, □)	*D*(□, □)
Image	*A*′(1, −2)	*B*′(□, □)	*C*′(□, □)	*D*′(□, □)

Zara and Sofia discuss the coordinates of the vertices of the object and its image.

Zara says:



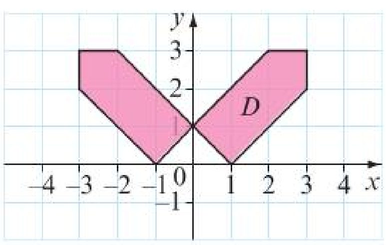
Sofia says:



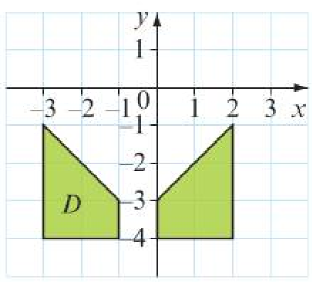
b Are Zara and Sofia correct? Explain why they are correct or are incorrect.

5 In which of these diagrams has shape *D* been correctly reflected in the y-axis?
If the reflection is incorrect, copy the diagram and draw the correct reflection.

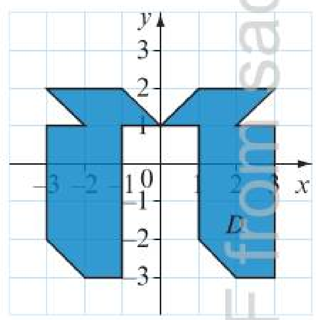
a



b



c

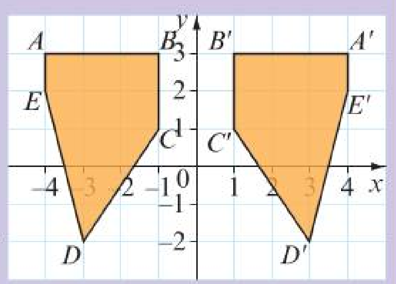


Think like a mathematician

6 Work with a partner, or in a small group, to answer this question.

Kurt reflects pentagon *ABCDE* in the y-axis.

The diagram shows the object, *ABCDE*, and its image, *A′B′C′D′E′*.



a The table shows the coordinates of the vertices of the object and its image. Copy and complete the table.

Object	Image
A(−4, 3)	A′(4, 3)
B(−1, 3)	B′(☐, ☐)
C(☐, ☐)	C′(☐, ☐)
D(☐, ☐)	D′(☐, ☐)
E(☐, ☐)	E′(☐, ☐)

b What can you say about the x-coordinates of the vertices of the object and its image?

c What can you say about the y-coordinates of the vertices of the object and its image?

d Will your answers to parts **b** and **c** always be true for whatever shape you reflect in the y-axis? Explain your answers.

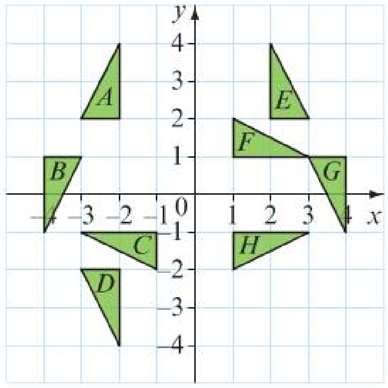
e In general, when you reflect a shape in the x-axis or the y-axis, will the object and the image always be congruent? Explain your answer.

Tip

Try to reflect some shapes of your own in the y-axis.

7 The diagram shows eight triangles, labelled A to H.

Copy and complete these statements. The first one has been done for you.



a E is a reflection of A in the y-axis.

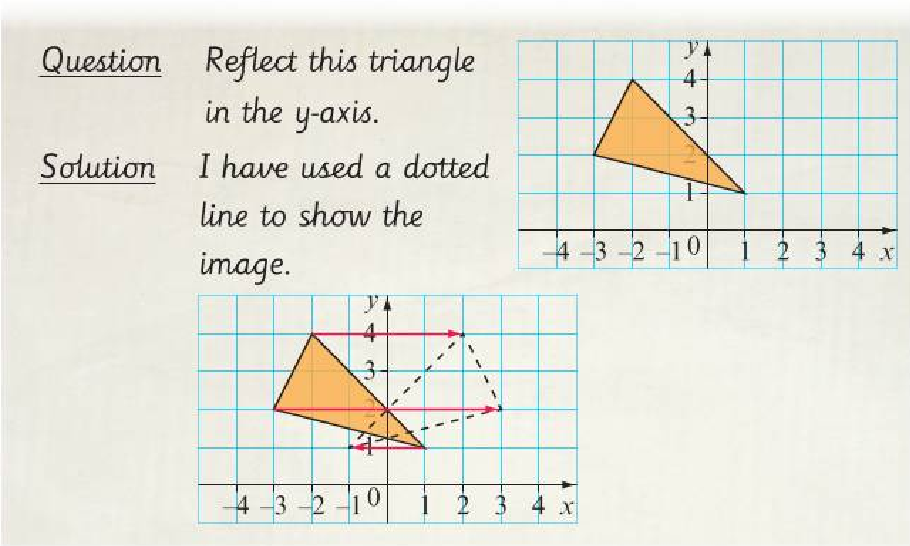
b F is a reflection of __ in the ______.

c C is a reflection of __ in the ______.

d G is a reflection of __ in the ______.

e D is a reflection of __ in the ______.

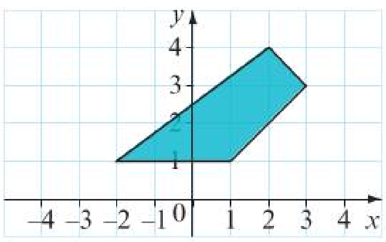
8 This is part of Marcus' homework.

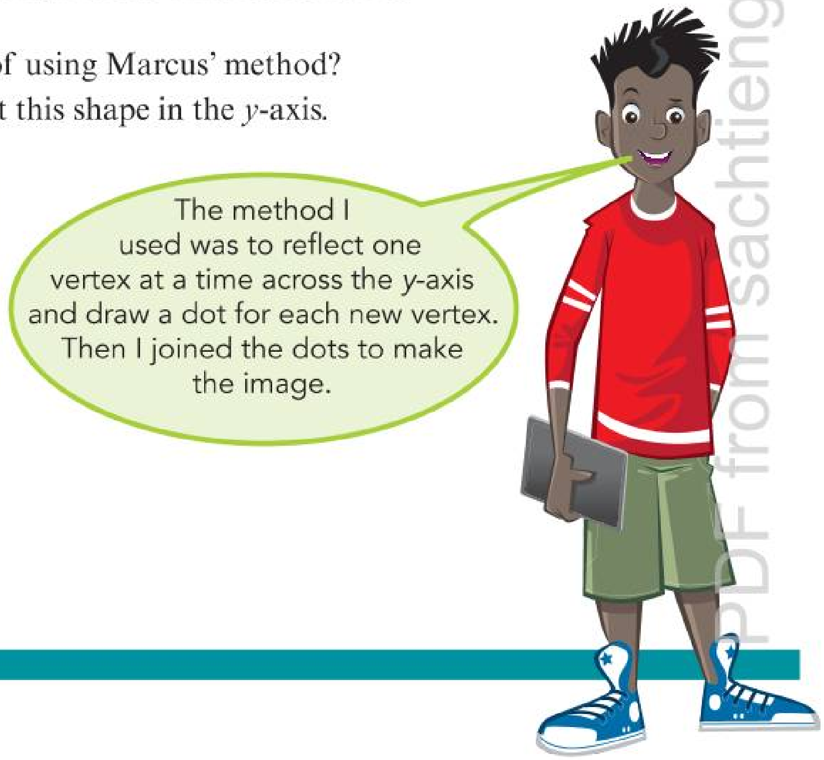


a What are the advantages of using Marcus' method to reflect the triangle?

b Are there any disadvantages of using Marcus' method?

c Use Marcus' method to reflect this shape in the y-axis.





Summary checklist

- [ ] I can reflect shapes in the x-axis or y-axis on a coordinate grid.

14.5 Rotating shapes

In this section you will …

- rotate shapes 90° and 180° around a centre of rotation on a coordinate grid.

Key words

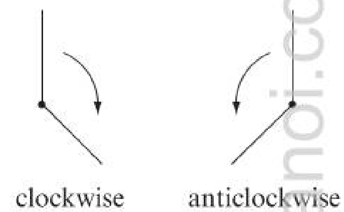
anticlockwise
centre of rotation
clockwise

When you rotate a shape, you turn it about a fixed point called the **centre of rotation**.

You can rotate a shape clockwise or **anticlockwise**.

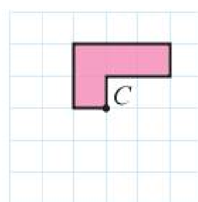
You must give the number of degrees by which you are rotating the object.

The rotations that are most often used are 90° and 180°. When a shape (the object) is rotated to a new position (the image), the object and the image are always congruent.

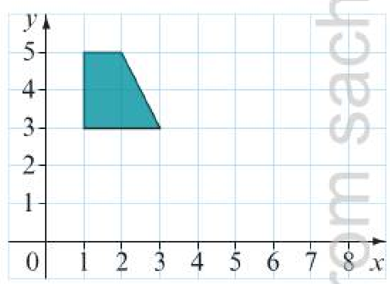


Worked example 14.5

a Draw the image of this shape after a rotation of 90° clockwise about the centre of rotation, which is marked C.



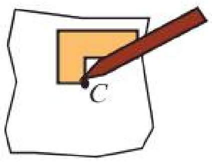
b Draw the image of this shape after a rotation of 180° about the centre of rotation at (4, 3).



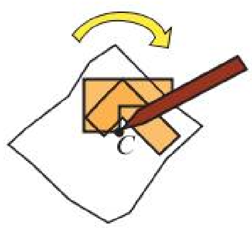
Continued

Answer

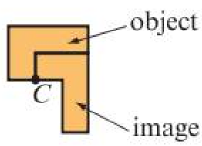
a



Start by tracing the shape onto tracing paper. Then put the point of your pencil on the centre of rotation, *C*.

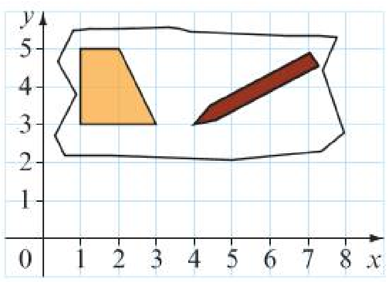


Turn the tracing paper 90° clockwise. Then make a note of where the image is.

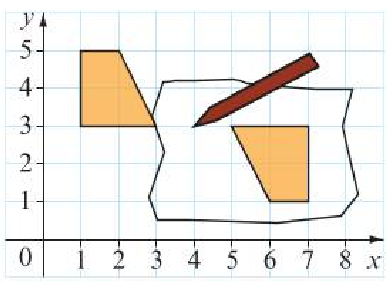


Draw the image onto the grid.

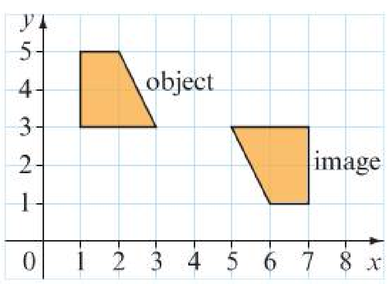
b



Start by tracing the shape onto tracing paper. Then put the point of your pencil on the centre of rotation at (4, 3).



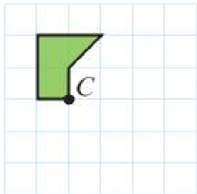
Turn the tracing paper 180°. Then make a note of where the image is.



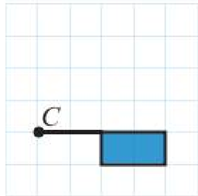
Draw the image onto the grid.

Exercise 14.5

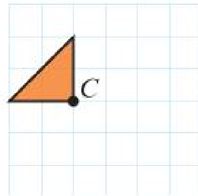
1 Copy each diagram and rotate the shape about the centre, C, by the given number of degrees.

a 

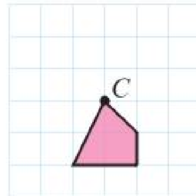
180°

b 

90° anticlockwise

c 

90° clockwise

d 

180°

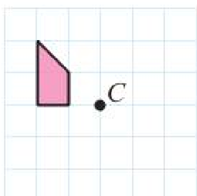
Think like a mathematician

2 Discuss, in pairs or groups, why these comments by Arun are true.

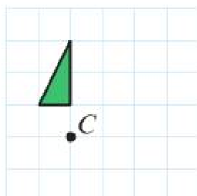
When you rotate a shape, the object and the image are always congruent.

When you rotate a shape through 180°, it doesn't matter whether you turn the shape clockwise or anticlockwise, as you will end up with the same image.

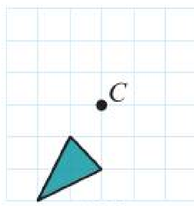
3 Copy each diagram and rotate the shape about the centre, C, by the given number of degrees.

a 

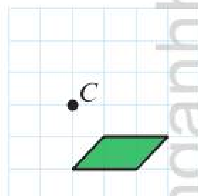
180°

b 

90° clockwise

c 

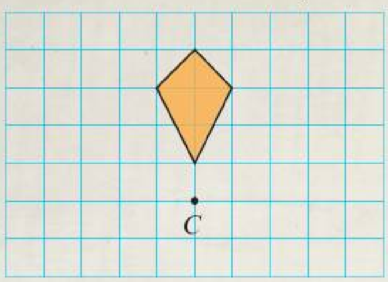
180°

d 

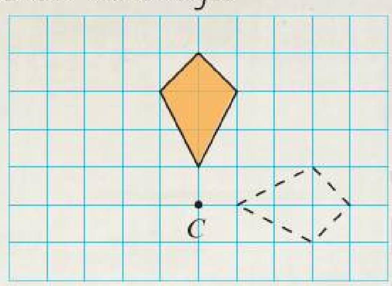
90° anticlockwise

4 This is part of Maksim's homework.

Question: Rotate this kite 90° anticlockwise about the centre, C.



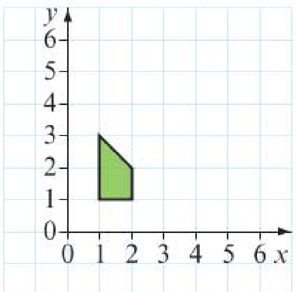
Solution: I have used a dotted line to show the image.



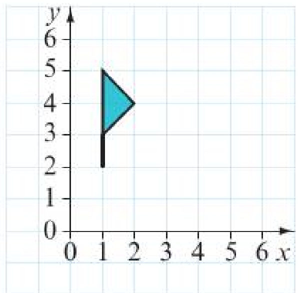
a What is incorrect about Maksim's solution?

b Copy the object onto squared paper and draw the correct image.

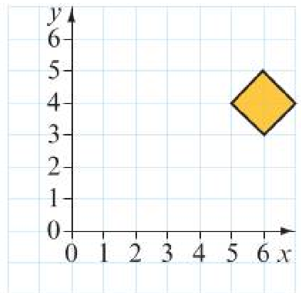
5 Copy each diagram and rotate the shape, using the information given.

a


90° anticlockwise centre (1, 3)

b


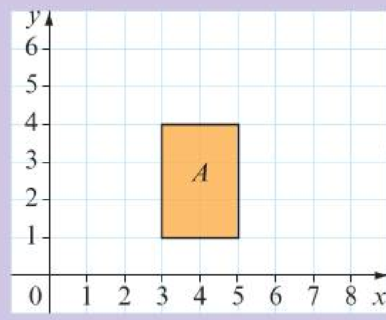
90° clockwise centre (1, 1)

c


180° centre (4, 3)

Think like a mathematician

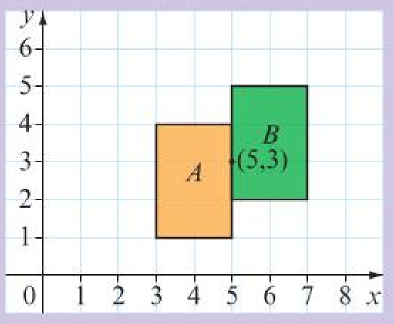
6 In groups, discuss the answers to these questions.
Arun draws rectangle *A* onto this coordinate grid.



Each square on the grid has an area of 1 cm^2.

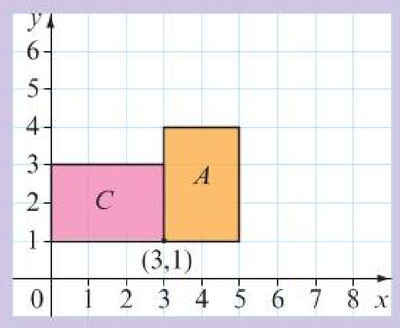
a What is the area of rectangle *A*?

Arun rotates rectangle *A* 180° about the centre (5, 3) to get rectangle *B*.



b What is the area of the combined shape of rectangle *A* and rectangle *B*?

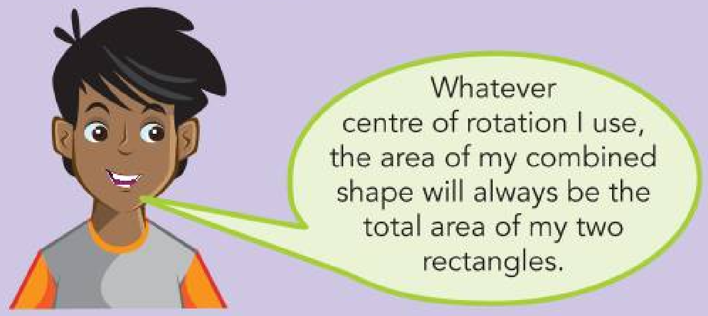
Arun rotates rectangle *A* 90° anticlockwise about the centre (3, 1) to get rectangle *C*.



Continued

c What is the area of the combined shape of rectangle *A* and rectangle *C*?

Arun makes this comment:



Tip

Use diagrams to help you prove or disprove Arun's comment.

d Is Arun correct? Give reasons for your answer.

7 Draw a coordinate grid that goes from 0 to 8 on the x-axis and 0 to 4 on the y-axis.

Draw the rectangle on your grid, which has vertices at (1, 1), (5, 1), (5, 3) and (1, 3).

a What is the area of the rectangle?

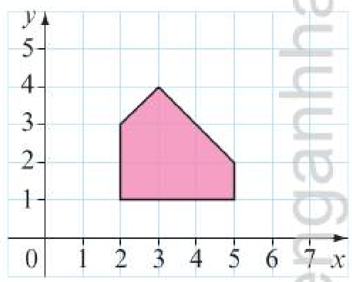
b Rotate the rectangle 180° about the centre (4, 2) and draw the image.

c The object and image together make one rectangle. What is the area of this rectangle?

8 Make two copies of this diagram.

a On the first copy, rotate the shape 90° clockwise about the centre (3, 3).

b On the second copy, rotate the shape 180° about centre (4, 2).



Summary checklist

- [ ] I can rotate shapes 90° and 180° around a centre of rotation on a coordinate grid.

14.6 Enlarging shapes

In this section you will …

- enlarge shapes, using a positive whole number scale factor.

Key words

enlargement
scale factor
similar shapes

An **enlargement** of a shape is a copy of the shape that changes the lengths, but keeps the same proportions.

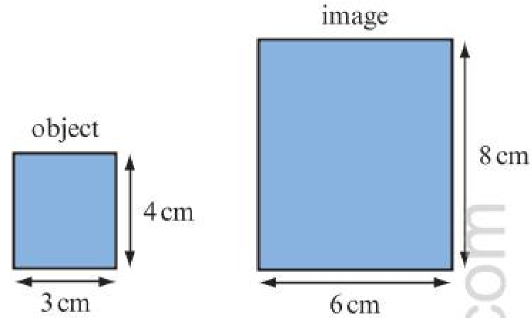
Look at these two rectangles.

The image is an enlargement of the object. Every length on the image is twice as long as the corresponding length on the object.

You say that the **scale factor** is 2.

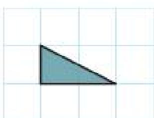
As the proportions are the same, the object and its image are called **similar shapes**.

In an enlargement all the angles stay the same size.



Worked example 14.6

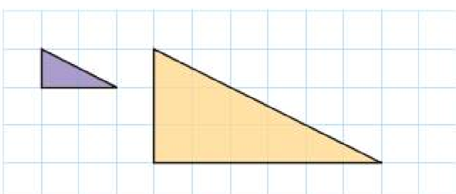
Draw the image of this triangle after an enlargement of scale factor 3.



Answer

The height of the triangle is 1 square, so the height of the image is $3 \times 1 = 3$ squares.

The base of the triangle is 2 squares, so the base of the image is $3 \times 2 = 6$ squares.



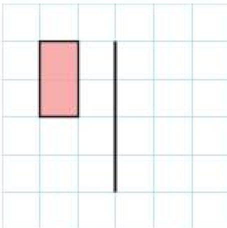
Draw both these sides onto the grid.

Draw the third side to complete the triangle.

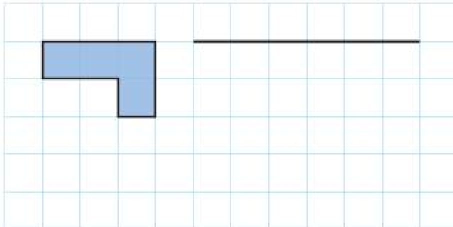
Exercise 14.6

1 Copy and complete these enlargements. Use a scale factor of 2.

a

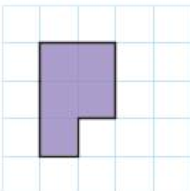


b



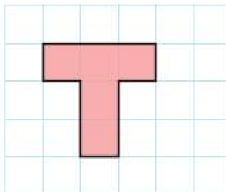
2 Copy each of these shapes onto squared paper. Enlarge each shape using its given scale factor.

a



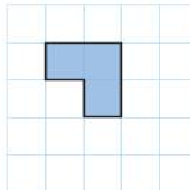
scale factor 2

b



scale factor 3

c



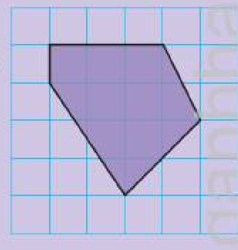
scale factor 4

Think like a mathematician

3 Work in groups to answer this question.

Inaya wants to enlarge this shape using a scale factor of 2.

Only one side of the shape is horizontal and only one side is vertical. The other three sides of the shape cut across squares on the grid.



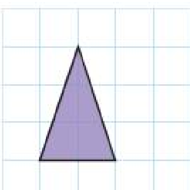
a What methods could Inaya use to enlarge this shape?

b Discuss the advantages and disadvantages of each method.

c What do you think is the best method to use?

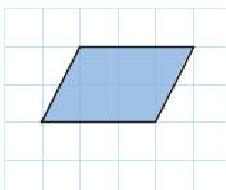
4 Copy each of these shapes onto squared paper. Enlarge each one, using its given scale factor.

a



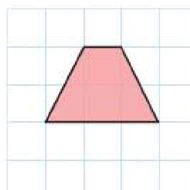
scale factor 2

b



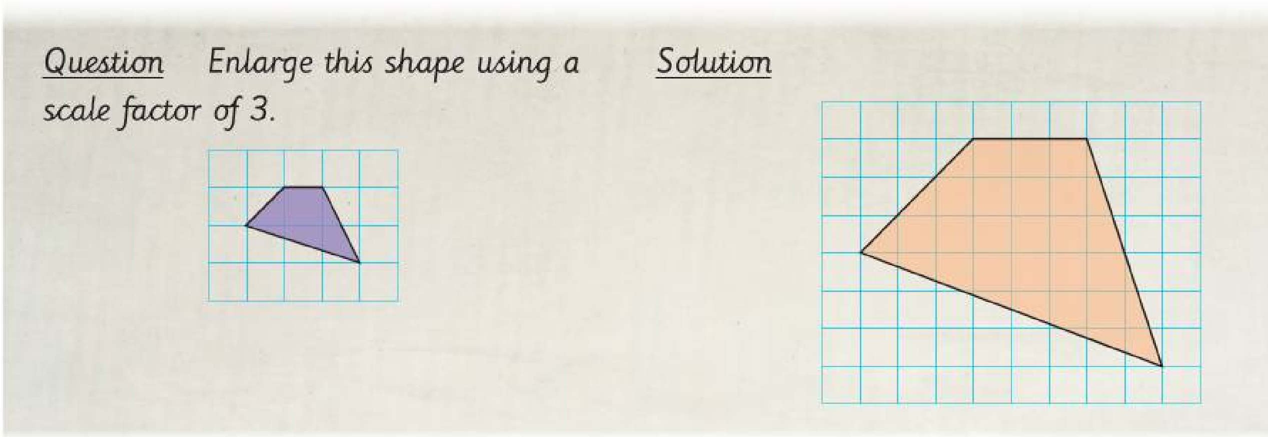
scale factor 3

c



scale factor 4

5 This is part of Rafael's homework.



a Rafael has made a mistake. Explain the mistake he has made.

b Draw the correct solution.

Activity 14.4

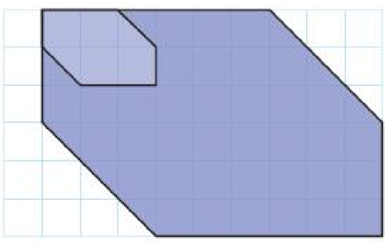
Work with a partner for this activity. Read the instructions before you start.

a On a piece of squared paper, draw a quadrilateral of your choice.

b Ask your partner to enlarge your quadrilateral by a scale factor of your choice. You must make sure that the enlarged shape will fit on the squared paper.

c Check each other's work and discuss any mistakes that you or your partner have made.

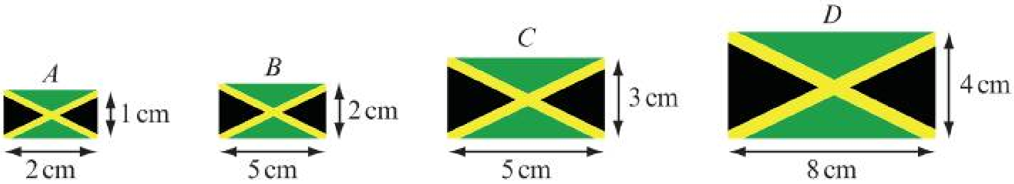
6 The diagram shows a shape and its enlargement.
What is the scale factor of the enlargement?



Tip

Make sure you check that the scale factor works for every side of the shape.

7 Marcus and Sofia are looking at these flags. The flags are <u>not</u> drawn accurately.



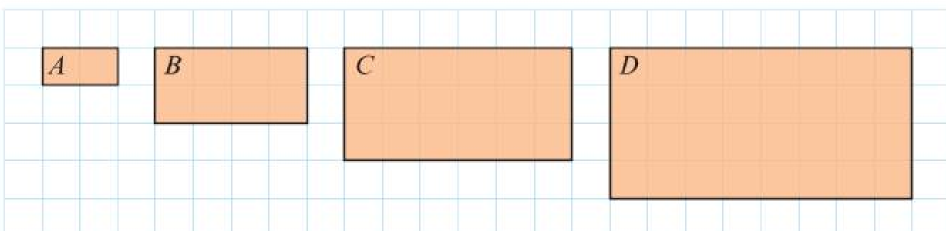
Marcus says:

a Explain why flag C is not an enlargement of flag A.

Sofia says:

b Explain why Sofia is correct.

8 The diagram shows four rectangles: A, B, C and D.



a Write down the scale factor of the enlargement of rectangles:

i A to B **ii** A to C **iii** A to D

b Work out the perimeter of rectangles:

i A **ii** B **iii** C **iv** D

c Copy and complete this table. Write all the ratios in their simplest form.

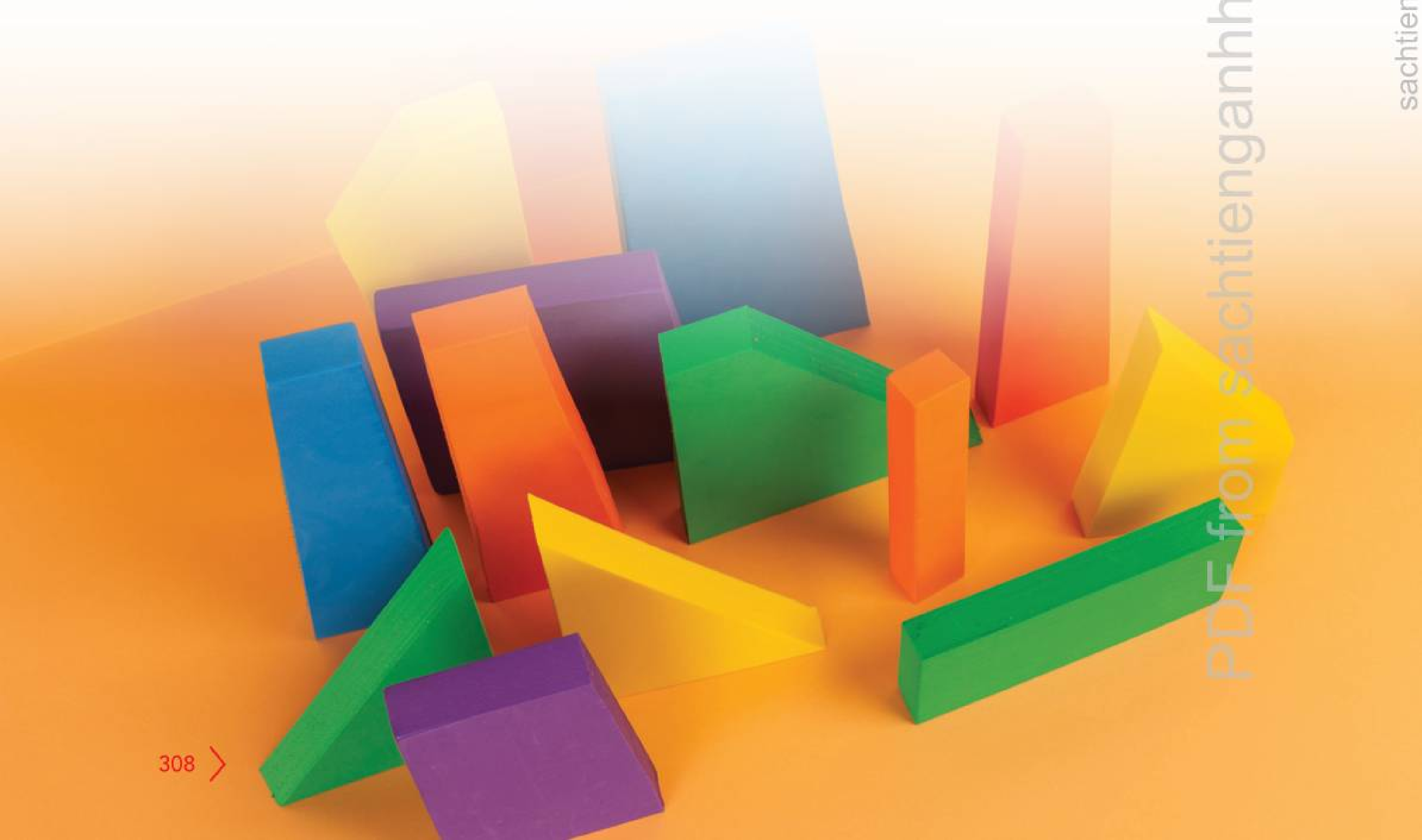
Rectangles	Scale factor of enlargement	Ratio of lengths	Ratio of perimeters
$A:B$	2	1 : 2	
$A:C$			
$A:D$			

d Write down a rule that connects the ratio of lengths to the ratio of perimeters.

e Will this rule work for any scale factor of enlargement? Will this rule work for any shape? Explain your answers.

Summary checklist

☐ I can enlarge shapes using a positive whole number scale factor.



Check your progress

1 Priyanka draws a scale drawing of a room. She uses a scale of 1 cm represents 40 cm.

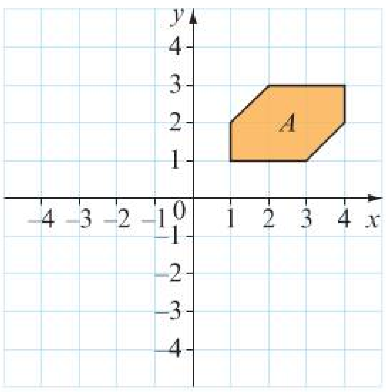
- a On her drawing the room is 12 cm long. How long is the room in real life? Give your answer in metres.
- b The room in real life is 5.6 m wide. How wide is the room on the scale drawing? Give your answer in centimetres.

2 Work out the distance between these points.

- a (8, 2) and (8, 11)
- b (4, 7) and (1, 7)

3 A rhombus, $PQRS$, has vertices at the points $P(3, 1)$, $Q(4, 3)$, $R(3, 5)$ and $S(2, 3)$.
$PQRS$ is translated 2 squares right and 5 squares up. The image of $PQRS$ is $P'Q'R'S'$.
Work out the coordinates of the vertices of $P'Q'R'S'$.

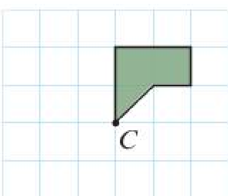
4 On square paper, make a copy of this diagram.



- a Reflect shape A in the x-axis. Label the shape B.
- b Reflect shape A in the y-axis. Label the shape C.

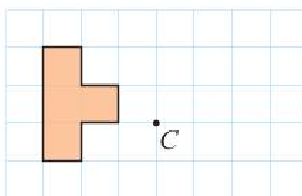
5 On square paper, copy each diagram and rotate the shape about the centre, C, by the given number of degrees.

a



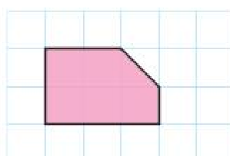
90° anticlockwise

b



180°

6 Copy this shape onto squared paper. Enlarge the shape using scale factor 3.



15 Shapes, area and volume

Getting started

1 Work out:

a 7×100 b 29.4×100 c 0.67×100

d $4500 \div 100$ e $250 \div 100$ f $7 \div 100$

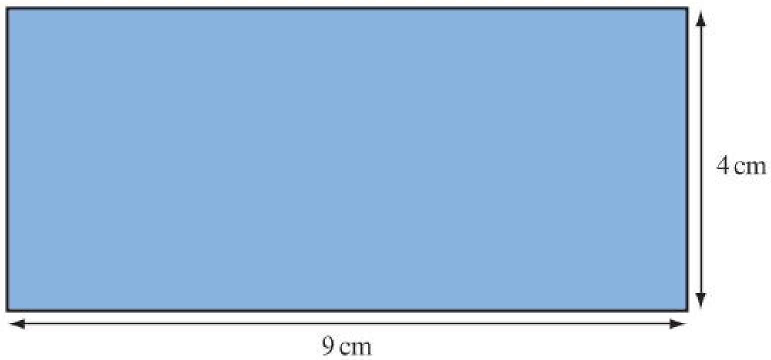
2 Work out:

a 2×1000 b $7.2 \times 10\,000$ c $0.08 \times 1\,000\,000$

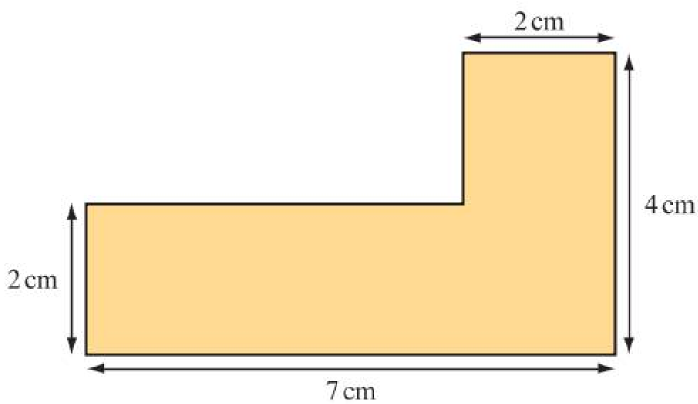
d $370\,000 \div 10$ e $67\,800 \div 10\,000$ f $54\,000 \div 100\,000$

3 Work out the area of these shapes.

a

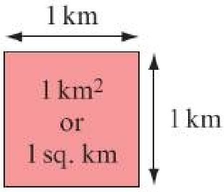


b



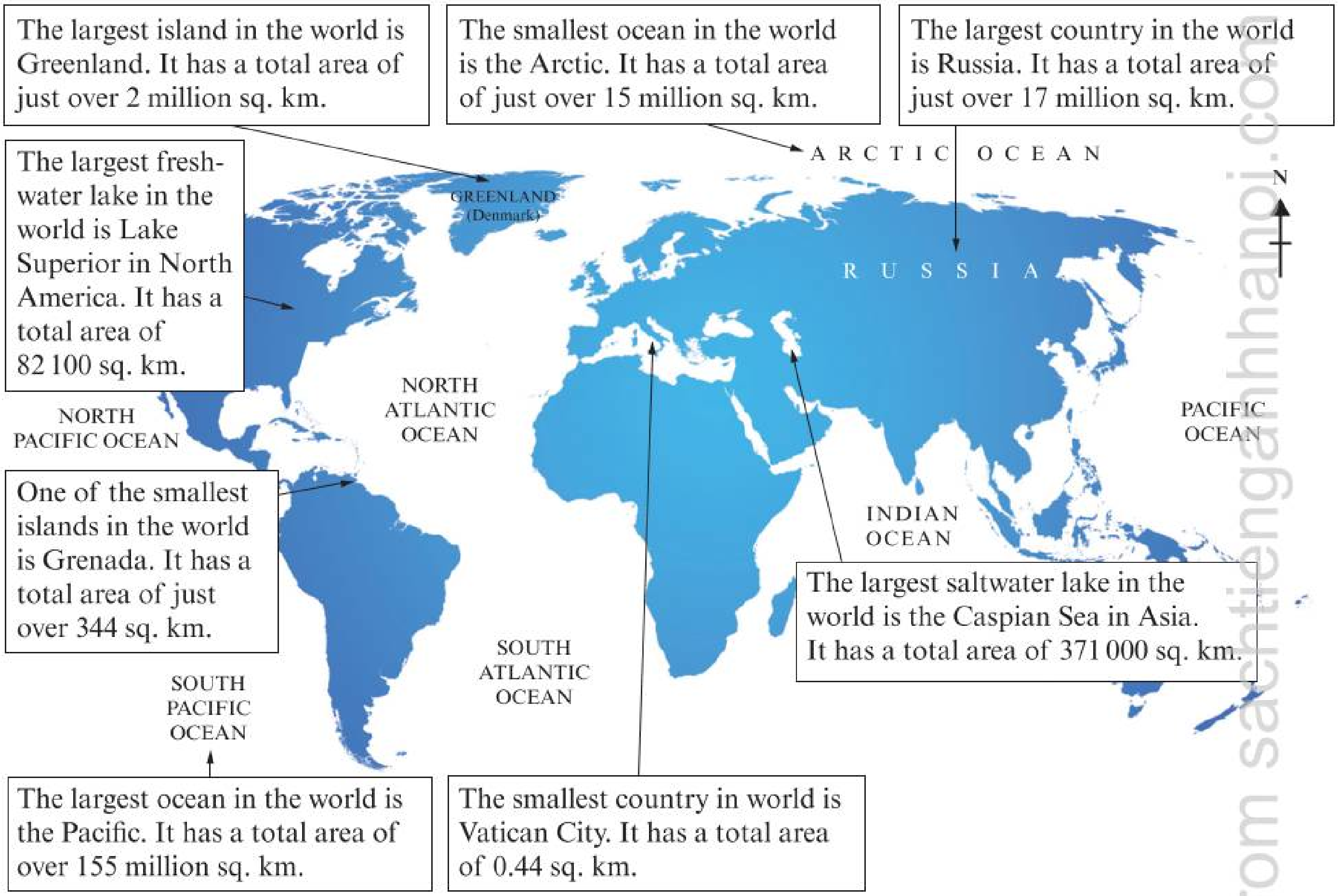
The **area** of a flat shape is the amount of space covered by the shape.

Imagine a square piece of land that is exactly 1 km long and exactly 1 km wide. The area of this piece of land is 1 **square kilometre**, which you can write as 1 **km²** or 1 sq. km.



The square kilometre is a very large unit of area. You will also use much smaller units when you are measuring smaller shapes. The unit you use must always be suitable for what you are measuring.

Here are some interesting area facts about places in the world.



15.1 Converting between units for area

In this section you will …

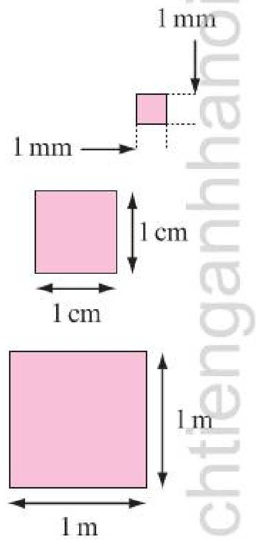
- convert between metric units of area.

Key words

area
centimetre (cm^2)
conversion factor
compound shape
dimensions
square millimetre (mm^2)
square square metre (m^2)

The diagram shows three squares.

- The first square has a side length of 1 mm.
- The second square has a side length of 1 cm.
- The third square has a side length of 1 m.
- The first square has an area of 1 **square millimetre (1 mm^2)**.
- The second square has an area of 1 **square centimetre (1 cm^2)**.
- The third square has an area of 1 **square metre (1 m^2)**.



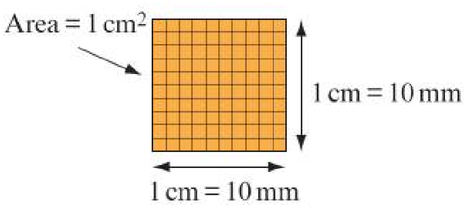
Not to scale

To convert between units of area you need to know the **conversion factors**.

Look at the square with a side length of 1 cm and area 1 cm^2.

If you divide the square into squares with side length 1 mm, you get $10 \times 10 = 100$ of these smaller squares.

This shows that: $1\,cm^2 = 100\,mm^2$

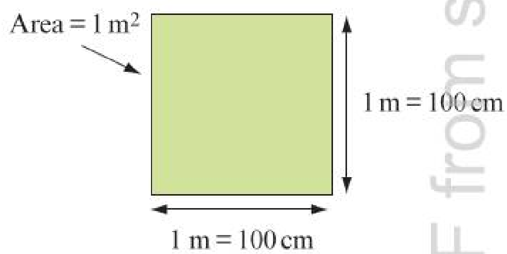


You can do the same with the square with a side length of 1 m and area 1 m^2.

If you divide the square up into squares with side length 1 cm, you get $100 \times 100 = 10\,000$ of these smaller squares.

This shows that: $1\,m^2 = 10\,000\,cm^2$

When you measure the area of a shape, decide which units you would use to measure a length of the shape; for example, cm. You then measure the area of the shape in those units squared, so cm^2.



Not to scale

Worked example 15.1

a What units for area would you use to measure the area of a football pitch?

b A shape has an area of 5 cm². What is the area of the shape, in square millimetres?

Answer

a square metres, m^2

You would measure the length of a football pitch in metres, so the area would be in square metres.

b $5 \times 100 = 500\ mm^2$

$1\ cm^2 = 100\ mm^2$, so $5\ cm^2$ would be five times as much.

Exercise 15.1

1 What units would you use to measure the area of:

a a postage stamp? b a bank note? c a tennis court? d a cinema screen?

2 Copy and complete these area conversions between cm^2 and mm^2.

a $8\ cm^2 = \square\ mm^2$ $8 \times 100 = \square\ mm^2$

b $0.75\ cm^2 = \square\ mm^2$ $0.75 \times \square = \square\ mm^2$

c $600\ mm^2 = \square\ cm^2$ $600 \div 100 = \square\ cm^2$

d $45\ mm^2 = \square\ cm^2$ $45 \div \square = \square\ cm^2$

Tip

Use the conversion factor $1\ cm^2 = 100\ mm^2$.

3 Copy and complete these area conversions between m^2 and cm^2.

a $3\ m^2 = \square\ cm^2$ $3 \times 10\,000 = \square\ cm^2$

b $8.1\ m^2 = \square\ cm^2$ $8.1 \times \square = \square\ cm^2$

c $70\,000\ cm^2 = \square\ m^2$ $70\,000 \div 10\,000 = \square\ m^2$

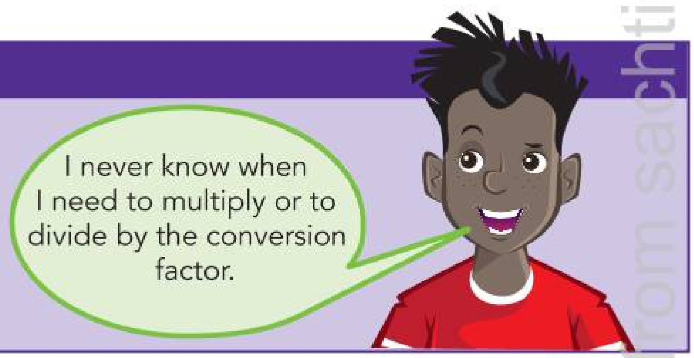
d $780\ cm^2 = \square\ m^2$ $780 \div \square = \square\ m^2$

Tip

Use the conversion factor $1\ m^2 = 10\,000\ cm^2$.

Think like a mathematician

4 Marcus says:



Discuss a strategy that Marcus could use to help him to decide when he should multiply and when he should divide by the conversion factor.

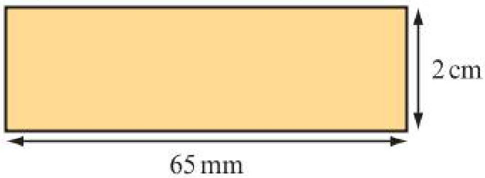
5 Copy and complete the following area conversions. Show your working.

a $6\,cm^2 = \square\,mm^2$ **b** $7.2\,cm^2 = \square\,mm^2$ **c** $3\,m^2 = \square\,cm^2$

d $5.4\,m^2 = \square\,cm^2$ **e** $900\,mm^2 = \square\,cm^2$ **f** $865\,mm^2 = \square\,cm^2$

g $20\,000\,cm^2 = \square\,m^2$ **h** $48\,000\,cm^2 = \square\,m^2$ **i** $125\,000\,cm^2 = \square\,m^2$

6 Suyin and Tam use algebra to work out the area of this rectangle, in cm^2.



This is what they write:

Suyin

Change length from mm to cm.
65 mm = 6.5 cm
$A = l \times w$
$= 6.5 \times 2$
$= 13\,cm^2$

Tam

Change width from cm to mm.
2 cm = 20 mm
$A = l \times w$
$= 65 \times 20$
$= 1300\,mm^2$
Use 10 mm = 1 cm.
So, $1300\,mm^2 = 130\,cm^2$

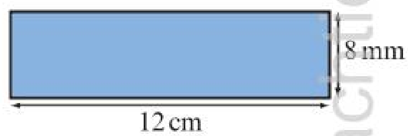
a Who has the correct answer, Suyin or Tam?

b Explain the mistake that the other person has made.

c Which method do you prefer: the method used by Suyin or the method used by Tam? Explain why.

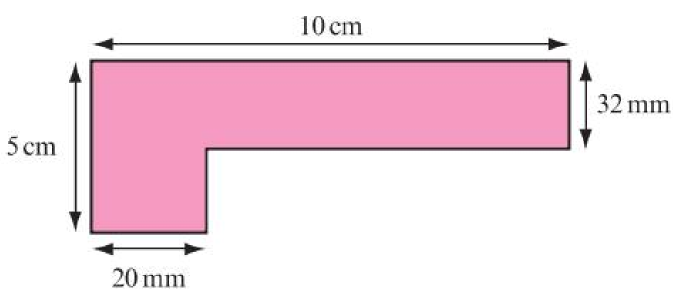
7 Work out the area of the rectangle shown. Give your answer in:

a mm^2 **b** cm^2



8 Work out the area of this **compound shape**. Give your answer in:

a mm^2 **b** cm^2

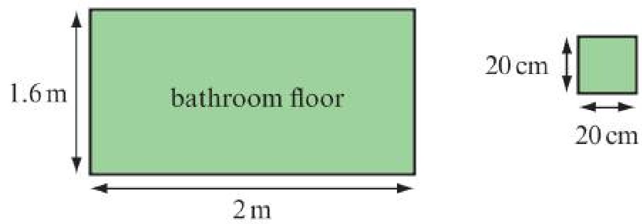


Tip

Start by dividing the compound shape into two rectangles.

9 Is Zara correct? Explain your answer.

10 Sven is going to lay tiles on the floor of his bathroom.
The diagram shows the **dimensions** of the floor and the dimensions of one tile.



a Show that Sven needs 80 tiles to cover the bathroom floor.

b Discuss with a partner and other members of the class the method you used to show part **a**.

c Did all of you use the same method? Which method do you think is the best?

Summary checklist

- [ ] I can convert between cm^2 and mm^2.
- [ ] I can convert between m^2 and cm^2.

15.2 Using hectares

In this section you will …

- convert between hectares and m^2.

Key word

hectares (ha)

You use **hectares** to measure areas of land.

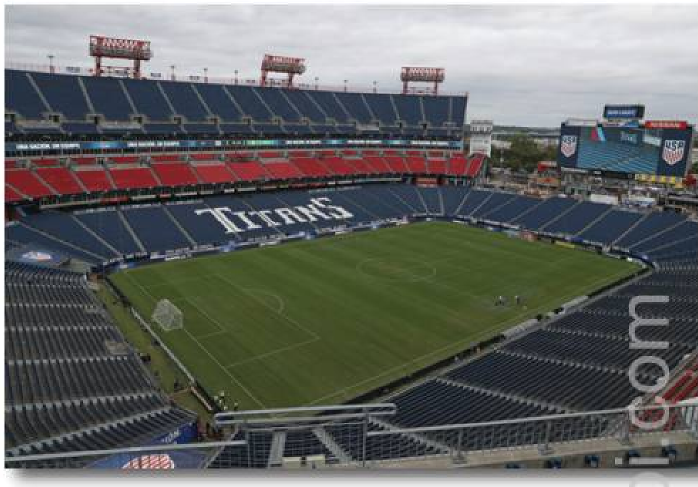
A hectare is the area of a square field of side 100 metres.

A football pitch is about half a hectare.

The abbreviation for hectare is **ha**.

You need to know this conversion:

1 hectare (ha) = $10\,000\,m^2$



Worked example 15.2

a Copy and complete these statements.

i $2.4\,ha = \square\,m^2$ **ii** $125\,000\,m^2 = \square\,ha$

b A rectangular piece of land measures 850 m by 1.4 km.

Work out the area of the land. Give your answer in hectares.

Answer

a **i** $2.4 \times 10\,000 = 24\,000$

$2.4\,ha = 24\,000\,m^2$

Multiply the number of hectares by 10 000 to convert to square metres.

ii $125\,000 \div 10\,000 = 12.5$

$125\,000\,m^2 = 12.5\,ha$

Divide the number of square metres by 10 000 to convert to hectares.

b $1.4\,km = 1400\,m$

First, find the area of the land, in square metres. Then change the answer to hectares. Start by converting 1.4 km to metres.

$\text{Area} = 850 \times 1400$

Then work out the area of the land.

$= 1\,190\,000\,m^2$

This answer is in square metres.

$1\,190\,000 \div 10\,000 = 119\,ha$

Divide by 10 000 to convert square metres to hectares.

Exercise 15.2

1 Copy and complete these conversions between hectares and m^2.

a $6\text{ ha} = \square\ \text{m}^2$ $\quad 6 \times 10\,000 = \square\ \text{m}^2$

b $11.2\text{ ha} = \square\ \text{m}^2$ $\quad 11.2 \times 10\,000 = \square\ \text{m}^2$

c $0.63\text{ ha} = \square\ \text{m}^2$ $\quad 0.63 \times 10\,000 = \square\ \text{m}^2$

2 Copy and complete these conversions.

a $4.6\text{ ha} = \square\ \text{m}^2$ **b** $0.8\text{ ha} = \square\ \text{m}^2$

c $0.75\text{ ha} = \square\ \text{m}^2$ **d** $0.025\text{ ha} = \square\ \text{m}^2$

3 Copy and complete these conversions between m^2 and hectares.

a $70\,000\text{ m}^2 = \square\ \text{ha}$ $\quad 70\,000 \div 10\,000 = \square\ \text{ha}$

b $135\,000\text{ m}^2 = \square\ \text{ha}$ $\quad 135\,000 \div 10\,000 = \square\ \text{ha}$

c $8000\text{ m}^2 = \square\ \text{ha}$ $\quad 8000 \div 10\,000 = \square\ \text{ha}$

4 Copy and complete these conversions.

a $89\,000\text{ m}^2 = \square\ \text{ha}$ **b** $240\,000\text{ m}^2 = \square\ \text{ha}$

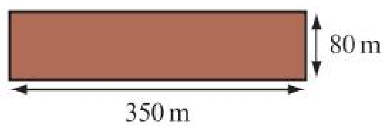
c $900\text{ m}^2 = \square\ \text{ha}$ **d** $1\,265\,000\text{ m}^2 = \square\ \text{ha}$

5 A rectangular piece of land measures 780 m by 550 m. Work out the area of the land, in:

a square metres **b** hectares

6 A builder buys a rectangular piece of land. The dimensions of the land are shown in the diagram.

a Work out the area of the land, in hectares.



The cost of the land is \$12 400 per hectare.

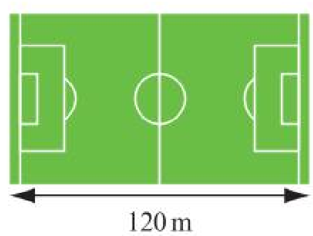
The builder says, 'This land will cost me more than \$34 000.'

b Is the builder correct? Explain your answer. Show your working.

7 A football pitch has an area of 0.78 ha.

a Work out the area of the football pitch, in m^2.

b The length of the football pitch is 120 m. Work out the width of the football pitch, in metres.



8 Alessia and Ben share a piece of land in the ratio 1 : 2. The area of the piece of land they share is 0.087 hectares. Work out the area of Ben's piece, in m^2.

> **Tip**
>
> In questions **5** to **7**, use the formula for the area of a rectangle. See Section 2.2 for a reminder on using formulae.

Think like a mathematician

9 Look back at Question **8**. In groups, discuss the methods that you used to answer the question.

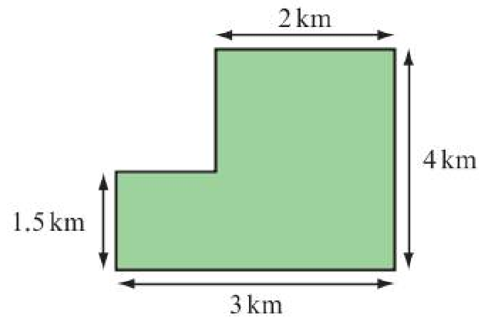
a What are the advantages and disadvantages of the different methods?

b Which method do you think is the best? Explain why.

10 A company wants to build a water park. The diagram shows a plan of the land the company wants to buy.

The price of the land is \$5200 per hectare. The company wants to pay less than \$5 million for the land.

Can the company afford to buy the land? Show all your working. Explain your answer.



Tip

Start by changing the measurements given on the plan to metres.

Look back at the questions in this exercise.

As well as converting between hectares and square metres, you have also used these topics:

- areas of rectangles
- areas of compound shapes
- working with money
- ratio
- converting kilometres to metres.

a Write down the topics that you easily remembered.

b Write down the topics that you needed help with.

Summary checklist

☐ I can convert between hectares and m^2.

15.3 The area of a triangle

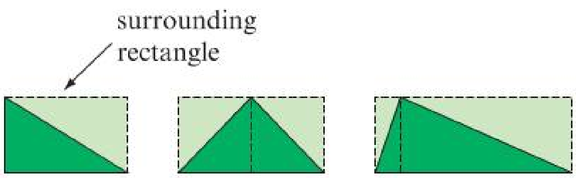
In this section you will …

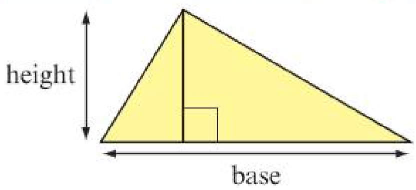
- derive and use the formula for the area of a triangle.

Key word

perpendicular height

The area of a triangle is always half of the area of the rectangle that surrounds it, as these diagrams show.





You find the area of a rectangle by multiplying the base by the height. So, the area of a triangle will be a half of the base multiplied by the height.

You can use algebra to write the formula as: Area $= \frac{1}{2} \times \text{base} \times \text{height}$

or simply: $A = \frac{1}{2}bh$

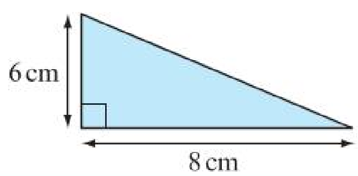
Note that the height measurement of the triangle <u>must</u> be the **perpendicular height**, from the base to the opposite vertex.

Tip

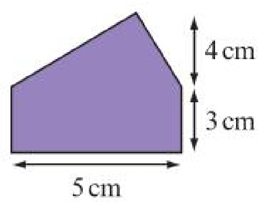
The height must be at right angles (90°) to the base.

Worked example 15.3

a Work out the area of this triangle.



b Work out the area of this compound shape.



Answer

a $A = \frac{1}{2}bh = \frac{1}{2} \times 8 \times 6$

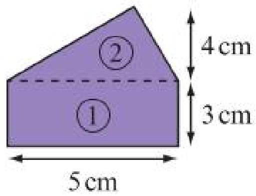
$= 24\,\text{cm}^2$

Write down the formula. Then substitute the values for b and h.

Work out the answer. Remember to include the units (cm^2).

Continued

b



Divide the compound shape into a rectangle and a triangle.

Shape ① is a rectangle.

Shape ② is a triangle.

Area ① $= 5 \times 3 = 15\text{ cm}^2$

Use $A = b \times h$ to work out the area of the rectangle.

Area ② $= \frac{1}{2} \times 5 \times 4 = 10\text{ cm}^2$

Use $A = \frac{1}{2}bh$ to work out the area of the triangle.

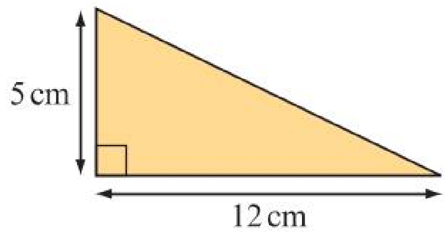
Total $= 15 + 10 = 25\text{ cm}^2$

Add the areas of shapes ① and ② to give the total area.

Exercise 15.3

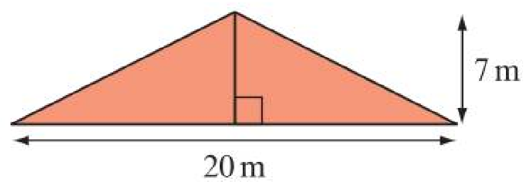
1 Copy and complete the workings to find the area of these triangles.

a



$A = \frac{1}{2}bh = \frac{1}{2} \times 12 \times \square = \square\text{ cm}^2$

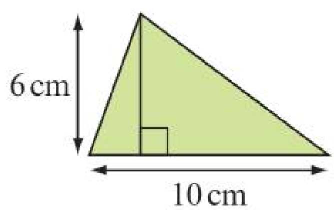
b



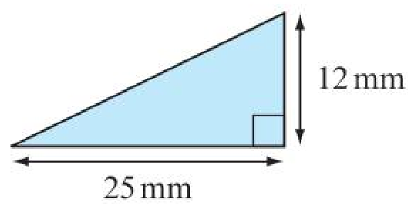
$A = \frac{1}{2}bh = \frac{1}{2} \times \square \times 7 = \square\text{ m}^2$

2 Work out the area of each triangle.

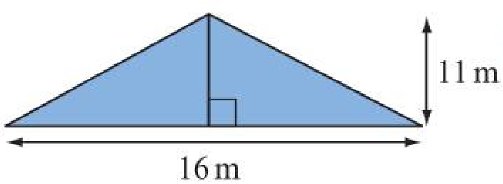
a



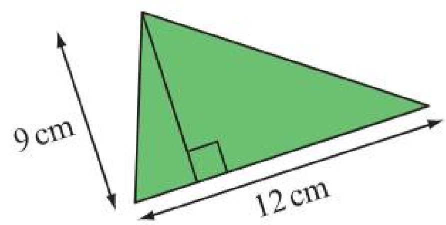
b



c

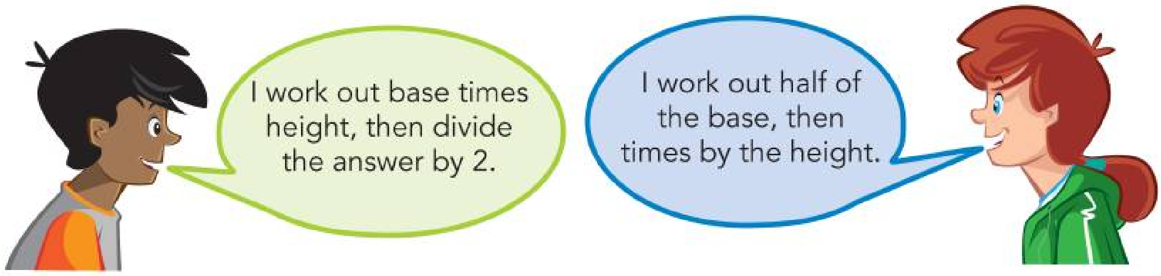


d

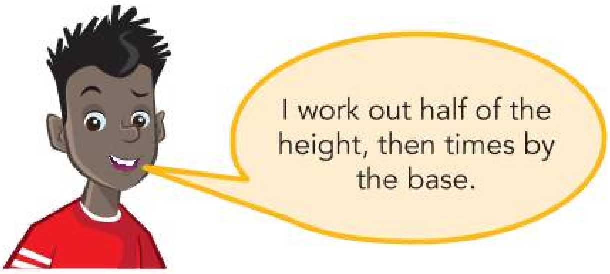


3 Arun, Marcus and Sofia are discussing the methods they use to find the area of a triangle.

Arun says: Sofia says:



Marcus says:



Will they all get the same answer? Explain why.

Think like a mathematician

4 Work with a partner to answer this question.

Look back at the methods that Arun, Marcus and Sofia used in Question **3**.

Whose method would it be best to use to find the areas of these triangles? Explain why.

a height = 16 cm and base = 9 cm

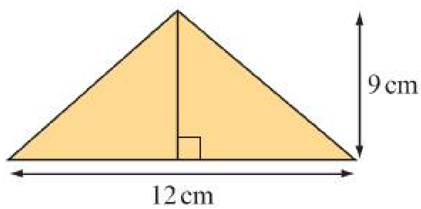
b height = 25 cm and base = 8 cm

c height = 7 cm and base = 9 cm

d height = 12 cm and base = 10 cm

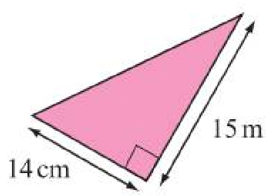
5 Work out the area of each triangle, using the method shown. Look back at Question **3** to check the method.

a



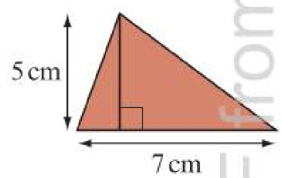
Sofia's method

b



Marcus method

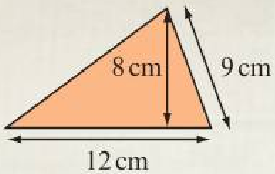
c



Arun's method

6 This is part of Budi's homework.

Question Work out the area of this triangle.



Solution $\text{Area} = \frac{1}{2}bh$

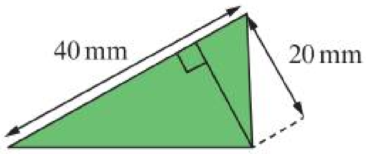
$= \frac{1}{2} \times 12 \times 9$

$= 54\ \text{cm}^2$

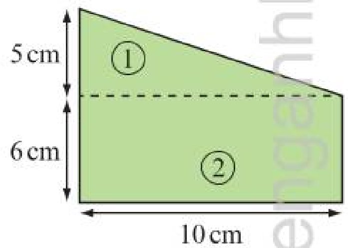
a Explain the mistake that Budi has made.

b Work out the correct answer.

7 Work out the area of this triangle.



8 Copy and complete the workings to find the area of this compound shape.



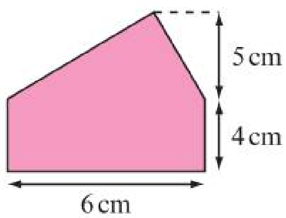
Area ① $= \frac{1}{2}bh = \frac{1}{2} \times 10 \times \square = \square\ \text{cm}^2$

Area ② $= bh = 10 \times \square = \square\ \text{cm}^2$

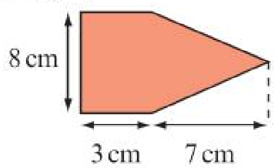
Total area $= \square + \square = \square\ \text{cm}^2$

9 Work out the areas of these compound shapes.

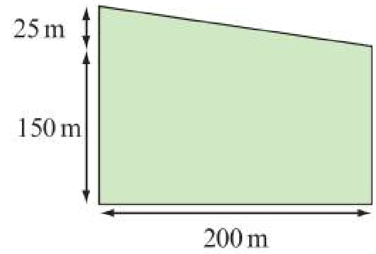
a



b



10 The diagram shows a farmer's field.



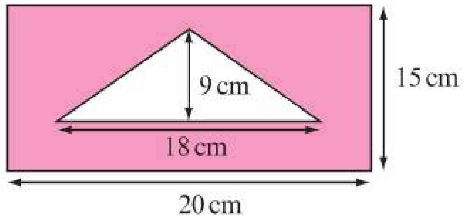
a Work out the area of the field.

The farmer adds fertiliser to the field.

b He uses 180 kg of fertiliser per hectare. How many kilograms of fertiliser does he use for this field?

c Fertiliser costs 80 cents per kg. How much does it cost the farmer for the fertiliser for this field? Give your answer in dollars.

11 Natasha works out that the shaded area in this diagram is 219 cm^2.



Tip

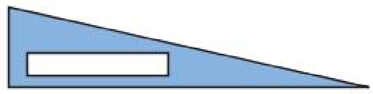
shaded area = area of rectangle − area of triangle

Is Natasha correct? Show your working.

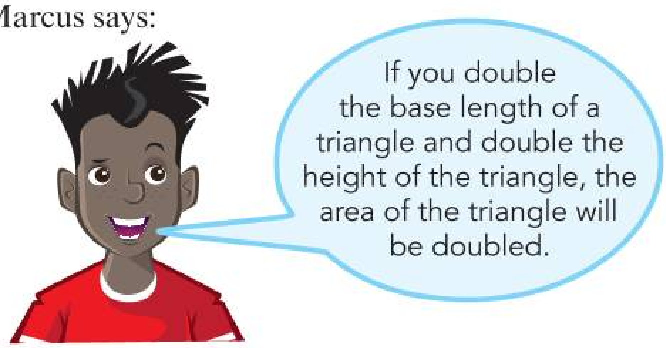
12 In this diagram, the rectangle has a base length of 55 mm and a height of 10 mm.

The triangle has a base length of 120 mm and a height of 40 mm.

Work out the shaded area, in square centimetres.



13 Marcus says:



Tip

Try different values for the base length and height.

Is Marcus correct? Explain your answer.

Summary checklist

☐ I can derive and use the formula for the area of a triangle.

15.4 Calculating the volume of cubes and cuboids

In this section you will …

- derive and use the formula for the volume of cubes and cuboids.

Key words

cubic centimetre (cm^3)

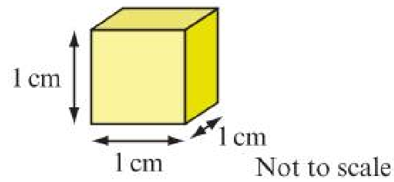
cubic millimetre (mm^3)

cubic metre (m^3)

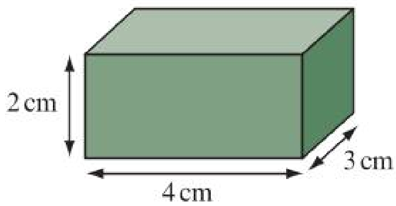
volume

Look at this cube. It has a length, a width and a height of 1 cm.

It is called a centimetre cube. You say that it has a **volume** of 1 **cubic centimetre** (1 cm^3).



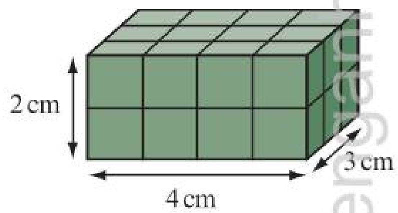
This cuboid is 4 cm long, 3 cm wide and 2 cm high.



If you divide the cuboid into centimetre cubes, it looks like this.

You can see that there are 12 cubes in each layer and that there are two layers. This means that the total number of centimetre cubes in this cuboid is 24.

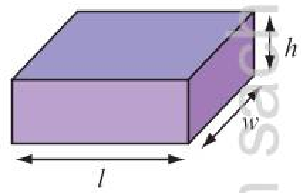
You say that the volume of the cuboid is 24 cm^3.



You can use algebra to work out the volume of a cuboid, using the formula:

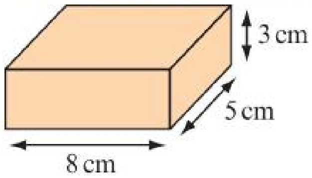
Volume = length × width × height or $V = l \times w \times h$

If the sides of a cuboid are measured in millimetres, the volume will be in **cubic millimetres (mm^3)**. If the sides of a cuboid are measured in metres, the volume will be in **cubic metres (m^3)**.

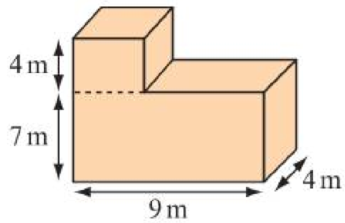


Worked example 15.4

a Work out the volume of this cuboid.



b This compound shape is made from a cube and a cuboid. Work out the volume of the compound shape.



Answer

a $V = 8 \times 5 \times 3$

$= 120\,\text{cm}^3$

Use the formula volume = length × width × height.

All the lengths are in cm, so the answer is in cm^3.

b Cube: $V = 4 \times 4 \times 4$

$= 64\,\text{m}^3$

Work out the volume of the cube. Use $V = l \times w \times h$.

Cuboid: $V = 9 \times 4 \times 7$

$= 252\,\text{m}^3$

Work out the volume of the cuboid. Use $V = l \times w \times h$.

Total $= 64 + 252$

The volume of the shape is the total volume of the cube and cuboid.

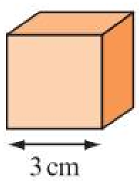
$= 316\,\text{m}^3$

All the lengths are in metres, so the answer is in m^3.

Exercise 15.4

1 Copy and complete the workings to find the volume of these cubes.

a

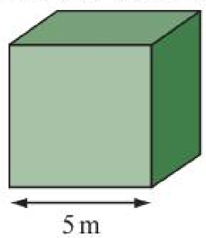


Volume = length × width × height

$= 3 \times 3 \times 3$

$= \square\,\text{cm}^3$

b



Volume = length × width × height

$= 5 \times \square \times \square$

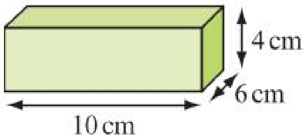
$= \square\,\text{m}^3$

Tip

Remember: in a cube the length, width and height are all the same.

2 Copy and complete the workings to find the volume of these cuboids.

a

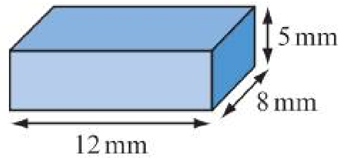


Volume = length × width × height

$= 10 \times 6 \times 4$

$= \square\ \text{cm}^3$

b



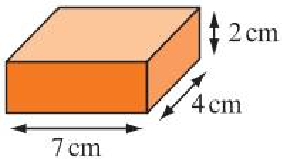
Volume = length × width × height

$= 12 \times \square \times \square$

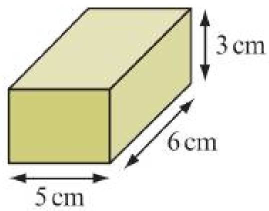
$= \square\ \text{mm}^3$

3 Work out the volume of each of these cuboids.

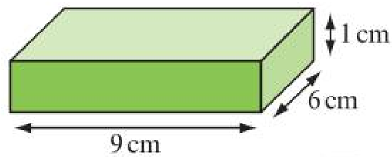
a



b



c



4 This is part of Steph's homework.

Question A cuboid has a length of 12 cm, a width of 9 cm and a height of 35 mm. What is the volume of the cuboid?

Solution Volume = 12 × 9 × 35

= 3780 cm³

Steph has got the solution incorrect.
Explain the mistake that Steph has made and work out the correct answer.

5 The table shows the length, width and height of four cuboids.

	Length	Width	Height	Volume
a	5 cm	12 mm	6 mm	$\square$ mm³
b	12 cm	8 cm	4 mm	$\square$ cm³
c	8 m	6 m	90 cm	$\square$ m³
d	1.2 m	60 cm	25 cm	$\square$ cm³

Copy and complete the table.

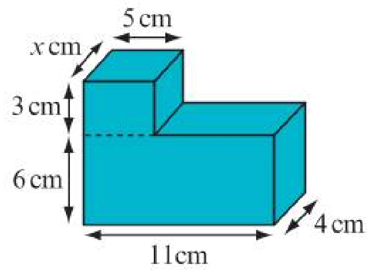
Tip

Make sure the length, width and height are all in the same units before you work out the volume.

6 Look at this compound shape.

a Write down the value of x.

b Copy and complete the workings to find the volume of the shape.



Top cuboid: $V = l \times w \times h = 5 \times \square \times 3$

$= \square \text{ cm}^3$

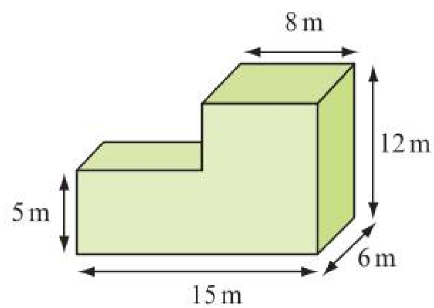
Bottom cuboid: $V = l \times w \times h = 11 \times 4 \times 6$

$= \square \text{ cm}^3$

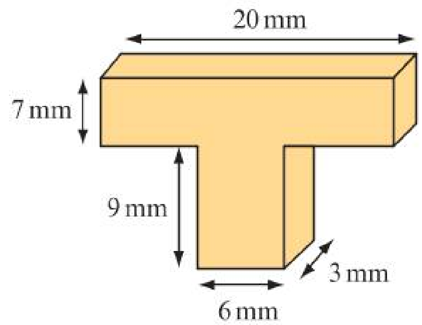
Volume of shape: $\square + \square = \square \text{ cm}^3$

7 Work out the volume of these compound shapes.

a



b



Activity 15.1

Work with a partner to complete this question.

On a piece of paper, draw one cuboid and one compound shape, like those in questions **3** and **7**. Make sure you write all the dimensions on your shapes.

On a different piece of paper, work out the volume of your two shapes. Do not let your partner see your working.

Swap shapes with your partner and work out the volume of their shapes.

Swap back your pieces of paper and mark each other's work. Discuss any mistakes that have been made.

Think like a mathematician

8 This is part of Amadeo's homework.

Question A cuboid has a length of 7 cm, a width of 3 cm and a volume of 168 cm^3.

What is the height of the cuboid?

Solution

I know that:	$7 \times 3 \times h = 168$	
So:	$21 \times h = 168$	
When $h = 10$:	$21 \times 10 = 210$	This is too big.
When $h = 9$:	$21 \times 9 = 189$	This is still too big.
When $h = 8$:	$21 \times 8 = 168$	This is correct, so the answer is $h = 8$ cm.

Amadeo tries different values for the height, until he gets the correct volume.

a What are the disadvantages of this method?

b Discuss different methods that Amadeo could use. What are the advantages of these methods? Which is the best method?

9 A metal cuboid has a length of 75 mm, a height of 4 mm and a volume of 1500 mm^3.

What is the width of the cuboid?

10 A cuboid has a length of 6 cm, a width of 4 cm and a height of 5 cm.

a What is the volume of the cuboid?

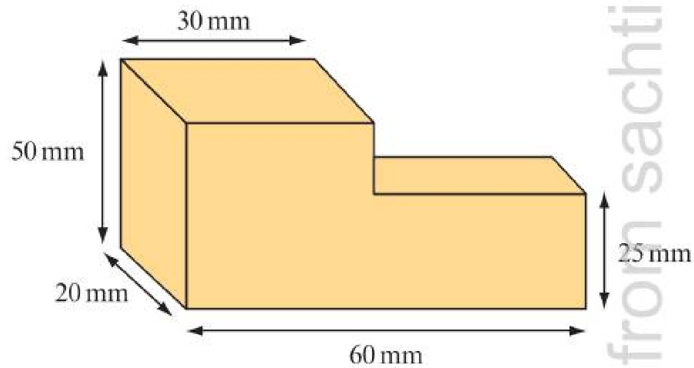
b Write down the dimensions of three other cuboids that have the same volume.

11 The diagram shows a shape made from gold.

The shape is melted and made into gold cubes.

The side length of each cube is 12 mm.

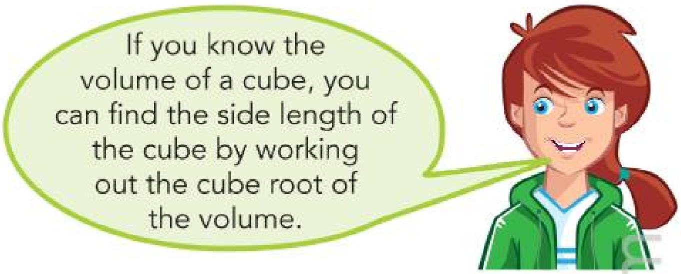
How many whole cubes can be made from this shape?



12 **a** Copy and complete this table.

Side length of cube	Volume of cube		
2 cm	2 × 2 × 2	2^3	8 cm³
3 cm	3 × 3 × 3	3^3	27 cm³
4 cm	4 × □ × □	□³	□ cm³
5 cm	□ × □ × □	□³	□ cm³

b Sofia says:



Is Sofia correct? Explain your answer.

Use your table in part **a** to help with your explanation.

c Work out the side length of a cube with volume:

i 1000 cm³ **ii** 216 cm³

Describe four situations when you would use volumes in real life.

Summary checklist

☐ I can derive and use the formula for the volume of a cube or cuboid.

15.5 Calculating the surface area of cubes and cuboids

In this section you will ...

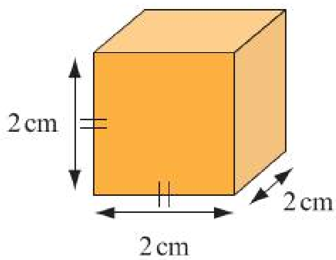
- calculate the surface area of cubes and cuboids.

Key words

net
surface area

The **surface area** of a cube or cuboid is the total area of all its faces. The units of measurement for surface area are square units; for example, mm^2, cm^2 or m^2.

This cube has six faces.



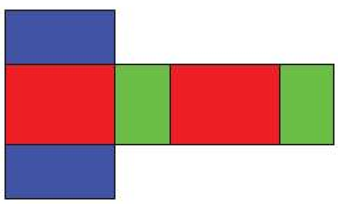
The area of one face $= 2 \times 2$

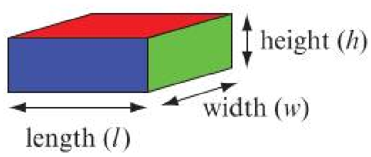
$= 4\,cm^2$

The surface area of the cube $= 6 \times 4$

$= 24\,cm^2$

This is a **net** of a cuboid. It can be folded up to make the cuboid shown.





You can see that the cuboid has two blue faces, two red faces and two green faces.

Area of one red face = length × width

Area of one blue face = length × height

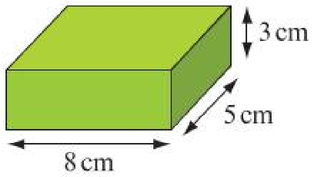
Area of one green face = width × height

To work out the surface area you add together the areas of all the faces.

This cuboid has two faces of each colour, so you multiply the area of each face by 2, before adding them all together.

Worked example 15.5

Work out the surface area of this cuboid.



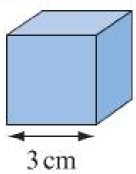
Answer

Area of top face $= 8 \times 5 = 40\,\text{cm}^2$	Use the formula: area = length × width.
Area of front face $= 8 \times 3 = 24\,\text{cm}^2$	Use the formula: area = length × height.
Area of side face $= 5 \times 3 = 15\,\text{cm}^2$	Use the formula: area = width × height.
Surface area $= 2 \times 40 + 2 \times 24 + 2 \times 15$	Multiply the area of each face by 2.
$= 80 + 48 + 30$	Add the areas to find the total surface area.
$= 158\,\text{cm}^2$	Remember to include the units in your answer.

Exercise 15.5

1 Copy and complete the workings to find the surface area of these cubes.

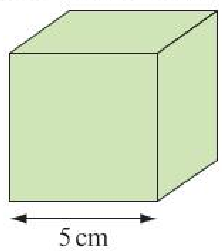
a



Area of one face = $\square \times \square = \square$ cm²

Surface area of cube = $6 \times \square = \square$ cm²

b

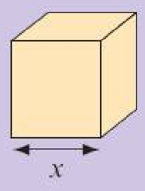


Area of one face = $\square \times \square = \square$ cm²

Surface area of cube = $6 \times \square = \square$ cm²

Think like a mathematician

2 This cube has a side length of x.



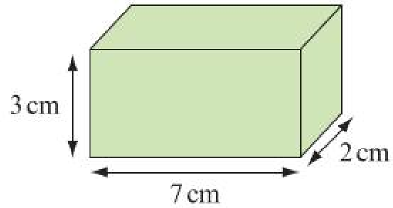
a Use algebra to write down an expression for the area of one face of the cube.

b Use algebra to write down a formula for the surface area of the cube.

c Discuss your answers with the rest of the class.

3 Copy and complete the workings to find the surface area of these cuboids.

a



Area of top face $= 7 \times 2 = \square\ \text{cm}^2$

Area of front face $= 7 \times 3 = \square\ \text{cm}^2$

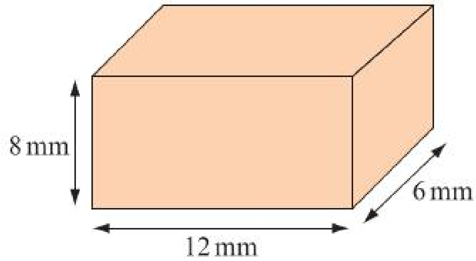
Area of side face $= 3 \times 2 = \square\ \text{cm}^2$

Surface area $= 2 \times \square + 2 \times \square + 2 \times \square$

$= \square + \square + \square$

$= \square\ \text{cm}^2$

b



Area of top face $= 12 \times \square = \square\ \text{mm}^2$

Area of front face $= 12 \times \square = \square\ \text{mm}^2$

Area of side face $= 8 \times \square = \square\ \text{mm}^2$

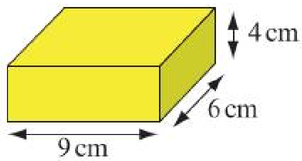
Surface area $= 2 \times \square + 2 \times \square + 2 \times \square$

$= \square + \square + \square$

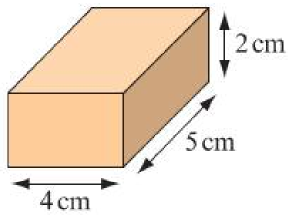
$= \square\ \text{mm}^2$

4 Work out the surface area of each of these cuboids.

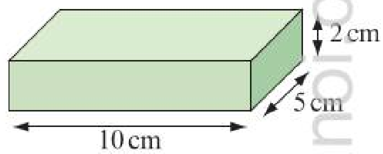
a



b

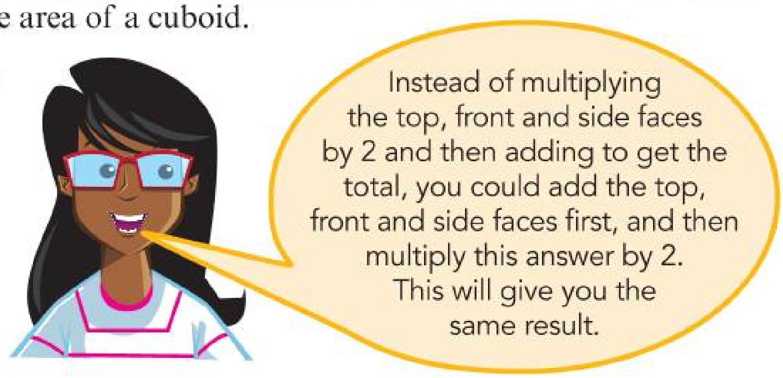


c



5 Zara is looking at the method given in Question **3** for working out the surface area of a cuboid.

Zara says:



Tip

If you are not sure, try Zara's method with one of the cuboids in Question **3**.

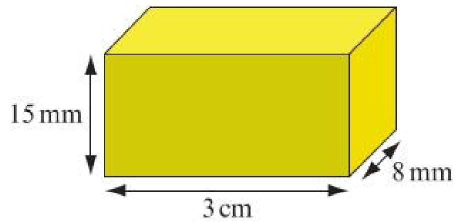
a Is Zara correct? Explain your answer.

b Do you prefer the method given in Question **3** or Zara's method? Explain why.

6 Work out the surface area of this cuboid.

Give your answer in:

a mm^2 b cm^2



7 Michiko has a metal container in the shape of a cuboid.

The container has a length of 2.5 m, a width of 2 m and a height of 3 m.

Michiko plans to paint all the outside faces of the container with two coats of metal paint.

a How many tins of paint does Michiko need to buy?

b What is the total cost of the paint?

Size of tin: 250 mL
Paint coverage: 4.5 m^2 per litre

Activity 15.2

The length of a cuboid is 20 cm.

The width of the cuboid is $\frac{3}{5}$ of the length of the cuboid.

The height of the cuboid is 75% of the width of the cuboid.

What is the surface area of the cuboid?

Swap your solution with a partner's. Follow the method your partner has used and check their working. Discuss what you think of each other's work.

Tip

Make sure that you set out your solution clearly so that it is easy for your partner to follow.

8 The surface area of a cube is 384 m^2.

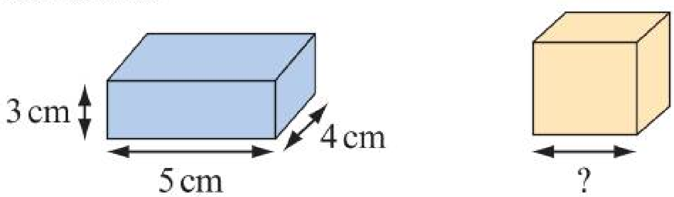
a What is the area of one face of the cube?

b What is the side length of the cube?

Tip

The area of six faces is 384 m^2.

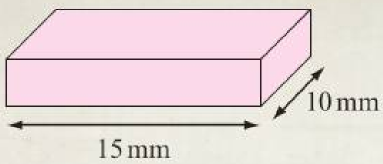
9 The surface area of this cube is 2 cm^2 more than the surface area of this cuboid.



What is the side length of the cube?

10 Marcus is working on this question.

Question This cuboid has a volume of $900\,mm^3$.



What is the surface area of the cuboid?

> **Tip**
>
> Start by using the formula for volume to work out the height of the cuboid.

Marcus says:

Is Marcus correct? Show your working.

Look back at the questions in this exercise.

What do you think is the most important thing to remember when working out the surface area of a cuboid?

Summary checklist

- [ ] I can calculate the surface area of cubes and cuboids.

Check your progress

1 Copy and complete the following area conversions.

a $8\,cm^2 = \square\, mm^2$ **b** $5\,m^2 = \square\, cm^2$ **c** $420\,mm^2 = \square\, cm^2$

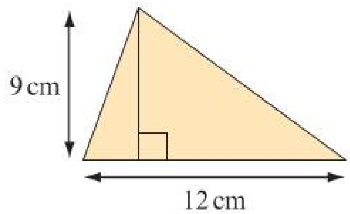
2 Copy and complete these statements.

a $3\,ha = \square\, m^2$ **b** $4.6\,ha = \square\, m^2$ **c** $0.8\,ha = \square\, m^2$

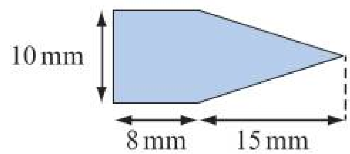
d $20\,000\,m^2 = \square\, ha$ **e** $94\,000\,m^2 = \square\, ha$ **f** $5600\,m^2 = \square\, ha$

3 Work out the area of these shapes.

a



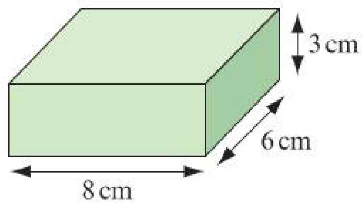
b



4 Work out the:

a volume of the cuboid

b surface area of the cuboid



5 This is part of Nawaf's homework.

Question A cuboid has a length of 15 mm, a width of 8 mm and a height of 12 mm.
What is the volume of the cuboid?

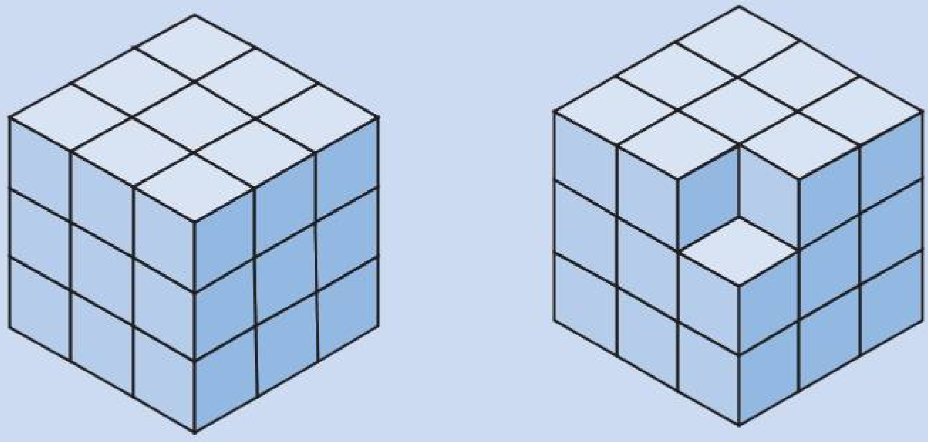
Solution Volume = 15 + 8 + 12
= 35 mm

Nawaf's solution is incorrect.
Explain the mistakes that he has made and work out the correct answer.

Project 6

Removing cubes

Shown are two solids made out of smaller cubes.



The solid on the left has a volume of $27\,cm^3$, and the solid on the right has a volume of $26\,cm^3$.

Work out the surface area of each solid. What do you notice?

Start with a $3 \times 3 \times 3$ cube and remove some cubes. Can you create solids with the same surface area as the original cube but with a volume of $25\,cm^3$? What about with a volume of $24\,cm^3$ or $20\,cm^3$ or …?

What is the minimum possible volume of a solid made in this way?

16 Interpreting results

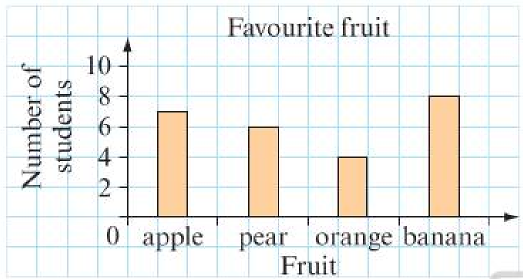
Getting started

1 The bar chart shows the favourite fruit of the students in Arun's class.

a How many students choose pear as their favourite fruit?

b What is the favourite fruit of the students?

c How many students are in the class?

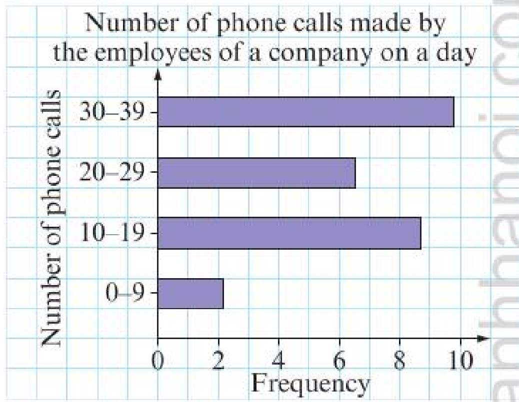


2 The frequency diagram shows the number of phone calls made by the employees of a company on a day.

a How many employees made 10–19 phone calls?

b How many more employees made 30–39 phone calls than employees who made 0–9 phone calls?

c How many employees are there in the company?



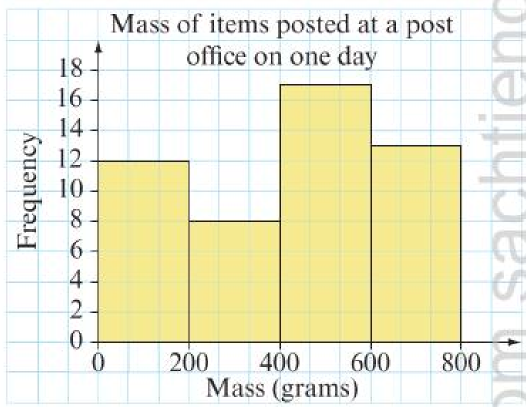
3 The frequency diagram shows the masses of the items posted at a post office on one day.

a How many items weighed 600–800 grams?

b What was the least common mass of items posted?

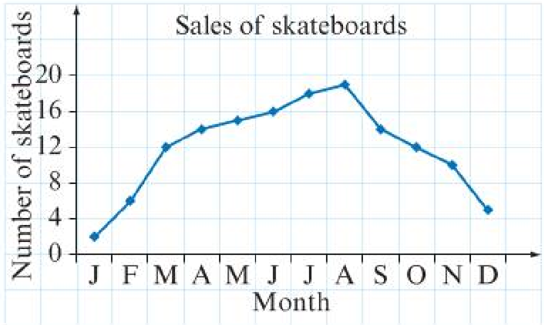
c How many fewer items were posted with a mass of 0–200 grams than items posted with a mass of 400–600 grams?

d How many items were posted altogether?



Continued

4 Ian records the sales of skateboards at his shop each month for one year. He joins the points with straight lines. The data are shown in the line graph.

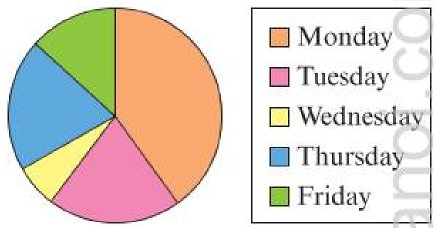


- **a** How many skateboards did Ian sell in:
  - **i** March? **ii** May?
- **b** In which month did he sell the most skateboards?
- **c** Between which two months did his sales double?
- **d** Describe how the sales of skateboards changed over the year.

> **Tip**
> 'Double' means '×2'.

5 The **pie chart** shows the number of emails Preety received in one week.

- **a** On which day did she receive the most emails?
- **b** On which day did she receive the fewest emails?
- **c** On which two days did she receive the same number of emails?
- **d** Can you tell from the pie chart how many emails Preety received on Friday? Explain your answer.



6 Fu writes down the ages of the five members of his family:

42	38	16	12	12

Work out the:

- **a** mean age **b** median age
- **c** modal age **d** range in ages

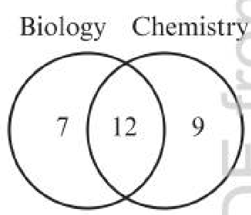
7 Here are the names of some quadrilaterals.

Rectangle Parallelogram Square
Trapezium Rhombus Kite

Copy the Carroll diagram and write the names of the shapes in their correct cells.

	Shape has two lines of symmetry	Shape doesn't have two lines of symmetry
All angles are 90°		
Not all angles are 90°		

8 The Venn diagram shows the number of students that study biology and chemistry.



How many students study:

- **a** biology **b** chemistry **c** only biology
- **d** biology and chemistry **e** only chemistry

Why do you draw charts or diagrams when you can just look at the data?

Looking at a 'picture' of the data makes it easier to see what the data are showing you.

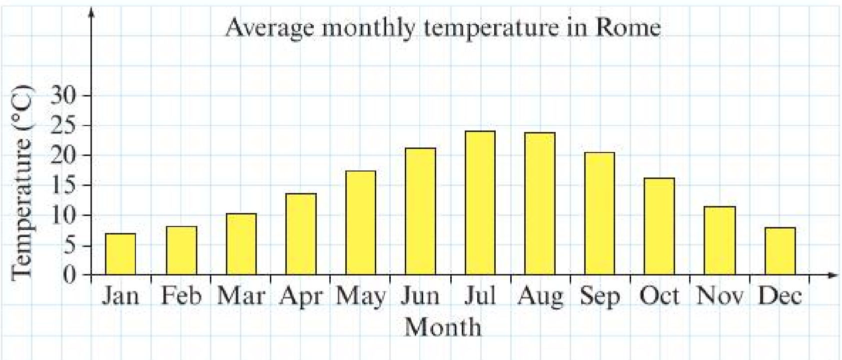
Imagine that you are going to book a holiday to Rome.

Travel guides often include graphs and charts that show you what the weather is usually like at holiday destinations. It is much easier to get the information you need from a graph or chart than from a long list of data.

This bar chart shows the usual temperatures in Rome throughout the year.

If you prefer the temperature to be lower than 20 °C, you can see that June, July, August and September are too hot, but the other months are suitable.

Charts like this can help you plan your perfect holiday.



16.1 Two-way tables

In this section you will …

- draw and interpret two-way tables.

Key words

discrete data
frequency table
two-way table

You already know how to draw and use frequency tables.

This **frequency table** shows the number of text messages sent by 30 people in one day.

Number of text messages sent	Frequency
0–9	8
10–19	12
20–29	7
30–39	3

This type of table is a really good way of showing one piece of information. In this case, the information shown is the number of text messages sent.

If you want to show more than one piece of information, you can use a **two-way table**.

You can use a two-way table to record two or more sets of **discrete data**.

In a two-way table you record different information in the rows and columns so that the information is easy to read.

Worked example 16.1

The two-way table shows the results of the games played by a hockey team in one season.

a How many home games did the hockey team lose?

b How many away games did the hockey team win?

c How many games did the hockey team draw altogether?

d What is the total number of games that the hockey team played in this season?

	Win	Draw	Lose	Total
Home games	7	3	2	12
Away games	3	4	5	12
Total	10	7	7	24

Answer

a 2 This is the number in the table where the 'Home games' row and the 'Lose' column meet.

b 3 This is the number in the table where the 'Away games' row and the 'Win' column meet.

c 7 This is the number in the table where the 'Total' row and the 'Draw' column meet.

d 24 This is the number in the table where the 'Total' row and the 'Total' column meet.

Exercise 16.1

1 The two-way table shows the hair colour of the girls and boys in Miss Jebson's class.

	Brown hair	Black hair	Other hair colour	Total
Girls	6	5	3	14
Boys	10	4	2	16
Total	16	9	5	30

- **a** How many of the boys have black hair?
- **b** How many of the girls have brown hair?
- **c** How many students are there altogether in Miss Jebson's class?
- **d** How many of the students do not have brown hair?

2 Some adults were asked if they like reading books. They answered either 'yes' or 'no'. The two-way table shows some of the results.
Copy and complete the table.

	Yes	No	Total
Men	16	7	
Women	22	5	
Total			

In here is the number of men.

In here is the number of women.

In here is the total number of adults.

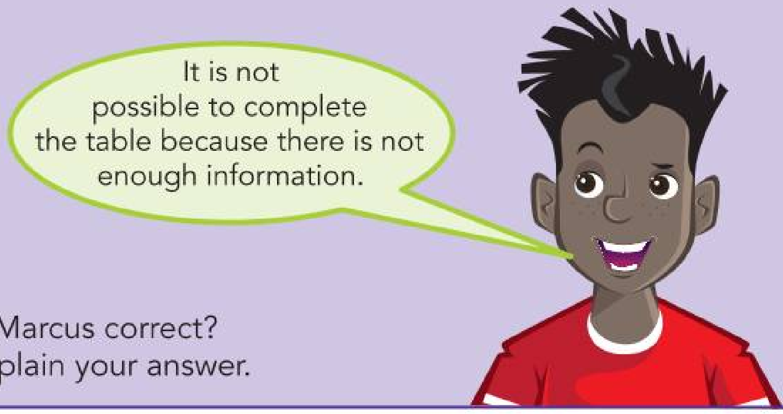
In here is the total number who answered 'yes'.

In here is the total number who answered 'no'.

Think like a mathematician

3 Discuss this question in pairs or small groups.
Marcus looks at this two-way table.
It shows the favourite rugby team of the students in his class.
Copy and complete the table.

	Scarlets	Blues	Dragons	Total
Girls		4	3	
Boys	5		4	
Total	13	12		



Is Marcus correct?
Explain your answer.

4 The two-way table shows the favourite subjects of the students in Mr Nguyen's class.

	Maths	Science	English	Other subject	Total
Girls	8	4		1	18
Boys	6		1		
Total		9			32

a Copy and complete the table.

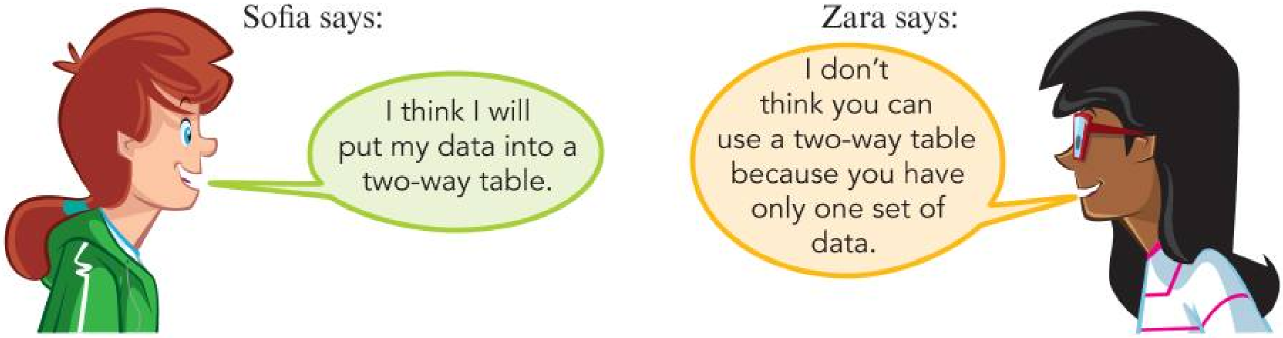
b How many of the boys chose Science as their favourite subject?

c How many of the students did not choose Maths, Science or English as their favourite subject?

Tip

Use the 'Total' row and 'Total' column to work out the missing values in the table.

5 Sofia keeps a record of the number of books she reads each month for one year. She would like to draw a table to represent her data.



a Do you agree or disagree with what Sofia and Zara say?

b What do you think is the best way for Sofia to represent her data? Explain your answer.

Activity 16.1

Work with a partner or in a small group.

A cinema records the number of adult tickets and child tickets sold.

On a Saturday the cinema shows the same film at 2 p.m. and 6 p.m.

One Saturday, a total of 350 adult tickets were sold.

110 adult tickets were for the 2 p.m. showing.

A total of 435 tickets were sold for the 2 p.m. showing.

125 child tickets were sold for the 6 p.m. showing.

1 Draw a two-way table to show this information.
2 Compare your two-way table with a different pair/group's two-way table.
3 Discuss any differences between your tables.
4 If your tables are different, who do you think has drawn the better table? Explain why.

6 A school has 42 teachers. All the teachers travel to school by car, bus or bicycle.

Twenty of the teachers are male.

Five of the male teachers and three of the female teachers cycle to school.

Seventeen of the teachers travel to school by bus.

Ten of the female teachers travel to school by car.

Copy and complete the two-way table to show the numbers of teachers that travel to school by either car, bus or bicycle.

	Car	Bus	Bicycle	Total
Male				
Female				
Total				

7 In a school there are 480 students.

The eye colour of the students is recorded as brown, blue, or 'other colour'.

$\frac{1}{2}$ of the students have brown eyes.

10% of the students have eyes of 'other colour'.

40% of the students with brown eyes are girls.

The number of girls with blue eyes is $\frac{2}{3}$ of the number of girls with brown eyes.

The ratio of girls : boys in the school is 2 : 3.

Copy and complete the two-way table to show this information.

	Brown	Blue	Other colour	Total
Girls				
Boys				
Total				

How would you describe a two-way table to someone who has never seen one before?

As well as drawing two-way tables, what other maths topics have you used in this exercise?

Summary checklist

- [ ] I can draw and interpret two-way tables.

16.2 Dual and compound bar charts

In this section you will …

- draw and interpret dual bar charts
- draw and interpret compound bar charts.

Key words

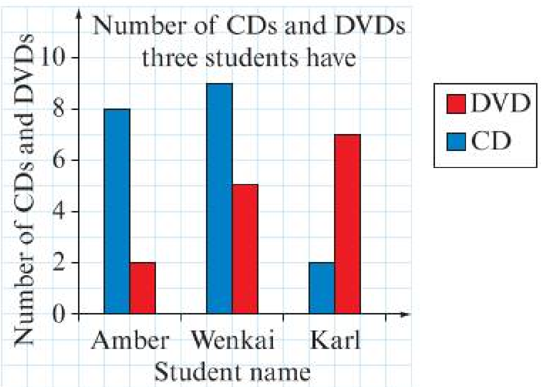
compound bar chart
dual bar chart

You already know how to draw and use bar charts. You use a bar chart to show one set of data.

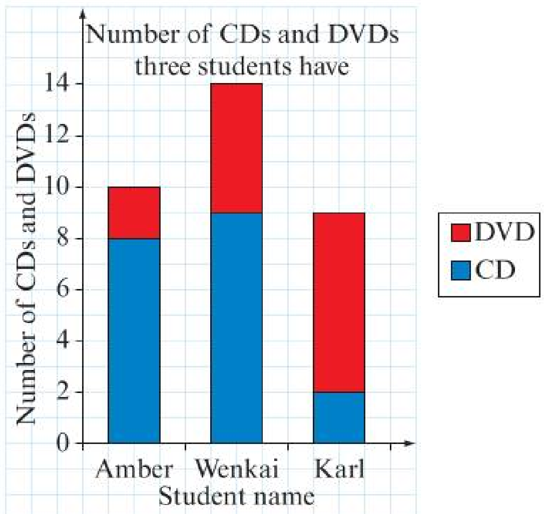
If you want to show more than one set of data, you can use a **dual bar chart** or a **compound bar chart**. A dual bar chart has the different sets of data presented side by side. A compound bar chart has the different sets of data combined into one bar.

Example: This dual bar chart and compound bar chart show how many CDs and DVDs three students have.

The dual bar chart shows the information like this:



The compound bar chart shows the information like this:



Worked example 16.2

a Use the dual bar chart shown in the previous example to answer these questions.

  i Who has the most number of DVDs?

  ii Amber has more CDs than DVDs. How many more does she have?

b Use the compound bar chart shown in the previous example to answer these questions.

  i Who has the most number of CDs and DVDs in total?

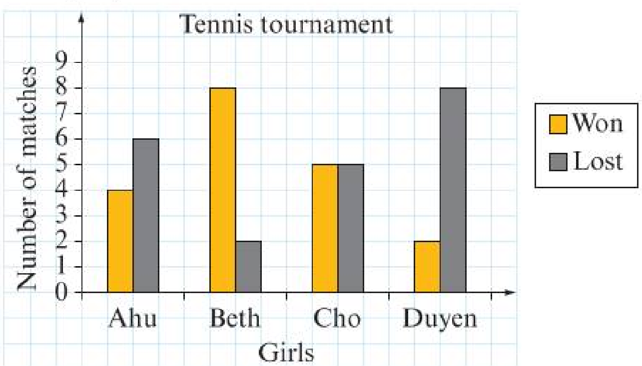
  ii How many CDs and DVDs does Karl have in total?

Answer

a	**i**	Karl	DVDs are represented by the red bars. Karl has the tallest red bar.
	ii	Number of CDs = 8	The blue bar (CDs) for Amber goes to 8.
		Number of DVDs = 2	The red bar (DVDs) for Amber goes to 2.
		$8 - 2 = 6$	Work out the difference to find out how many more CDs
		She has 6 more	than DVDs Amber has.
b	**i**	Wenkai	Wenkai has the tallest combined bar, so he has the most number of CDs and DVDs in total.
	ii	9	The height of Karl's combined bar is at 9.

Exercise 16.2

1 This dual bar chart shows the number of matches won and lost by four players at a tennis tournament.



a Which player won the most number of matches?

b Which player lost the most number of matches?

c How many more matches did Ahu win than Duyen?

d Make two other comments about what the bar chart shows.

e Did all the players play the same number of matches? Explain how you worked out your answer.

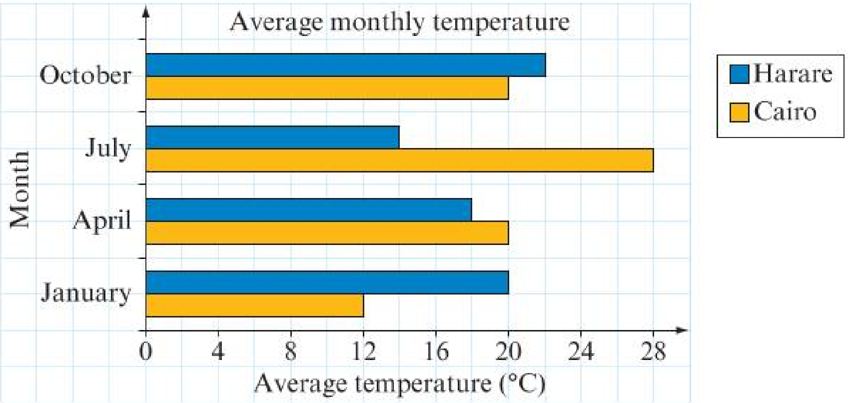
Tip

For part **d**, for example, you could compare the number of matches won or lost between different players, or you could compare the number of matches won and lost by one player.

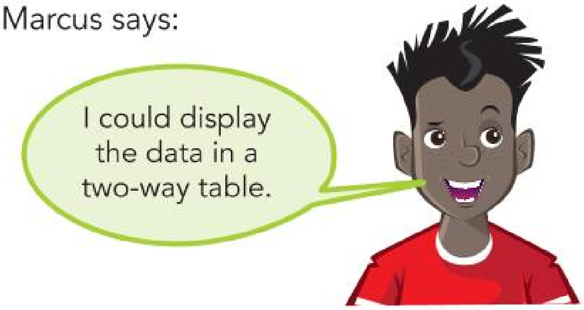
Activity 16.2

Work with a partner or in a small group to answer this question.

The dual bar chart shows the average monthly temperature in Cairo and Harare.



Marcus says:



Show that Marcus is correct by drawing a two-way table.

Compare your two-way table with a different group's two-way table.

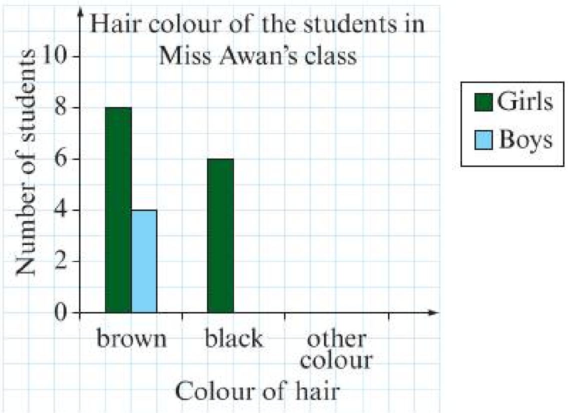
Discuss any differences between your tables.

If your tables are different, who do you think has drawn the better table? Explain why.

2 This two-way table shows the hair colour of the girls and boys in Miss Awan's class.

	Brown hair	Black hair	Other hair colour
Girls	8	6	2
Boys	4	9	1

a Copy and complete the dual bar chart to show this information.



b Make two comments about what the bar chart shows.

3 The two-way table shows how the students in class 7P travel to school.

	Walk	Car	Bus	Bicycle
Girls	10	2	4	3
Boys	5	1	8	5

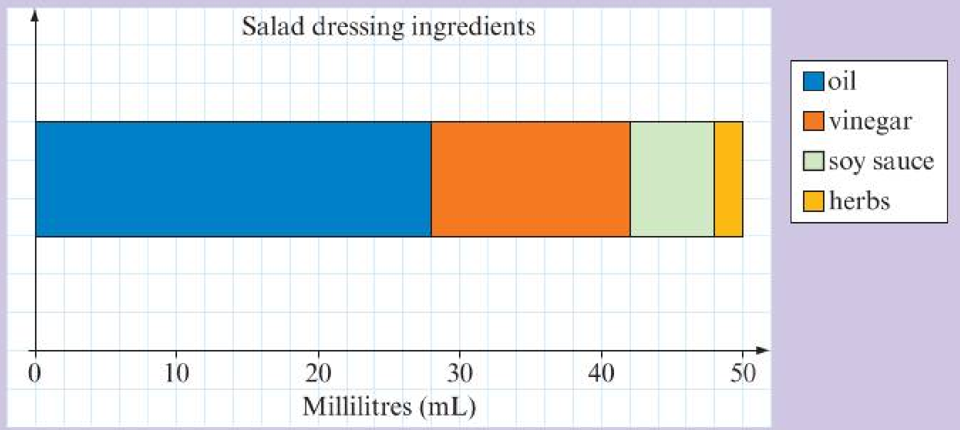
a Draw a dual bar chart to show this information.

b Make two comments about what the bar chart shows.

Think like a mathematician

4 Work with a partner or in a small group to answer this question.

The compound bar chart shows the ingredients in 50 millilitres (mL) of salad dressing.



Tip

A salad dressing is a sauce you can make to put on a salad.

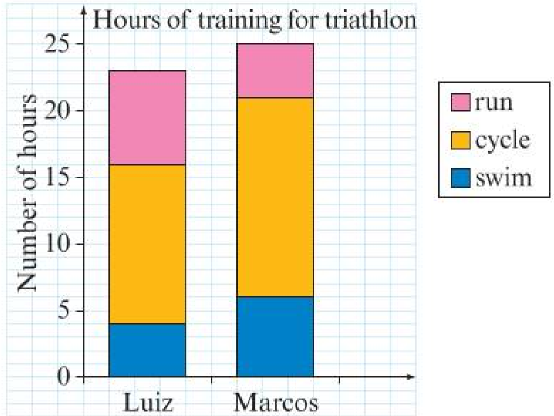
Sofia uses this method to work out how many millilitres of vinegar are in the salad dressing.

a What do you think of Sofia's method?

b Discuss other methods that you could use to work out the amount of vinegar in the salad dressing.

c Of the different methods discussed in parts **a** and **b**, which do you think is the best method and why?

5 This compound bar chart shows the number of hours of triathlon training Luiz and Marcos do one week.



a How many hours does:

i Luiz swim?

ii Marcos run?

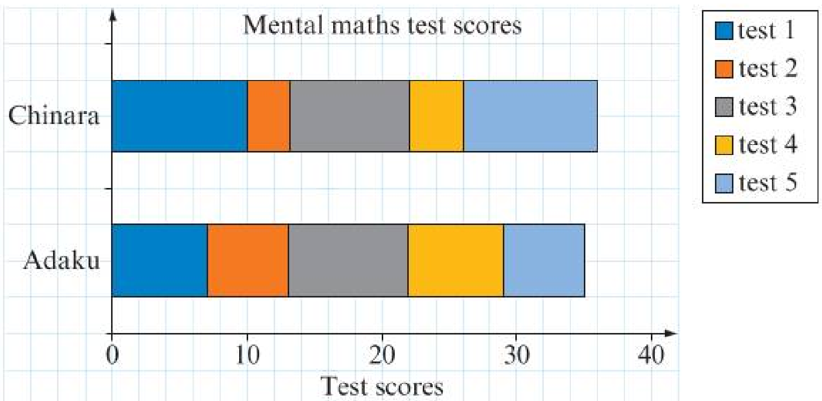
b How many more hours does Marcos cycle than Luiz?

c How many more hours does Luiz run than Marcos?

Tip

A triathlon is a competition in which people swim, then cycle, then run.

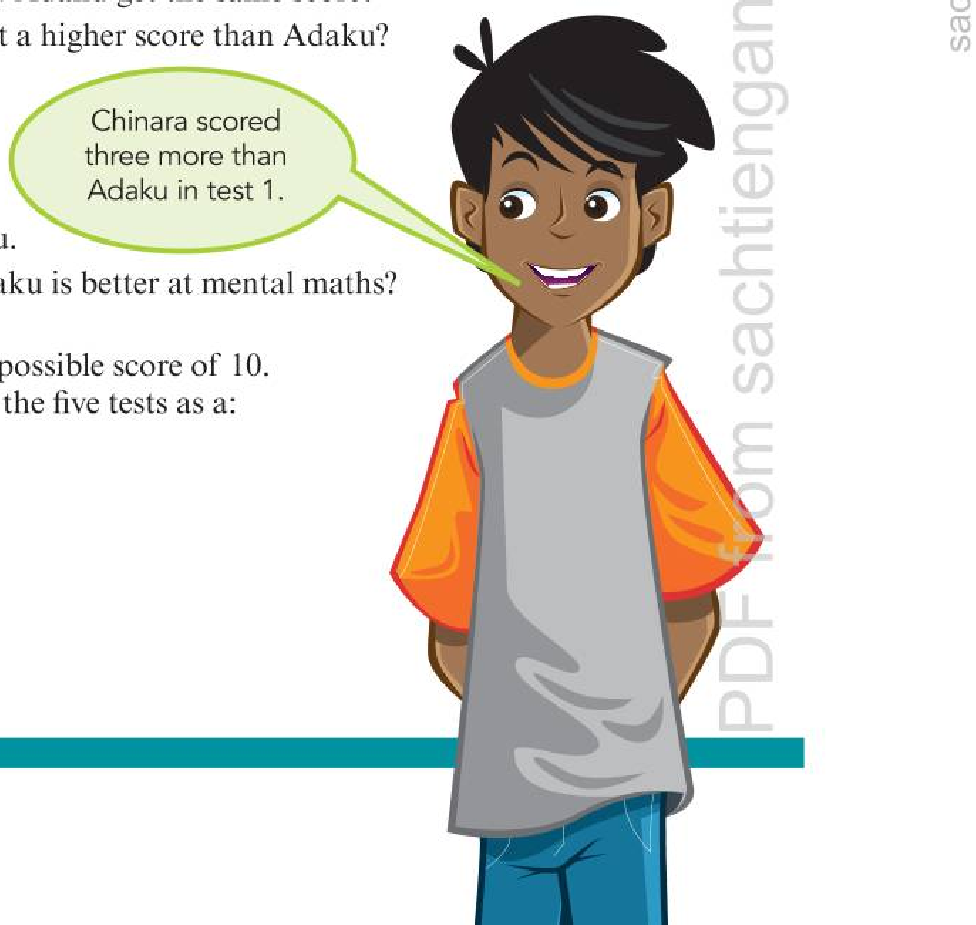
6 Chinara and Adaku compare their scores from five mental maths tests. The compound bar chart shows their test scores.



a In which test did Chinara and Adaku get the same score?

b In which tests did Chinara get a higher score than Adaku?

c Arun says:



Write down two comments that are similar to Arun's that compare the test results of Chinara and Adaku.

d Do you think Chinara or Adaku is better at mental maths? Explain your answer.

e In each test there was a total possible score of 10. Write Adaku's total score for the five tests as a:

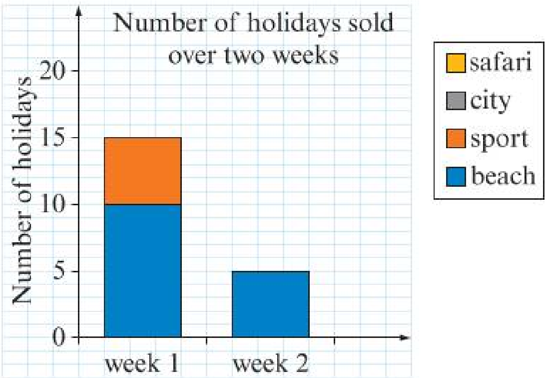
i fraction

ii percentage

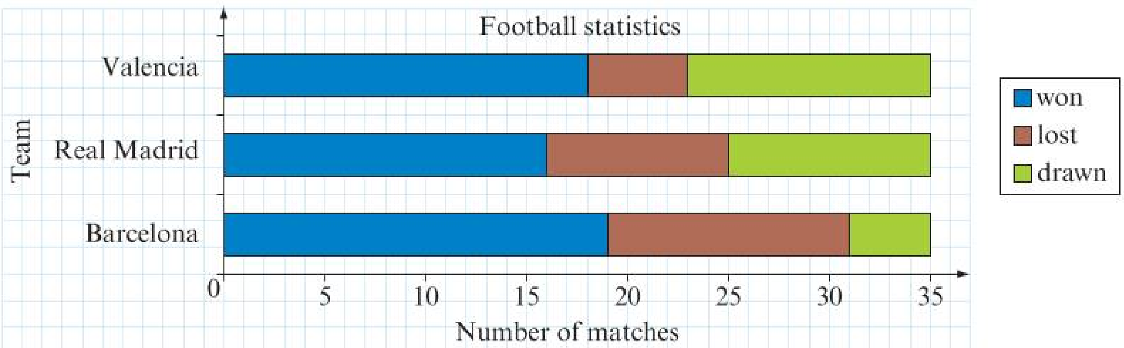
7 This two-way table shows the number of holidays sold by a travel agency in a period of two weeks.

	Type of holiday			
	beach	sport	city	safari
Week 1	10	5	4	1
Week 2	5	1	8	2

Copy the compound bar chart shown and use the information in the two-way table to complete the chart.



8 The compound bar chart shows the number of football matches won, lost and drawn by three teams.



Copy this two-way table and use the information in the chart to complete the table.

	Won	Lost	Drawn	Total
Barcelona				
Real Madrid				
Valencia				
Total				

9 This two-way table shows the number of cakes sold by a café one Saturday and Sunday.

	Type of cake	
	chocolate	vanilla
Saturday	18	7
Sunday	9	14

a i Draw a dual bar chart to show this information.
ii Make one comment on what your chart shows.

b i Draw a compound bar chart to show this information.
ii Make one comment on what your chart shows.

c Which chart do you think is the best to use to display this information? Give reasons for your answer.

Summary checklist

- ☐ I can draw and interpret dual bar charts.
- ☐ I can draw and interpret compound bar charts.

› 16.3 Pie charts and waffle diagrams

In this section you will …

- draw and interpret pie charts and waffle diagrams.

Key words

label
pie chart
proportions
sector
waffle diagram

You can use a pie chart to display data showing how an amount is divided or shared. It shows **proportions**, not actual amounts.

You draw a pie chart as a circle divided into sections called sectors. The angles at the centres of all the sectors add up to 360°. When you draw a pie chart, you must make sure that you label each sector and draw the angles accurately.

Worked example 16.3

a Ninety people were asked what type of holiday they had last year. The table shows the results.

i Draw a pie chart to represent the data.

ii What percentage of the people went on a beach holiday?

Type of holiday	Number of people
activity	32
beach	27
city break	24
other	7

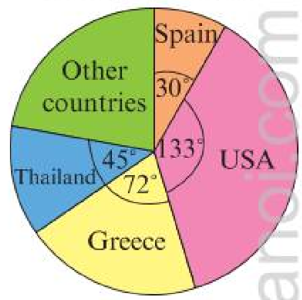
b The pie chart shows where the 90 people went on holiday last year.

i What fraction of the population went to Spain?

ii What percentage of the population went to Greece?

iii How many people went to other countries?

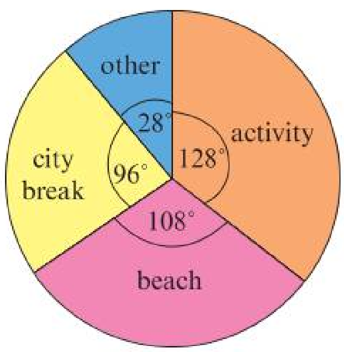
Holiday destination



Answer

a i

Type of holiday



First, work out the number of degrees per person:

$360° \div 90$ people $= 4°$ per person

Now work out the number of degrees for each sector:

Activity: $32 \times 4° = 128°$ Beach: $27 \times 4° = 108°$

City break: $24 \times 4° = 96°$ Other: $7 \times 4° = 28°$

Check that the total of all the sectors is 360°:

$128° + 108° + 96° + 28° = 360°$ ✓

Draw the pie chart. Remember to use a protractor to measure each sector accurately. Give the pie chart a title and label each sector.

ii $\frac{27}{90} \times 100 = 30\%$

27 out of the 90 people went on a beach holiday.

The pie chart also shows that the beach sector is 108° which is 30% of a full circle.

Continued

b **i** $\frac{30}{360} = \frac{1}{12}$	30° out of 360° represents Spain. Cancel the fraction to its simplest form.
ii $\frac{72}{360} = \frac{1}{5}, \frac{1}{5} \quad \frac{2}{10} = 20\%$	72° out of 360° represents Greece. Cancel the fraction to its simplest form. Change the fraction to a percentage.
iii $30 + 133 + 72 + 45$ $= 280°$	Add up the degrees that are shown for the four countries.
$360 - 280 = 80°$	Subtract this total from 360° to find out how many degrees are left.
$\frac{80}{360} = \frac{2}{9}$	80° out of 360° is for other countries. Cancel the fraction to its simplest form.
$\frac{2}{9} \times 90 = 20$ people	Multiply the fraction by 90 to work out the number of people.

Exercise 16.3

1 The table shows the number of different makes of car in a car park.

Make of car	Frequency
Ford	12
Vauxhall	18
Toyota	10
Nissan	20

a Copy and complete the calculations below to work out the number of degrees for each sector of a pie chart, to show the information given in the table.

> *Total number of cars* = 12 + 18 + 10 + 20 = □ *cars*
> *Number of degrees per car* = 360 ÷ □ = □°
> *Number of degrees for each sector:*
> *Ford* = 12 × □ = □°   *Vauxhall* = 18 × □ = □°
> *Toyota* = 10 × □ = □°   *Nissan* = 20 × □ = □°

b Draw a pie chart to show the information in the table.
Remember to label each sector and to give the pie chart a title.

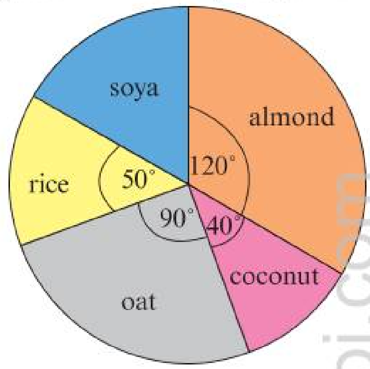
2 A group of 40 people are asked which type of music they prefer. The table shows the results.

Draw a pie chart to show the information in the table.

Type of music	Frequency
soul	5
classical	20
pop	8
other	7

3 A supermarket sells five types of milk made from plants. The pie chart shows the proportion of the different plant milks the supermarket sold one day.

Types of milk sold by a supermarket



a Which milk was the most popular?

b What fraction of the different plant milks sold was almond?

c What percentage of the different plant milks sold was oat?

d Altogether, the supermarket sold 180 litres on this day. How many litres of soya milk was sold on this day?

Think like a mathematician

4 Work with a partner or in a small group to complete this question.

Alexi asked some people their favourite type of film and worked out the number of degrees for each sector of a pie chart.

The table shows his results, but some of the numbers are covered with spilt orange juice.

Favourite type of film	Frequency	Number of degrees
action	2	40
romantic	7	
science fiction		80
comedy		100

a Draw a pie chart to show the information given in the table.

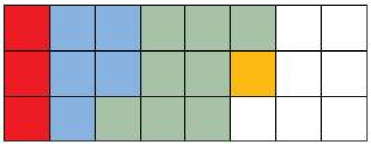
b Discuss with other groups in your class the answer to these questions.

  i Did you need to work out the missing frequencies for science fiction and comedy to be able to draw the pie chart?

  ii How can you work out the missing frequencies for science fiction and comedy?

  iii How many people did Alexi ask?

5 The **waffle diagram** shows the colours of the cars in a school's staff car park.



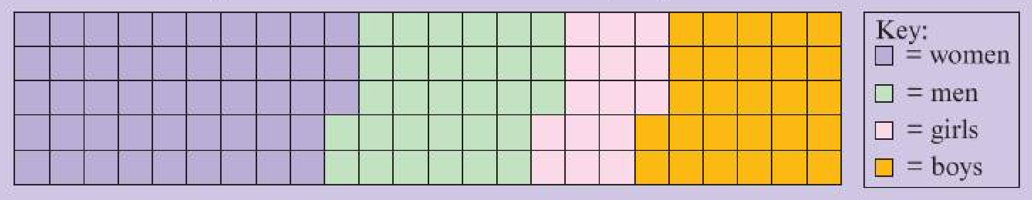
a Copy the table and use the waffle diagram to complete it, showing the number of each colour car.

Colour of car	Number of cars
red	3
blue	
green	
yellow	
white	

b Draw a pie chart to show the information given in the completed table.

Think like a mathematician

6 Work with a partner or in a small group to answer this question.
The waffle diagram shows the number of people at a tennis tournament.

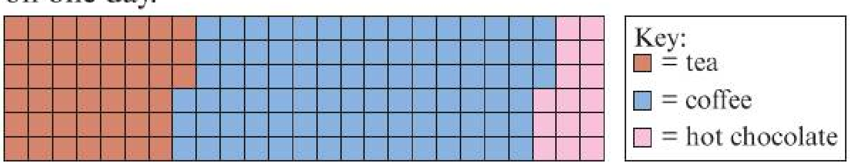


a Draw a pie chart to show the information given in the waffle diagram.

b Discuss with other groups in your class the answer to these questions:

- **i** What method did you use to work out the angles for the pie chart?
- **ii** Will your method work in general, with any number of squares in the waffle diagram?
- **iii** Now that you have compared your method with those of other learners in the class, what do you think is the best method to use? Explain why.

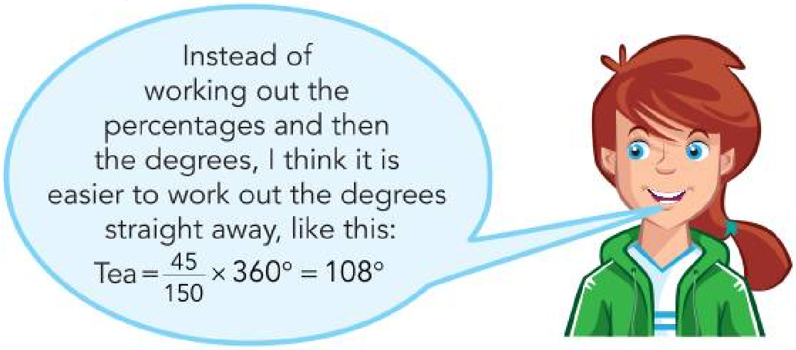
7 The waffle diagram shows the number of hot drinks sold in a café on one day.



a Copy the table and use the diagram to complete it.

Hot drink	Number of drinks	Percentage of total	Number of degrees
tea	45	$\frac{45}{150} \times 100 = 30\%$	30% of $360 = 108°$
coffee			
hot chocolate			
Total	150	100%	360°

b Sofia says:



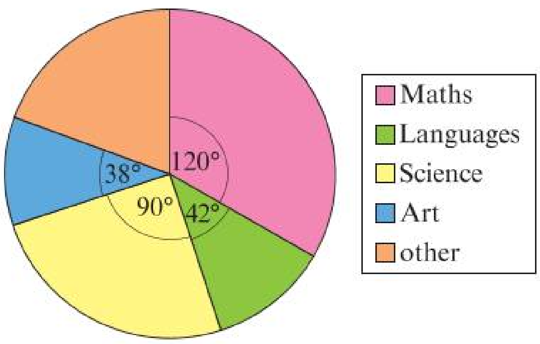
Do you agree with Sofia or would you rather work out the percentages and then the degrees? Explain why.

c Draw a pie chart to show the information given in the waffle diagram.

8 The pie chart shows the results of a survey of students' favourite subject. 180 students chose Maths.

Show that 105 students chose 'other'.

Favourite subject



Summary checklist

☐ I can draw and interpret pie charts and waffle diagrams.

16.4 Infographics

In this section you will …

- draw and interpret infographics.

Key word

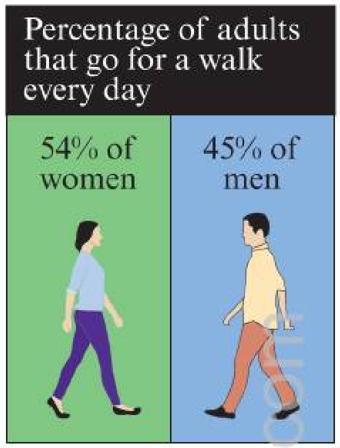
infographic

The word **infographic** is short for 'information graphic'.

An infographic is a visual representation of information or data.

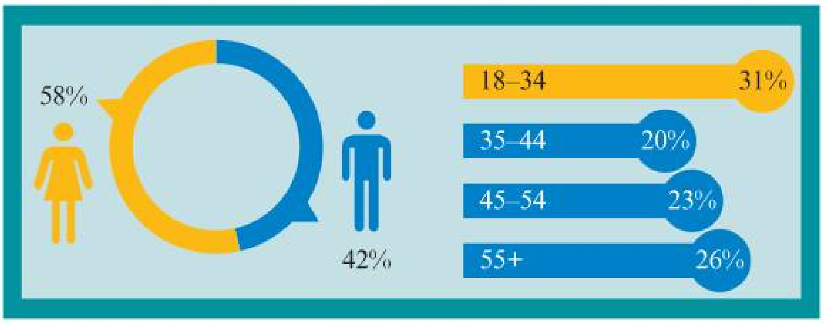
An infographic shows information quickly and clearly.

For example, this infographic clearly shows that over a half (50%) of women go for a walk every day, whereas less than a half of men go for a walk every day.



Worked example 16.4

This infographic shows the ages of the adult men and women living in a village.



a What percentage of the adults are women?

b What percentage of the adults are 55 years old or older?

c In which age group is the highest percentage of adults?

d There are 650 adults in the village. How many of them are 35–44 years old?

Answer

a 58%

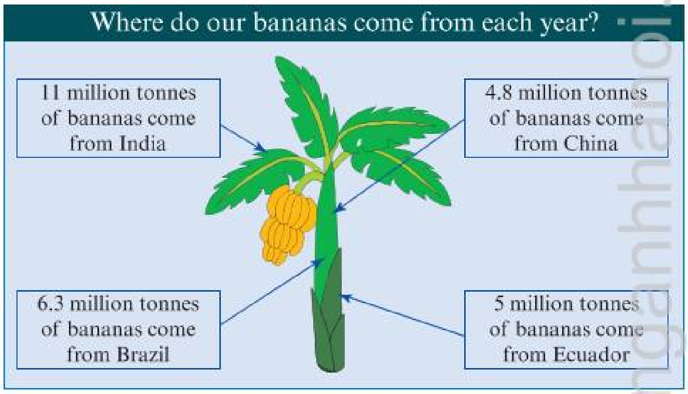
The chart on the left of the infographic shows the percentages of adult men and women. It shows that 58% of the adults are women and 42% are men.

Continued

b	26%	The chart on the right of the infographic shows the age percentages. The bottom bar shows that 26% of the adults are in the age group 55+.
c	18–34 years old	The longest bar in the age chart is highlighted in yellow. This shows that the 18–34 age group has the highest percentage of adults; that is, 31%.
d	$10\% = 65$	The chart shows that 20% of the adults in the village are 35–44 years old.
	$20\% = 2 \times 65$	First, find 10% of 650. Then multiply 65 by 2 to find 20%.
	$= 130$	130 adults in the village are 35–44 years old.

Exercise 16.4

1 This infographic shows the mass of bananas produced by different countries.



a Which country produces the most bananas?

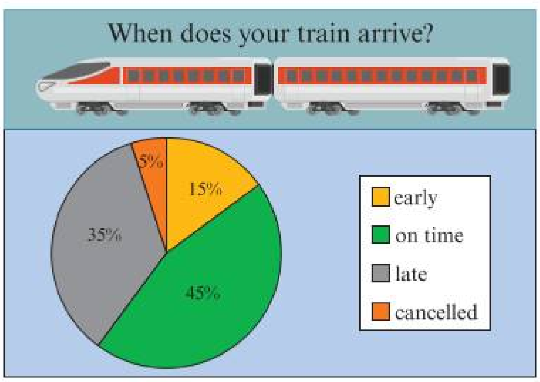
b What mass of bananas does China produce?

c Copy and complete this sentence:

India produces ___ million tonnes more bananas than Brazil.

2 Viktor sees this infographic at a train station.

It shows the percentage of trains that arrive early, on time or late or are cancelled.



a What percentage of trains arrive early?

b What percentage of trains are cancelled?

c The train station manager says: 'Over half of our trains arrive on time.'

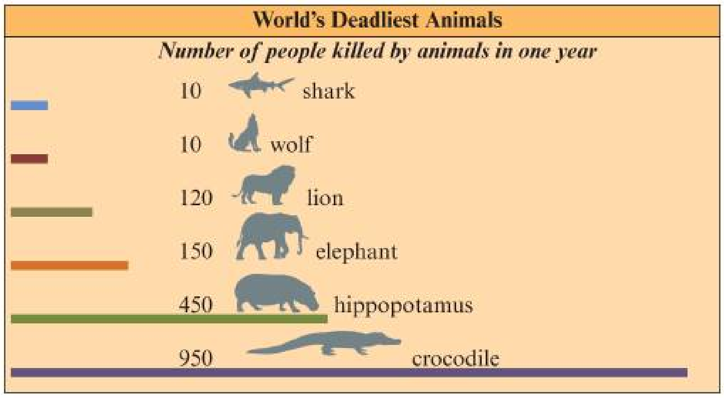
Is the manager correct? Explain your answer.

3 This infographic shows the number of people killed by animals in one year.

a How many people were killed by a lion?

b How many more people were killed by an elephant than by a wolf?

c Which animal killed the most number of people?



4 This infographic shows information about cars.

a What percentage of people:

i always wear their seat belt?

ii do not always wear their seat belt?

iii got their first car when they were 18 years old?

iv have never owned a car?

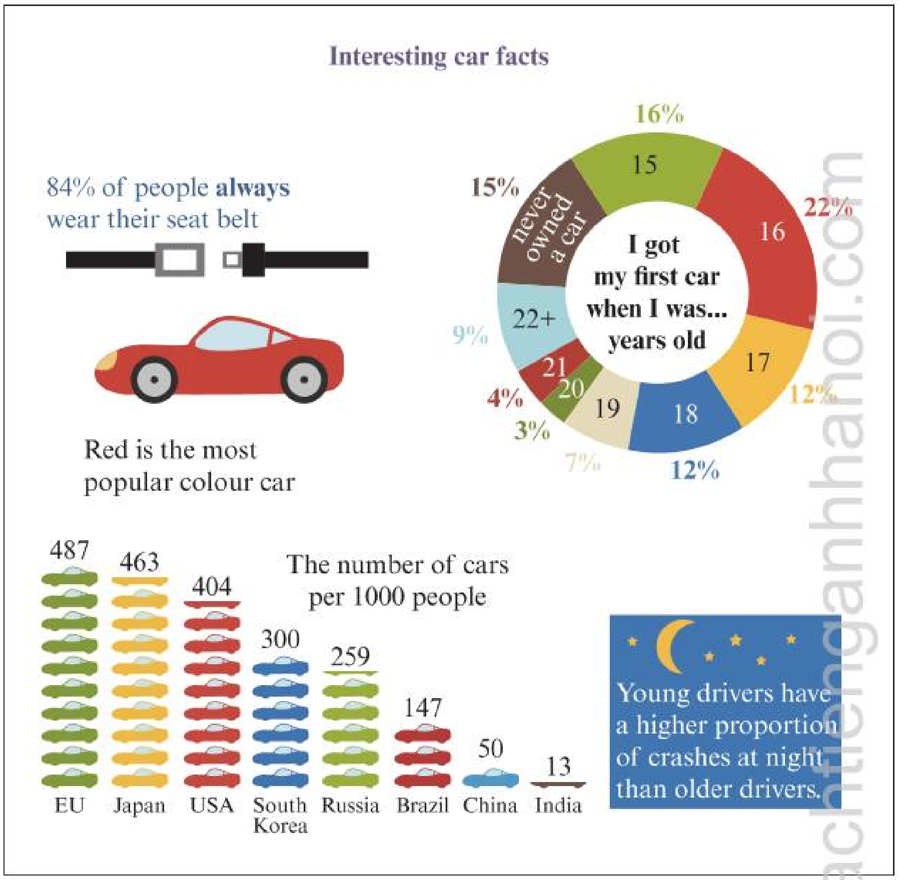
b At what age is it most popular for people to get their first car?

c Which country has 259 cars per 1000 people?

d Which country listed on the chart has the least number of cars per 1000 people?

e Copy and complete this sentence with the names of the correct countries:

____ has six times as many cars per 1000 people as ____.



5 This infographic shows the results of a survey.

The speech bubbles show the topics that people ask about online.

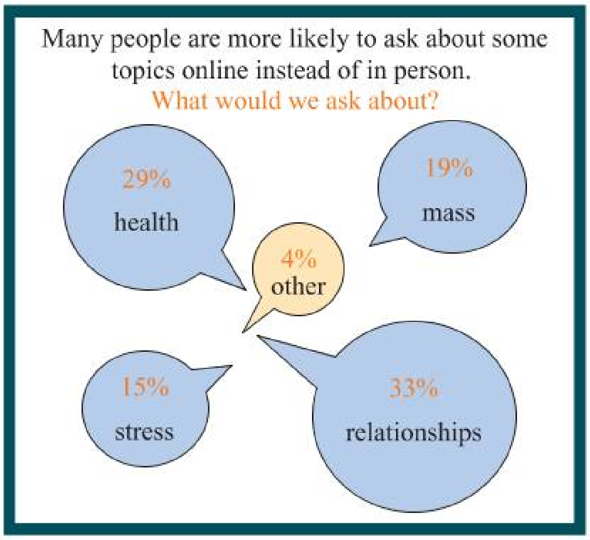
They also show the percentage of people that ask about each topic.

a What percentage of people ask about health?

b Which topic is asked about the most?

c i What do you notice about the size of the speech bubbles and the percentages?

ii Do you think it is a good idea to have speech bubbles of different sizes? Explain why.



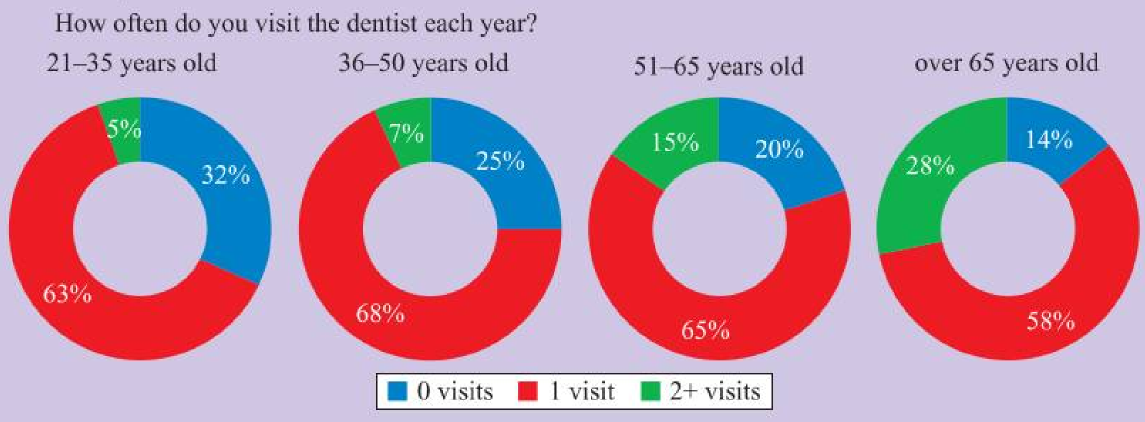
Think like a mathematician

6 Work with a partner or in a small group to discuss this question.

This infographic shows the number of times people of different ages visited the dentist in one town.

What do you notice about how the percentages change as the age groups get older?

Discuss possible reasons for these changes.

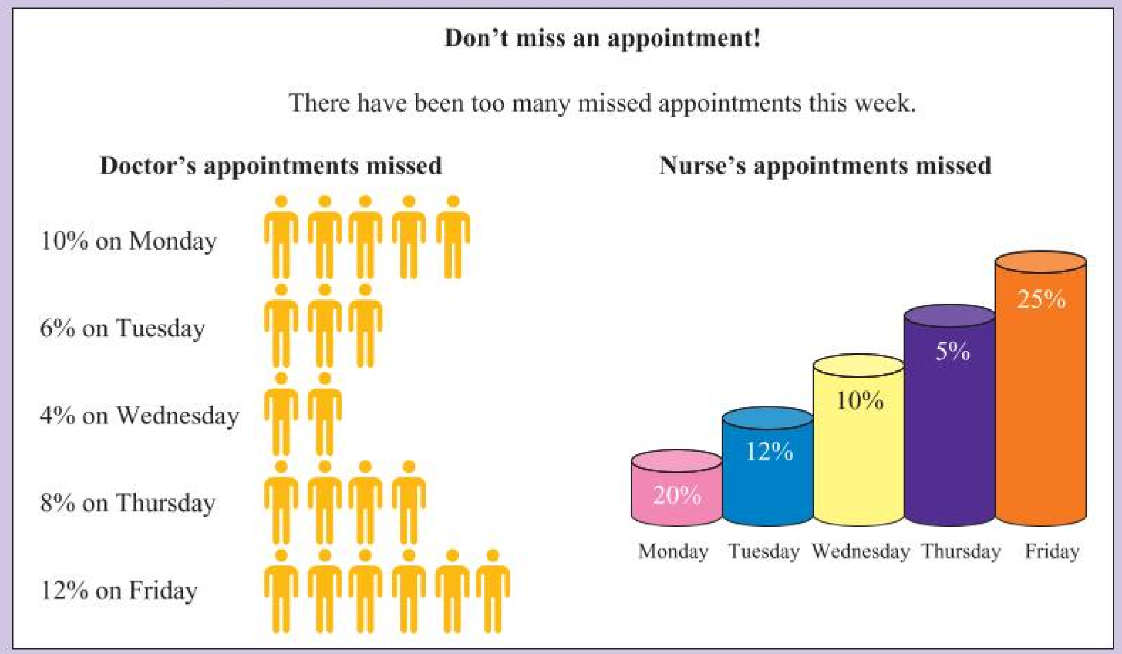


Think like a mathematician

7 Work with a partner or in a small group to discuss this question.
Adira works in a health centre. She wants to make an infographic to display the information given in this table.

Percentage of appointments missed to see the:	Monday	Tuesday	Wednesday	Thursday	Friday
doctor	10%	6%	4%	8%	12%
nurse	20%	12%	10%	5%	25%

This is the infographic Adira makes:



What do you think of Adira's infographic?
Do you think it shows the information in the table correctly?
How do you think you could improve the infographic?

Activity 16.3

Work with a partner to answer this question.

One hundred Year 7 students in a school are asked three questions about sport.
The questions and results tables are shown.

Question 1: Do you like to play sport?

	Yes	No	Total
Boys	34	16	50
Girls	38	12	50
Total	72	28	100

Question 2: What is your favourite sport?

	Cricket	Football	Hockey	Tennis	Other	Total
Boys	16	14	8	10	2	50
Girls	2	12	18	14	4	50
Total	18	26	26	24	6	100

Question 3: How many times do you play sport in a week?

	Number of times you play sport in a week				
	0	1	2	3+	Total
Boys	5	6	15	24	50
Girls	8	16	21	5	50
Total	13	22	36	29	100

Make an infographic to show the information given in the tables.
Display your infographic on a poster.
Look at the posters made by other learners.
Do you think their posters show the information correctly?
Critique their posters. Discuss what you like and dislike about their poster.
Now that you have seen other learners' posters, what do you think of your own poster?
Is there anything you could do to improve your poster?

Summary checklist

☐ I can draw and interpret infographics.

16.5 Representing data

In this section you will …

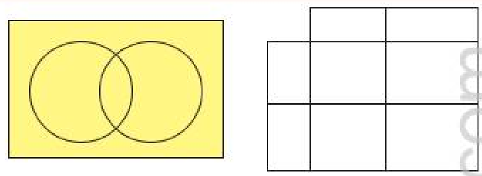
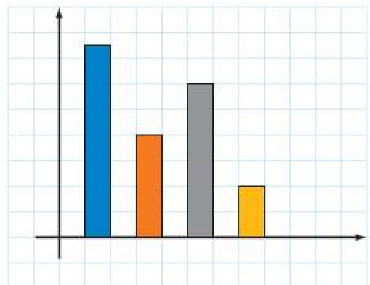
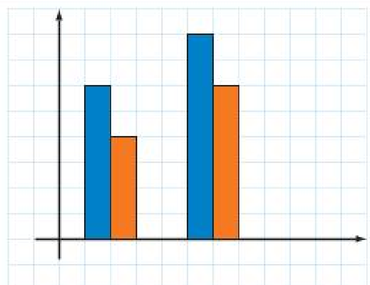
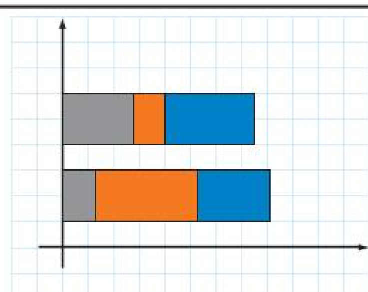
- choose how to represent data.

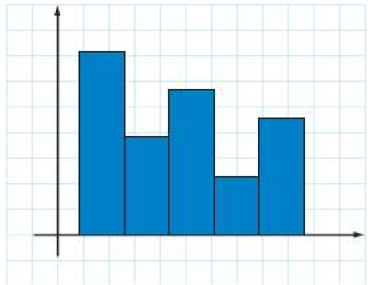
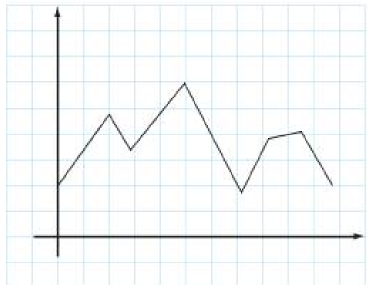
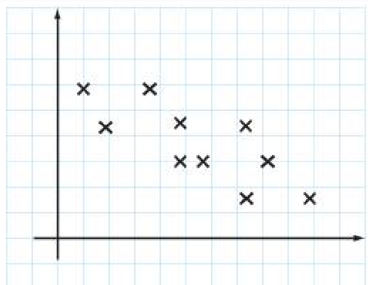
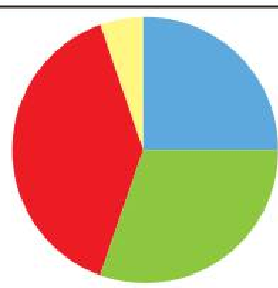
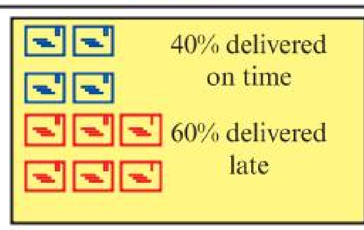
Key words

continuous data
discrete data
justify

When you represent data using a diagram, graph or chart, you must decide which type of diagram is best to use.

This table will help you decide.

Type of diagram/graph/chart	When to use it	What it looks like
Venn or Carroll diagram	When you want to sort data or objects into groups that have some common features.	
bar chart	When you want to compare **discrete data**.	
dual bar chart	When you want to compare two sets of discrete data.	
compound bar chart	When you want to combine two or more quantities into one bar in order to look at individual amounts and the total amounts.	

Type of diagram/graph/chart	When to use it	What it looks like
frequency diagram	When you want to compare **continuous data**.	
line graph	When you want to see how data changes over time.	
scatter graph	When you want to compare two sets of data points.	
pie chart	When you want to compare the proportions of each sector with the whole amount.	
infographic	When you want to show some information in a quick way that is easy to understand.	

Worked example 16.5

Look at the following sets of data. Which type of diagram, graph or chart do you think is best to use to display the data? Justify your choice.

a The depth of snow at a ski resort at the end of every week.

b The maths test scores and science test scores of 20 students.

c The number of gold, silver and bronze medals won by two countries at the Olympic Games.

Answer

a	line graph	You can see how the depth of snow changes over time.
b	scatter graph	You can plot the maths test score against the science test score for each student and see if there is any connection (correlation).
c	dual bar chart	You can draw bars for the number of gold, silver and bronze medals for each country side by side. You can then easily compare the heights of the bars.

Exercise 16.5

1 Look at the following sets of data. Which type of diagram, graph or chart do you think is best to use to display the data? Justify your choice.

  a The number of icecreams sold in a shop each day for one week.

  b The height and the shoe size of 20 students.

  c The total number of cakes, sandwiches and drinks sold in a café on two different days.

  d The proportion of students that travel to college by car, bus, bicycle or on foot.

2 Ten students are asked which sports they play out of a choice of football, hockey and cricket.

The ten students are: Aaron, Brad, Chloe, Dian, Eralia, Fayard, Guang, Harper, Irine and Jengo.

The students that play football are: Aaron, Chloe, Eralia, Irine and Jengo.

The students that play hockey are: Brad, Chloe, Eralia, Fayard and Harper.

The students that play cricket are: Chloe, Guang, Harper and Jengo.

  a Draw a diagram, graph or chart to represent the data.

  b Justify your choice.

  c Make one comment about what information your diagram, graph or chart shows.

3 The table shows how the amount of air in a scuba tank changes during a dive.

Amount of air (litres)	15	14	12	8	7	4
Time (minutes)	0	10	20	30	40	50

a Draw a diagram, graph or chart to represent the data.

b Justify your choice.

c Make one comment about what information your diagram, graph or chart shows.

Tip

A scuba tank is a metal cylinder used to store air for a diver to use under water.

Think like a mathematician

4 The table shows the number of dentist appointments for two dentists on one day.

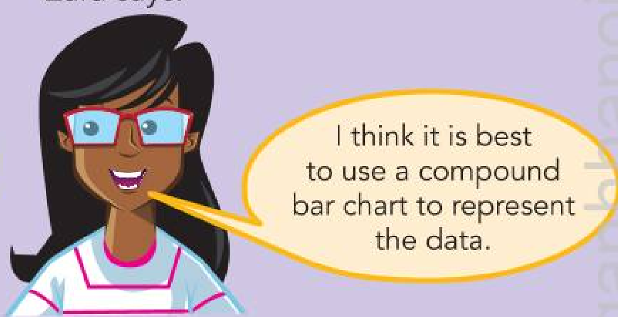
Type of appointment	check-up	filling	extraction
Dentist A	14	8	2
Dentist B	7	10	3

Tip

An extraction is when the dentist removes a tooth.

Arun says:

Zara says:



Discuss the answers to these questions in a small group, and then with other groups in the class.

a Using a dual bar chart:

i What parts of the data are easier to compare?

ii What parts of the data are more difficult to compare?

b Using a compound bar chart:

i What parts of the data are easier to compare?

ii What parts of the data are more difficult to compare?

c Copy these statements and complete them using either 'dual bar chart' or 'compound bar chart'.

i In general, to compare total amounts it is best to use a ....

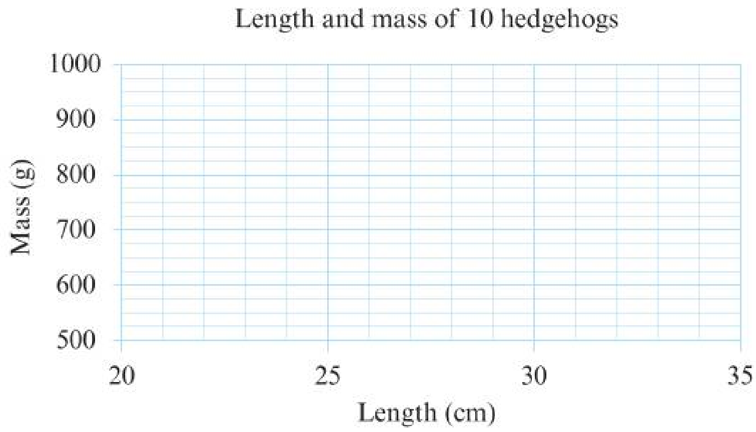
ii In general, to compare individual amounts it is best to use a ....

5 The table shows the length and mass of 10 hedgehogs.

Length (cm)	26	31	34	32	22	31	30	24	33	27
Mass (g)	625	800	975	875	525	850	750	550	950	700

a Explain why a scatter graph is the best way to represent this data.

b Copy the grid and draw a scatter graph to show this data.



c Draw a line of best fit on your scatter graph.

d Use your line of best fit to estimate the length of a hedgehog with a mass of 725 g.

6 Javed records the distances he cycled each day in May.
This frequency table shows his results.

Distance in km	Frequency
0 – 5	4
5 – 10	7
10 – 15	14
15 – 20	6

a Draw a diagram, graph or chart to represent the data.

b Justify your choice.

c Make one comment about what information your diagram, graph or chart shows.

Tip

If Javed cycled 5 km he would record this in the 5 – 10 km group. If he cycled 10 km he would record this in the 10 – 15 km group, etc.

Activity 16.4

Work with a partner to answer this question.
Dinesh carries out a survey about holidays.
He asks 40 people these four questions.
Question 1: Who plans your holiday?
Question 2: What type of holiday do you prefer?

Continued

Question 3: What is the length of your holiday?
Question 4: How do you travel to your holiday?

Here are the results of his survey.

Question 1: 55% women 45% men

Question 2:

Type of holiday	beach	city	activity	camping
Frequency	9	12	15	4

Question 3:

Length of holiday in days	0 – 5	5 – 10	10 – 15	15 – 20	20+
Frequency	3	18	10	7	2

Question 4: Eight people always travel by car; six people always travel by plane; three people always travel by bus; seven people travel by car or plane; two people travel by plane or bus; and four people travel by car or bus

Discuss and decide which diagrams, graphs or charts are best to use to represent the data.
Display all the information on a poster.
Look at the posters made by other learners in your class.
Do you think their posters show all the information correctly?
Critique their posters. Discuss what you like and dislike about their posters.
Now that you have seen other learners' posters, what do you think of your own poster?
Is there anything you could do to improve your poster?

7 Eight people were asked to run 100 m, and their time was recorded in seconds. They were also given a spelling test of ten words. The table shows their results.

Time to run 100 m (seconds)	16	18	20	22	19	23	24	17
Spelling test result (out of 10)	7	10	6	4	3	9	7	2

a Draw a diagram, graph or chart to represent the data.

b Justify your choice.

c Make one comment about what information your diagram, graph or chart shows.

Summary checklist

☐ I can choose how to represent data.

16.6 Using statistics

In this section you will …

- use mode, median, mean and range to describe sets of data.

Key words

bimodal
mean
median
mode
range
statistical measures

You already know how to work out some **statistical measures**, such as the mode, median, mean and range.

The mode is the most common value or number.

If a set of data has two modes, it is called **bimodal**.

The median is the middle value when they are listed in order of increasing size.

The mean is the sum of all the values divided by the number of values.

The **range** is the largest value minus the smallest value.

In a real situation, you must decide which measure to use.

If you want to measure how spread out a set of measurements is, the range is the most useful statistic.

If you want to find a representative measurement, you need an average.

But should the average be the mode, the median or the mean? Which average to use depends on the particular situation.

Here is a summary to help you decide which average to choose.

- Choose the mode when you want to know which is the most commonly occurring number or numbers..
- The median is the middle value when the data values are put in order of increasing size. Half the numbers are greater than the median and half the numbers are less than the median.
- The mean depends on every value. When you change one number, you change the mean.

Worked example 16.6

Here are the ages, in years, of the players in a football team.

16, 17, 18, 18, 19, 20, 20, 21, 21, 32, 41

a Work out the mode, median and mean age.

b Which average best represents the data? Give a reason for your choice of average.

c Work out the range in ages of the players.

d A different football team has a range in ages of 14 years. Which team, the first or the second, has more variation in the ages of the players?

Answer

a Mode = 18, 20 and 21 years old.

There are three numbers that appear the most often, occurring two times each.

16, 17, 18, 18, 19, 20, 20, 21, 21, 32, 41

There are 11 players. Their ages are already in order of increasing size in the list. The age in the middle is 20 years old.

Median = 20 years old

16 + 17 + 18 + 18 + 19 + 20 + 20 + 21 + 21 + 32 + 41 = 243

243 ÷ 11 = 22.1

The total of all the ages is 243.

Mean = 22.1 years old

There are 11 players, so 243 ÷ 11 gives a mean of 22.1 years old.

b The median is the best average to use in this case.

The mode is not a good choice because there are three of them.

The mean is affected by the two much older members of the team. Only two players are older than the mean, but nine players are younger than the mean.

There are five players younger than the median and four players older than the median, so it is in the middle of the data.

c 41 – 16 = 25

Range = largest value – smallest value

Range = 25 years

Oldest player is 41 and youngest player is 16.

d Range for first team = 25, range for second team = 14

25 > 14, so the ages of the first team are more spread out, or varied, than the ages of the second team.

The first team has more variation in ages.

Exercise 16.6

1 Zaralia works for 20 days each month. She records the time she waits in line for lunch each day in May.

Here are the times, in minutes.

2	5	3	8	5	2	10	7	8	8
4	7	2	2	3	6	10	3	4	7

a Work out the:

i mode ii median iii mean time

b Which average best represents the data? Give a reason for your choice of average.

c Work out the range in Zaralia's waiting times.

d In June, Zaralia's range in waiting times is 3 minutes. In which month, May or June, is there more variation in her waiting times?

Tip

Start by writing the list of times in order of size from smallest to largest.

Range = largest value – smallest value

Tip

The month with the larger range has more variation in the waiting times.

2 These are the ages, in years, of the members of a fitness class.

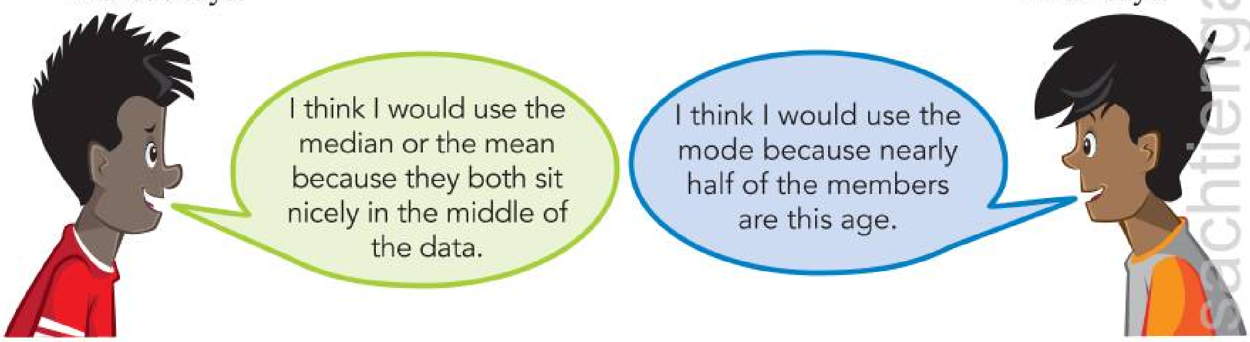
57	56	51	59	51	56	58	58	51	53	50	51	54	51

a Work out the:

i mode ii median iii mean age

b Marcus and Arun discuss which average best represents the data.

Marcus says: Arun says:



What do you think? Which average would you use? Give a reason for your choice.

c Work out the range in ages of the members of the fitness class.

d A different fitness class has a range in ages of 16 years. Which fitness class, the first or the second, has less variation in ages of the members?

Think like a mathematician

3 Work with a partner to answer this question.

The table shows the number of days of rain in the first week of May in a town, recorded over 35 years.

Days of rain	0	1	2	3	4	5	6	7
Frequency	13	7	5	2	0	3	2	3

a Copy and complete the workings to find the mode, median and mean number of days of rain.

Mode: The greatest frequency in the table is 13. So the mode is ☐ days of rain.

Median: There are 35 years of data. $\frac{35+1}{2} = \frac{36}{2} = 18$, so the 18th year is the one in the middle when the years are listed from 0 days of rain to 7 days of rain.

The first 13 years have 0 days of rain, the next 7 years have 1 day of rain.

So the 14th to 20th years have 1 day of rain. This means the median is ☐ day of rain.

Mean: First, work out the total number of days of rain over the 35-year period.

Days of rain (d)	Frequency (f)	$d \times f$
0	13	$0 \times 13 = 0$
1	7	$1 \times 7 = 7$
2	5	$2 \times 5 = 10$
3	2	$3 \times 2 = 6$
4	0	$4 \times 0 = 0$
5	3	$5 \times 3 =$
6	2	$6 \times 2 =$
7	3	$7 \times 3 =$
		Total =

$$\text{Mean} = \frac{\text{total number of days of rain}}{\text{total number of years}} = \frac{\square}{35} = \square \text{ days of rain}$$

Continued

b Petra thinks that the mean best represents the data. What do you think? Which average would you use? Give a reason for your choice.

c Describe how you used algebra to help you work out the answers in part **a**.

4 This table shows the number of men's belts sold in a store during one month.

Length (cm)	80	85	90	95	100	105	110	115
Frequency	6	16	28	41	17	18	10	13

Use an appropriate average to decide which size of belt the store owner should always try to keep in stock.

5 Arun records the number of people in 60 passing cars.

Here are his results.

Number of people	1	2	3	4	5	6
Frequency	28		3	6	2	1

a Find the missing frequency.

b Arun says:

Explain the mistake that Arun has made.

c How can you tell, by looking at the table, that the median is 2 people per car?

d Arun works out that the mean is 1.95 people per car. Show that Arun is correct.

e Which average best represents the data? Give a reason for your choice of average.

Tip

Remember: You can say 'the <u>mode</u> is 28' or 'the <u>modal value</u> is 28'. They mean the same thing.

6 A test has ten questions. A total of 120 students take the test.
The table shows the students' test scores.

Questions answered correctly	4	5	6	7	8	9	10
Frequency	3	5	12	13	17	30	40

a How many students scored:
  i more than the median?
  ii more than the mode?
  iii more than the mean?

b Which average best represents the data? Give a reason for your choice of average.

Think like a mathematician

7 Work with a partner or in a small group to answer this question.

You are going to roll two dice and add the numbers on the dice to give the score.

For example, if you roll these numbers, you get a score of 7.

a What is the smallest score you can get?

b What is the largest score you can get?

You are going to roll the dice 40 times.

c Draw a table ready to record the scores you will get. Your table needs to have a 'Tally' column and a 'Frequency' column.

d Now roll the dice 40 times and record all your scores. When you have finished, make sure your frequency column adds up to 40.

e For your set of data, work out the:
  i mode ii median iii mean score

f Which average best represents your data? Give a reason for your choice of average.

g Compare your data and averages with those of other learners in your class. Do you have different averages? Do you have the same averages? Discuss why.

Summary checklist

☐ I can use mode, median, mean and range to describe sets of data.

Check your progress

1 The two-way table shows the favourite colour of the students in Mr Byrd's class.

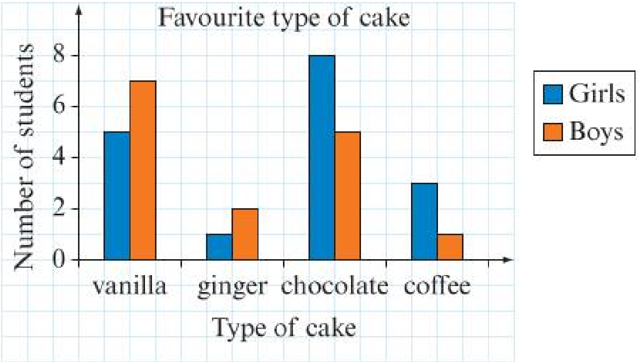
	Red	Blue	Green	Other colour	Total
Girls	9		3	2	
Boys	4	8			16
Total			4		34

a Copy and complete the table.

b How many of the boys chose green as their favourite colour?

c How many of the students did not choose red, blue or green as their favourite colour?

2 The dual bar chart shows the favourite type of cake of the students in class 7P.



a How many of the girls chose vanilla as their favourite cake?

b How many of the boys chose ginger as their favourite cake?

c How many more girls than boys chose coffee as their favourite cake?

d How many students are in class 7P?

e Which type of cake was chosen most by the:

i girls? ii boys? iii whole class?

3 Look at the following sets of data. Which type of diagram, graph or chart do you think is best to use to display the data? Justify your choice.

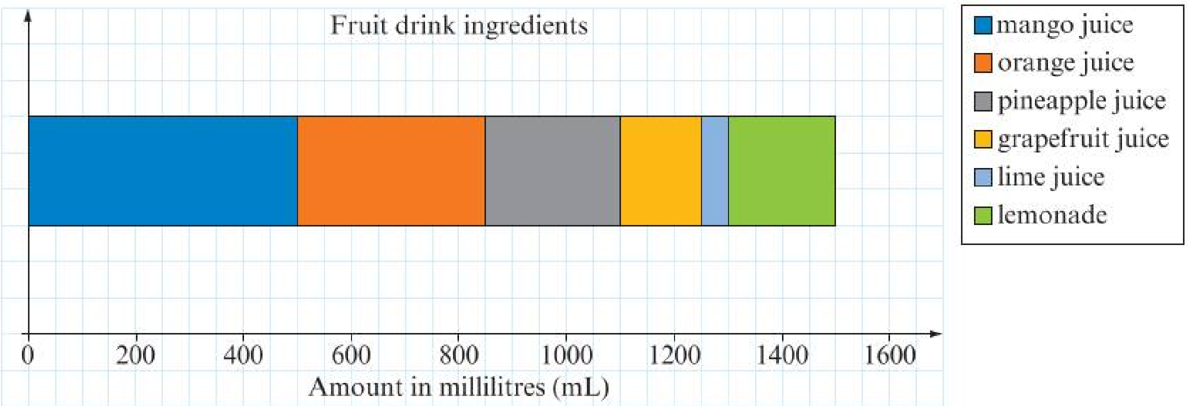
a The proportion of different flavour icecreams sold in a shop one day.

b The sales of DVDs each month for a year.

c The number of people going to hospital with a broken arm each day for one week.

d The heights of 100 trees in a forest.

4 The compound bar chart shows the ingredients in 1500 mL of fruit drink.



- **a** Which is the most used ingredient?
- **b** Which is the least used ingredient?
- **c** How many millilitres of grapefruit juice are used in the fruit drink?
- **d** How many more millilitres of orange juice than pineapple juice are used in the fruit drink?
- **e** What fraction of the fruit drink is mango juice? Give your answer in its simplest form.

5 A group of 60 people are asked which type of transport they prefer to use. The table shows the results. Draw a pie chart to show the information given in the table.

Type of transport	Frequency
car	15
bus	22
train	5
bicycle	18

6 A garage has eight used cars for sale.

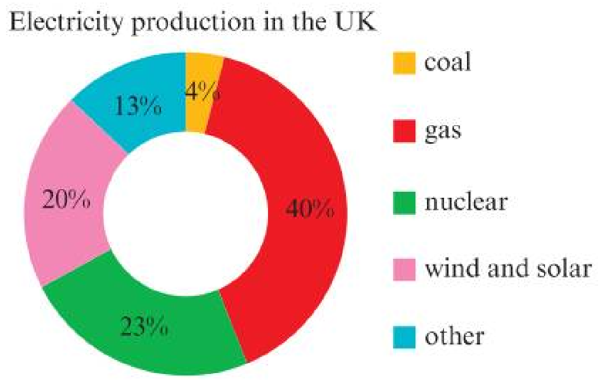
The table shows the price of each car and the number of kilometres it has already travelled.

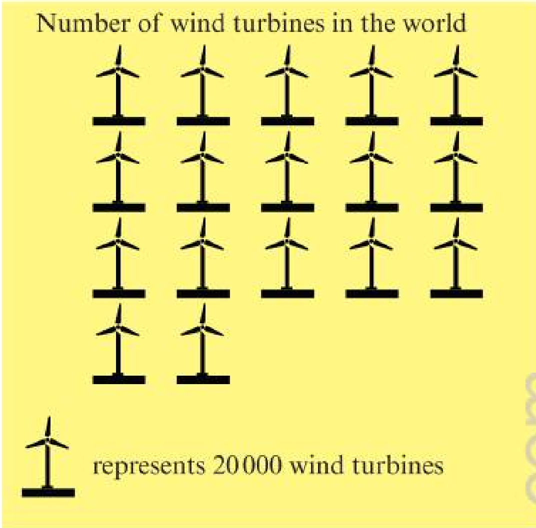
Price ($)	3800	5800	3200	6400	5200	6800	6200	4600
Distance (km)	52000	30000	58000	20000	32000	10000	28000	45000

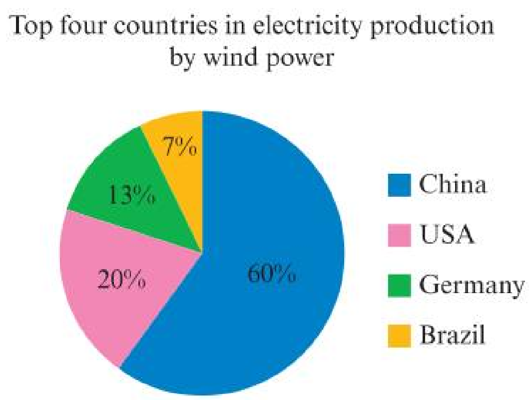
- **a** Draw a diagram, graph or chart to represent this data.
- **b** Justify your choice.
- **c** Make one comment about what your diagram, graph or chart shows you.

7 This infographic shows information about electricity production in the United Kingdom.

Where does our electricity come from?







a What percentage of electricity production in the UK comes from:

i gas? **ii** wind and solar?

b How many wind turbines are there in the world?

c Copy and complete this sentence with the names of the correct countries:

'__________ produces three times as much electricity from wind power as __________.'

8 These are the annual salaries (to the nearest thousand dollars) of each employee of a small company.

23 000	26 000	26 000	29 000	29 000	30 000	30 000	32 000	46 000	56 000

a Work out the mode, median and mean salary.

b Which average best represents the data? Give a reason for your choice of average.

Glossary and index

advantages	the good points	49
anticlockwise	turning in the opposite direction as the hands of a clock	299
area	the amount of space covered by a flat shape	33
axes	number lines drawn in a coordinate grid	233
bimodal	when a set of data has two modes	370
brackets	symbols used to enclose items that are to be seen as a single expression	14
cancel	to find an equivalent fraction by dividing both the numerator and denominator by a common factor	143
categorical data	data that is descriptive, rather than numerical	126
centre of rotation	the point that remains still when rotating a shape	299
chord	a straight line that starts and finishes on the circumference of a circle	171
classify	organise into groups of the same type	37
clockwise	turning in the same direction as the hands of a clock	278
coefficient	a number in front of a variable in an algebraic expression; the coefficient multiplies the variable	34
collecting like terms	gathering, by addition and subtraction, all like terms	43
common denominator	a denominator that is the same in all the fractions to be added or subtracted	139
common factor	a number that is a factor of two different numbers; e.g. 3 is a common factor of 15 and 24	19
common multiple	a number that is a multiple of two (or more) different numbers; 24 is a common multiple of 2 and 3	17
compare	to notice what is the same and what is different between two things	13
compound bar chart	a chart that has different sets of data combined into one bar	345
compound shape	a shape made up from simpler shapes	314
congruent	identical in shape and size	177
conjecture	to form a mathematical question or idea	170
consecutive	two numbers are consecutive if they are next to each other when written in order	28
consecutive terms	terms that are next to each other in a sequence	192

constant	a number on its own (with no variable)	34
continuous data	data that can take any value within a given range	126
conversion factor	a multiplier for converting from one unit to another	312
coordinates	two numbers used to identify a point, written in brackets; e.g. (4, 6)	225
corresponding angles	the angles that are the same in relation to two congruent shapes	177
corresponding sides	the sides that are the same in relation to two congruent shapes	177
corresponds	in the same relative position	290
cube number	the result of multiplying three 'lots' of the same whole number together; 125 is a cube number equal to $5^3 = 5 \times 5 \times 5 = 125$	27
cube root	the number that produces the given number when three 'lots' of the number are multiplied together; the cube root of 125 is $\sqrt[3]{125} = 5$	27
cubic centimetre (cm^3)	unit of volume; volume of a cube of side 1 centimetre	324
cubic metre (m^3)	unit of volume; volume of a cube of side 1 metre	324
cubic millimetre (mm^3)	unit of volume; volume of a cube of side 1 millimetre	324
data	facts, numbers or measurements collected about someone or something, used for reference or analysis and to produce useful information	124
decimal	a number in the counting system based on 10; the part before the decimal point is a whole number, and the part after the decimal point is a decimal fraction	64
decimal number	a number with at least one digit after the decimal point	64
decimal part	the part of the decimal number to the right of the decimal point	79
degree of accuracy	the level of accuracy in any rounding	70
denominator	the number below the line in a fraction	47
derive	construct a formula or work out an answer	38
different orientation	positioned in a different direction	178
digits	the individual numbers 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9.	18
dimensions	the measurements of a shape, such as length, height and width	315
direct proportion	two quantities are in direct proportion when the ratio of one to the other stays the same	257

disadvantages	the bad points	49
discrete data	data that can take specific, exact values only	125
distance between two points	the length of a line segment that joins two points	103
divide	to split up into parts	14
divisible	one whole number is divisible by another whole number when it is a multiple of that other whole number	22
dual bar chart	a chart that has different sets of data presented side by side	345
enlargement	a transformation that increases the size of a shape to produce a mathematically similar shape	304
equally likely	equally likely outcomes or events have the same chance of happening. When you throw a dice, getting a 2, a 4 or a 6 are equally likely outcomes.	267
equation	two different mathematical expressions, both having the same value, separated by an equals sign (=)	34
equivalent	equivalent means same value but written differently; e.g. 2.25 is equivalent to $2\frac{1}{4}$	26
equivalent expression	an expression that means the same thing as another expression	37
equivalent fractions	fractions that have the same value; e.g. $\frac{3}{6}$ and $\frac{1}{2}$	97
estimation	approximation of an answer, based on a calculation that used rounded numbers	95
even chance	a probability of 0.5	263
event	an action that can have different outcomes; getting an even number when you throw a dice is an event	263
expand	multiply all parts of the expression inside the brackets by the term outside of the brackets	48
experiment	an activity to collect data	126
experimental probability	a probability estimated from data	272
expression	a collection of symbols representing numbers and mathematical operations, but not including an equals sign (=)	34
factor	a factor of a whole number will divide into the whole number without a remainder; e.g. 6 and 8 are factors of 24	9
fewest	the smallest number or the least number	205
fill in	write in the missing numbers	32
finite sequence	a sequence that does not continue forever	192
formula (plural formulae)	an equation that shows the relationship between two or more quantities	33

fourth	in position number 4	79
fraction	a part of a whole, such as $\frac{1}{4}$ or $\frac{2}{3}$	20
fractional part	the part of a mixed number that is a fraction and not a whole number	139
frequency table	a table that shows how many times an event occurs	339
front view (elevation)	a 2D drawing of a solid shape seen from the front	183
full turn	a turn of 360°	166
function	a relationship between two sets of numbers	206
function machine	a method of showing a function	206
graph	a line drawn on a coordinate grid	225
half as much	halved or divided by 2	257
hectares (ha)	a standard metric unit used to measure large areas; 1 ha = 10 000 square metres	316
highest common factor	the largest factor of two (or more) other numbers. You can abbreviate highest common factor to HCF.	19
horizontal line	a line that goes from left to right that is parallel to level ground	166
hundredths	the second digit after the decimal point in a decimal number	78
image	a shape after a transformation	279
improper fraction	a fraction in which the top number is larger than the bottom number	47
index (plural indices)	a number used to show a power; in 3^4, 4 is the index	26
inequality	a relationship between two expressions that are not equal	56
inequality symbols	symbols used to represent a range of numbers	56
infinite sequence	a sequence that continues forever	192
infographic	a visual representation of information or data	358
input	a number to be acted upon by a function	206
integer	the whole numbers; e.g. …, −3, −2, −1, 0, 1, 2, 3, …	9
intersect	two lines intersect if they meet or cross	109
inverse	the operation that has the opposite effect; the inverse of 'add 5' is 'subtract 5'	11
inverse calculation	a method of checking your answer by working backwards through the calculation	95
inverse operation	the operation that reverses the effect of another operation	11

irregular polygon	a polygon that doesn't have all sides and angles the same size	164
justify	give reasons to support your decision	12
label (verb)	to write what something (in this case, the sector) represents	113
like terms	terms containing the same letter(s)	43
likelihood	the probable chances of an event	262
likely	more than an even chance	262
line symmetry	a shape has line symmetry if you can draw a line that divides the shape into two parts, where each part is the mirror image of the other part	166
lowest common multiple	the smallest possible common multiple of two (or more) numbers; 24 is the lowest common multiple of 6 and 8. You can abbreviate lowest common multiple to LCM.	17
mapping diagram	a type of diagram that represents a function	206
maps	the process of changing an input number to an output number by the use of a function	206
mean	an average of a set of numbers, found by adding all the numbers and dividing the total by how many numbers there are in the set	124
median	the middle number when a set of numbers is put in order of increasing size	124
mentally/ using a mental method	work out in your head, without writing down your working	82
mirror line	a line dividing a diagram into two parts, each being a mirror image of the other	278
mixed number	a number expressed as the sum of a whole number and a proper fraction	140
mode	the most common number in a set of numbers	124
multiple	the result of multiplying a number by a positive integer; the first four multiples of 3 are 3, 6, 9 and 12	17
mutually exclusive	two outcomes are mutually exclusive when they cannot both happen at the same time	262
negative gradient	general trend when a graph slopes downwards from left to right	239
negative infinity	negative numbers going on forever	56
negative integers	the whole numbers less than zero: $-1, -2, -3, -4, \ldots$	10
net	a flat diagram that can be folded to form a 3D shape	330
***n*th term**	the general term of a sequence, where n represents the position number of the term	201

number line	a line used to show numbers in their correct position	9
numerator	the number above the line in a fraction	47
object	a shape before a transformation	279
once	occurs one time	167
one-step function machine	a function machine that has only one mathematical operation	206
open interval	a range of numbers that does not include its endpoints	56
opposite angles	angles opposite each other when two lines cross	109
order	to arrange from the smallest to the largest	9
order of size	to arrange from the smallest to the biggest	78
outcome	a possible result of an event; getting an even number when you throw a dice is an event. There are three different outcomes for this event: throwing a 2, a 4 or a 6	262
output	the result after a number has been acted upon by a function	206
parallel	straight lines where the shortest (perpendicular) distance between the lines is always the same; straight railway lines are parallel	110
partitioning	a method of multiplying two numbers where the units, tens, hundreds, etc., in one of the numbers is multiplied separately by the other number	97
percentage	a fraction written out of 100, as 'per cent'; e.g. a quarter is 25%	213
perpendicular	lines meeting or crossing at right angles	110
perpendicular height	the height of a shape or object measured at 90° to its base	319
pie chart	a circle divided into sectors; each sector represents its share of the whole	338
place value	the value of the digit in a number based on its position in relation to the decimal point	65
polygon	a 2D shape with three or more straight sides	164
population	the total set of people, things or events being investigated	63
position number	the position of a term in a sequence of numbers	201
positive gradient	when a graph slopes upwards from left to right	239
positive infinity	a number greater than any number you can think of	56
positive integers	the whole numbers greater than zero: 1, 2, 3, 4, …	10
power	a number written using an index; you write '3 to the power 4' as $3^4 = 3 \times 3 \times 3 \times 3 = 81$	64
powers of 10	the number 10 being multiplied by itself a number of times	64

prediction	a statement that may be true or false	124
probability	a number between 0 and 1 used to measure the chance that something will happen	263
product	the result of multiplying two numbers; the product of 9 and 7 is $9 \times 7 = 63$	14
proper fraction	a fraction in which the numerator is smaller than the denominator	143
proportion	two pairs of numbers are in the same proportion when the ratio formed by the first pair is the same as the ratio formed by the second pair	247
proportions	fractions (or percentages) of the whole	352
protractor	equipment used to measure and draw angles	115
quadrilateral	a flat shape with four straight sides	104
range	the difference between the largest and smallest numbers in a set	370
rate	the change of a quantity in relation to time; e.g. speed is a change in distance in relation to time	238
ratio	a number compared to another number, using the symbol :	34
reciprocal	the multiplier of a number that gives one as the result. For example $\frac{2}{3}$ is the reciprocal of $\frac{3}{2}$ because $\frac{2}{3} \times \frac{3}{2} = 1$	152
recurring decimals	in a recurring decimal, a digit or group of digits is repeated forever	138
reflected	the image as seen in a mirror	294
regular polygon	a polygon that has all sides and angles the same size	164
rotational symmetry	a shape has rotational symmetry if, during one full turn, it fits exactly onto its original position at least twice	164
round	make an approximation of a number, to a given accuracy	11
sample	a selection from a large population	129
sample size	the number of people or objects in the sample	133
scale	the ratio between the lengths on a scale drawing and the actual lengths of the original object	280
scale drawing	an accurate drawing in which the lengths in the drawing are in a given ratio to the lengths of the original object	280
scale factor	the ratio by which a length is increased	304
scalene triangle	a triangle in which all three sides are different lengths	174
sector	part of a circle that has, as its perimeter, two radii and the arc of the circle that joins them	269
sequence	a set of numbers arranged in order, according to a rule	191
sequence of patterns	patterns made from shapes; the number of shapes in each pattern forms a sequence of numbers	197
set square	equipment used to measure and draw right angles	115

share	to split up into parts	32
short division	a method of division where you simply place the remainders in front of the following digit (see Worked example 4.5)	92
side view	a 2D drawing of a solid shape seen from the side	183
similar shapes	shapes in which corresponding angles are equal and corresponding sides are in the same proportion	304
simplest form	a fraction in which the numerator and denominator do not have a common factor (apart from 1)	44
simplify (like terms)	gathering, by addition and subtraction, all like terms to give a single term	43
simplify (ratio)	divide all parts of the ratio by a common factor	248
sketch	a simple, quickly made diagram that isn't drawn accurately	113
solution	the value of any unknown letter(s) in an equation	52
solve	calculate the value of any unknown letter(s) in an equation	52
square (verb)	to multiply a number by itself	149
square centimetre (cm^2)	unit of area; area of a square of side 1 centimetre	312
square metre (m^2)	unit of area; area of a square of side 1 metre	312
square millimetre (mm^2)	unit of area; area of a square of side 1 millimetre	312
square number	the result of multiplying a whole number by itself; 81 is a square number equal to $9^2 = 9 \times 9 = 81$	9
square root	the square root of a number, multiplied by itself, gives that number; the square root of 36 is $\sqrt{36} = 6$	26
statistical measures	single numbers, such as the mode, median, mean and range, which can be used to represent a complete set of data	370
statistical question	a question that is answered by collecting and analysing data	124
substitute	replace part of an expression, usually a letter, by another value, usually a number	38
sum	the result when values are added together	22
surface area	the area of the faces of a solid or 3D shape	330
tangent	a straight line that touches the circumference of a circle at only one point	171
tenths	the first digit after the decimal point in a decimal number	78
term	a single number or variable, or numbers and variables multiplied together	34
term-to-term rule	a rule to find a term of a sequence, when given the previous term	192

tests for divisibility	tests you can use to decide if one number is divisible by another number	22
theoretical probability	a probability found using equally likely outcomes	267
top view (plan)	a 2D drawing of a solid shape seen from above	183
translate	transform a shape by moving each part of the shape the same distance in the same direction	278
transversal	a line that crosses two or more parallel lines	110
trial	a single action, such as a throw of a dice or a flip of a coin	133
twice	occurs two times	35
twice as much	two times as much or doubled	253
two-way table	a table that has rows and columns showing different groups of discrete data	340
unitary method	used to solve direct proportion questions by first calculating the value of a single item	258
unknown	a letter (or letters) in an equation, for which the value (or values) is yet to be found	34
unlikely	less than an even chance	263
upside down	when you turn a fraction upside down you swap over the numerator and the denominator; e.g. $\frac{2}{3}$ turned upside down is $\frac{3}{2}$	152
variable	a symbol, usually a letter, that can represent any one of a set of values	34
vertex (plural vertices)	a point where edges of a shape meet	118
vertical line	a line that goes straight up, at 90° to level ground	105
visualise	to form a picture in your mind	183
volume	the space occupied by a solid or 3D shape	324
waffle diagram	a diagram divided into coloured squares; each colour represents a proportion of the whole	356
whole-number part (fraction)	the part of a mixed number that is a whole number and not a fraction	139
whole-number part (decimal)	the part of the decimal number to the left of the decimal point	78
written method	work out on paper, showing your working	83
***x*-axis**	the axis containing the values of x	225
***x*-coordinate**	the horizontal position of a point on a coordinate grid	285
***y*-axis**	the axis containing the values of y	234
***y*-coordinate**	the vertical position of a point on a coordinate grid	235
zero gradient	when a graph is horizontal	239